Access and Lobbying in Legislatures

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April 29, 2019

Abstract

With which legislators does an interest group want to form relationships that facilitate lobbying? I study a model of legislative policymaking where interest groups can get access to particular legislators. Access provides opportunities to lobby those legislators if they control the agenda. In equilibrium, persistent access has spillover effects. It changes legislature-wide expectations, thereby changing which policies pass today and, in turn, can change proposals by other legislators. These endogenous spillovers encourage access to some legislators, but discourage access to others. Under broad conditions, groups forgo access to a range of more centrist legislators. But they are keen to access more extreme legislators. The results have implications for campaign finance and revolving door hiring. Additionally, I show that equilibrium lobbying expenditures increase with several measures of legislature polarization.

*I am indebted to Tasos Kalandrakis for many helpful discussions. I also thank Avi Acharya, Emiel Awad, Peter Bils, Peter Buisseret, Richard DiSalvo, John Duggan, Wiola Dziuda, Mark Fey, Michael Gibilisco, Cathy Hafer, Kei Kawai, Brenton Kenkel, Chris Li, Zhao Li, Adam Meirowitz, MaryClare Roche, Larry Rothenberg, Greg Sasso, Brad Smith, Yannis Vassiliadis, Craig Volden, Alan Wiseman, Stephane Wolton, Jan Zápal, attendees of the 2018 Vanderbilt Frontiers of Formal Theory Conference, and audiences at Chicago Harris and Princeton for comments and suggestions.

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Special interests and lobbying are widely maligned. A popular, cynical view is that wealthy interest groups frequently lobby key politicians to get favorable policy. Implicit in many discussions of this view are the relative preferences of groups and the politicians they influence. In particular, the possibility that ideologically extreme groups pull centrist politicians away from majority interests arouses widespread concern.

To lobby effectively, interest groups typically must get access by developing good working relationships with politicians.\(^1\) Anticipating who influences whom, and the resulting welfare implications, thus requires an understanding of where such relationships form. Widely perceived benefits of lobbying suggest groups always crave access. Yet, in large legislatures, groups cannot feasibly influence every legislator at all times. In such settings, groups may need to account for the possibility that access to particular legislators indirectly affects behavior by other legislators. These spillovers could push in either direction, potentially discouraging or encouraging access.

To develop our understanding of these forces, I study the connections interest groups cultivate in legislatures. More precisely, which combinations of groups and legislators form relationships yielding access? In pursuing this question, I also address two related questions. First, given connections that form, how do various political conditions influence observed levels of lobbying? Second, what are the policy and welfare effects of access and lobbying?

I study a game-theoretic model where access provides interest groups with opportunities to influence legislative policy proposals. Although expanding the scope of application for the canonical legislative bargaining framework has independent theoretical interest, I model a rich legislative environment to disentangle access-seeking incentives from lobbying. There are four primary contributions in this direction. First, I provide a microfoundation for spillover effects from access, as they arise endogenously in equilibrium. Beyond highlighting features of legislative policymaking that can produce spillovers, this microfoundation also permits characterization of how their magnitude and direction depend on various political considerations. Second, I highlight that the nature of access-driven spillovers depends on group and legislator ideology, relative to the legislature’s ideological distribution. Specifically, for different group-legislator pairs, I shed light on whether these spillovers encourage or discourage access compared to a setting without spillovers. Third, I show that some groups optimally forgo access to particular legislators and characterize when this behavior occurs. It does not require

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\(^1\)See, e.g., Wright (1989, 1990); Hall and Wayman (1990); Hansen (1991); Ainsworth (1993); de Figueiredo and Silverman (2006) and Powell (2014).
costly access, as negative spillovers alone can dominate gains from access. Fourth, I discuss how forgoing access, although counterintuitive, is consistent with several empirical regularities in US campaign finance.

The model has three key features. First, lobbying allows groups to influence policy proposals before they reach the floor. Second, to study which connections form, I distinguish between access and lobbying. Interest groups choose whether to access particular legislators before policymaking and, if they do so, have chances to lobby those legislators if they control the agenda. Finally, I unpack the legislative black box by modeling a canonical legislative bargaining environment where failed proposals can be revisited, forward-looking legislators anticipate outside influence, agenda power can change hands unpredictably, and passage requires majority approval.²

To illustrate how lobbying affects legislative policymaking, I first study a baseline model with exogenous access. I establish equilibrium existence and provide a sharp characterization that has clear connections to equilibria in models without lobbying. As expected, groups pull policy in their favored direction whenever they can lobby the proposer. Groups may be constrained, however, because successful policy proposals must satisfy a legislative majority. The equilibrium characterization also yields clear comparative statics about relationships between lobbying expenditures and various legislative conditions.

The baseline analysis produces a key insight: access can have indirect effects on equilibrium policymaking under broad conditions. Access-driven spillovers arise because all legislators account for where connections form and anticipate potential lobbying facilitated by those connections. The direction and magnitude of such spillovers depends on the relative extremism of groups and their associated legislators, as lobbying can increase or decrease policy extremism. Thus, there are qualitative differences in how groups value connections to different legislators.

To study the consequences of access-driven spillovers, I extend the baseline model so that groups choose access before policymaking. Substantively, this extension reflects that access must be acquired well ahead of time to facilitate lobbying opportunities. The sharp characterization of legislative behavior pins down how access affects a group’s welfare. Access-driven spillovers can present interest groups with an important tradeoff. On the one hand, access provides more opportunities to lobby during policymaking. On the other hand, access can have a negative indirect effect on proposals by legislators

²The legislative setting follows Banks and Duggan (2006a) and Cho and Duggan (2003), which integrate key features of Baron and Ferejohn (1989) and Romer and Rosenthal (1978).
the group does not access.

The logic is as follows. Forward-looking legislators anticipate lobbying behavior following rejected proposals. Thus, a group’s access to some legislator affects every legislator’s expectation about policymaking. More precisely, access affects each legislator’s reservation value, which is generated endogenously by equilibrium expectations about future policymaking. This effect can change which policies pass. In turn, access can indirectly change proposals of legislators constrained by majority approval. From a group’s perspective, this indirect effect can be good or bad. Furthermore, the magnitude depends on various legislative conditions, including ideological polarization and the distribution of agenda power.

I show that some interest groups optimally forgo access to a range of legislators, even if that access is free. Specifically, groups that are not too extreme forgo access to neighboring, more centrist legislators. Access-driven spillovers increase policy extremism in these cases, and this indirect effect outweighs the group’s gain from more lobbying opportunities. Thus, these connections are foregone even if they are free. In such cases, groups face a time inconsistency problem. They always want to lobby when given the opportunity. Ex ante, however, they forgo access because it polarizes the policymaking environment too much relative to their expected gain from lobbying.

To illustrate the logic, consider the following stylized example. A regional energy interest group anticipates national legislation regulating emissions. It prefers moderately tighter regulations to capitalize on recent investments in clean technology. The group’s local congressman wants to tighten existing regulations more than the group. If the group gets access, its chances to lobby the congressman increase. Then, moderate and pro-environment legislators are less optimistic about the eventual regulatory outcome. They know that if the congressman drafts policy, then the group is more likely to lobby. If so, the resulting policy will be more extreme than if the congressman had acted alone. Consequently, rejecting proposals is less attractive and these other legislators are willing to approve a wider range of policies. The group’s access thus indirectly allows extreme pro-energy legislators to pass weaker emissions regulations if they draft policy. Such policies would reduce the group’s benefits from its recent technological investments. I show that this threat of greater extremism can worsen the group’s expectations about policymaking so that it prefers to forgo access.

On the other hand, groups always want access to nearby, more extreme legislators. In this case, access increases lobby opportunities and also favorably constrains extreme
legislators. The analysis thus suggests that centrist and moderate interest groups have especially strong incentives to acquire access to a broad spectrum of legislators.

The analysis provides implications for welfare and empirical work. First, important consequences of access depend on relative preferences of groups and targeted politicians. Many worry that groups pull otherwise public-minded politicians away from majority interests. But some groups may moderate otherwise extreme politicians. Studying which combinations of groups and legislators form connections highlights when society may want to restrict access, encourage it, or do nothing. Second, empirical evidence suggests that two ways groups get access are campaign contributions and hiring lobbyists with *revolving door* connections (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kalla and Broockman, 2015). Identifying who groups want to access provides direct implications for both how groups allocate contributions and which lobbyists they hire.

I contribute to a literature studying strategic lobbying by interest groups. Lobbying has been modeled in many ways. I focus on lobbying to influence policy content. Specifically, I study *lobbying as exchange* in the spirit of Grossman and Helpman (1994), where groups provide resources to shape policy proposals. The lobbying technology here is similar to Bils, Duggan and Judd (2019), which studies lobbying in a model of repeated elections. There, officeholder ideology exogenously determines access and groups can always lobby their affiliated officeholders. In contrast, I study whether groups want access and allow them to choose.

Existing work also explores implications of less cynical perspectives on lobbying, such as groups providing useful information (Austen-Smith, 1995; Prat, 2002) or services to politicians and voters (Hall and Deardorff, 2006). Studying these perspectives is worthwhile, but I focus on a cynical form of influence for several reasons. First, it aligns with a widespread outlook and underlies public concern about special interests. Second, I aim to strengthen positive theory under this perspective to parse its empirical implications and welfare consequences.

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3 Other work studies lobbying to influence voting on a fixed agenda. Many have analyzed vote buying in legislatures, mostly studying distributive policies or public goods (Snyder Jr., 1991; Groseclose and Snyder, 1996; Banks, 2000; Dal Bó, 2007; Dekel et al., 2009). Others allow groups to influence votes by strategically providing information (Bennedsen and Feldmann, 2002; Jackson and Tan, 2013; Schnakenberg, 2015, 2017; Alonso and Câmara, 2016; Awad, 2018).

4 See Grossman and Helpman (2002) for an extensive overview of this setting, which they apply to campaign contributions. Also see Martimort and Semenov (2008) and an extension in Acemoglu et al. (2013) for recent studies using this lobbying technology.

5 Another notable difference is that I allow partial access, which does not guarantee lobbying opportunities. Furthermore, I consider legislative, rather than executive, policymaking.
Scholars have studied access acquisition in static, informational lobbying environments (Austen-Smith, 1995; Cotton, 2012, 2016). Closest to this paper is Schnakenberg (2017), where groups can buy access in a legislature. There, groups try to influence a legislative vote over exogenous policy proposals in a static setting. Access allows groups to provide information. As in this paper, influencing a legislature is quite different from influencing a solitary policymaker. In contrast, I study a complete information setting where lobbying affects endogenous policy proposals and policymaking continues after failed proposals. Moreover, groups sometimes optimally forgo free access in this paper, which never happens in Schnakenberg (2017).

I also contribute to a literature taking access as given and analyzing lobbying to influence the agenda within various legislative institutions. Helpman and Persson (2001) introduce interest groups into a static version of Baron and Ferejohn (1989). They compare the consequences of lobbying in different legislative institutions. As in this paper, groups can lobby particular legislators when they control the agenda and lobbying influences proposals. Unlike this paper, they study distributive policies, groups can also lobby to influence votes, and bargaining does not continue after rejected proposals. Moreover, they do not study access acquisition and, here, the prospect of future bargaining creates endogenous spillovers from access.


Other work incorporating interest groups into the non-static Baron and Ferejohn (1989) framework allows groups to buy agenda control (Yildirim, 2010, 2007; Ali, 2015). Most papers in this vein analyze distributive policies. An exception is Levy and Razin (2013), who study a dynamic setting with an endogenous status quo in a one-dimensional policy space. In each period, a continuum of groups compete in an all-pay auction for temporary agenda control. They provide conditions for policies to moderate over time. They do not address which connections form, as they do not model politicians and instead implicitly treat them as homogeneous. Furthermore, they do not study persistent access, as groups instead vie for temporary access throughout policymaking. In contrast, I study targeted and persistent access acquired before bar-

\[^6\text{See Eraslan and McLennan (2013) for a thorough discussion of models using the Baron and Ferejohn (1989) framework.}\]
gaining. There are several other differences. Here, bargaining ends when a proposal passes and I abstract from head-to-head competition for access.

Finally, a key result of this paper is that groups may forgo access to more centrist legislators. The logic connects to moderation results in spatial models of dynamic bargaining with endogenous status quo (Baron, 1996; Zápal, 2014; Forand, 2014; Buisseret and Bernhardt, 2017). There, legislators prefer proposing more centrist policies to constrain future proposers in equilibrium. They forgo the full power of their current agenda control to constrain the scale of policy changes by future proposers who may have substantially different preferences. I study a different setting in which policymaking ends once a proposal passes, but incentives to forgo access arise from the same desire to constrain potential future proposers who are ideologically distant.

Model of Legislative Bargaining with Lobbying

To study access, it is important to firmly understand its downstream effects through lobbying. Thus, I first present and analyze the legislative environment with access fixed exogenously, having implicitly arisen from previous efforts to create connections. This handle on legislative behavior sets the stage to subsequently study access acquisition and then, given access, how lobbying expenditures vary with legislative features.

In the model, legislators bargain to set a common policy. Throughout policymaking, ideological interest groups may receive opportunities to influence policy by providing favors. The logic for the main results can be illustrated in a streamlined setting with four legislators and one interest group. I provide several comments after describing the baseline model.

There are four legislators: a left partisan $L$, a moderate $M$, a right partisan $R$, and a generic legislator $\ell$. The interest group is denoted $g$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Each legislator $i$ has associated ideal point $\hat{x}_i \in X$ and $g$’s ideal point is $\hat{x}_g \in X$. Throughout, I normalize $\hat{x}_M = 0$. Furthermore, I assume $\hat{x}_L < 0 < \hat{x}_R$. To reflect that partisans are staunchly ideological, I maintain $\min\{|\hat{x}_L|, \hat{x}_R\} > |q|$. Although not crucial, this assumption clarifies key tradeoffs.

Legislative bargaining occurs over an infinite horizon, with periods discrete and indexed by $t \in \{1, 2, \ldots\}$. Let $\rho_i > 0$ denote the probability legislator $i$ is chosen.

\footnote{Although Forand (2014) is cast as a model of elections, it can be interpreted as a spatial bargaining model with an endogenous status quo and endogenous proposers.}

\footnote{Appendix A presents a more general setting.}
to propose in any period $t$. Then $\rho = (\rho_L, \rho_M, \rho_R)$ denotes the distribution of recognition probabilities, which sum to one.

The interest group, $g$, has opportunities to influence $\ell$’s policy proposals. Specifically, $g$’s *access* to $\ell$ is $\alpha \in [0, 1]$, which is the probability that $g$ can lobby $\ell$, conditional on $\ell$ being recognized to propose. This technology reflects the standard view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). To reflect targeted access, $g$ does not have access to legislators other than $\ell$. Although, $g$’s access is exogenously endowed for now, later on I let $g$ choose $\alpha$ to study when groups seek access.

In each period $t$, bargaining proceeds as follows. If no policy has passed before $t$, then each legislator $i$ is recognized as the period-$t$ proposer with probability $\rho_i$. The identity of the period-$t$ proposer, $i_t$, is publicly observed. If $i_t \neq \ell$, then $g$ is not active and $i_t$ proposes any policy $x_t \in X$. If $i_t = \ell$, then $g$ can lobby $\ell$ with probability $\alpha$. If $g$ lobbies, then $g$ offers $\ell$ a binding contract $(y_t, m_t)$ consisting of a policy $y_t \in X$ and a transfer $m_t \geq 0$. After observing $g$’s offer, $\ell$ decides to accept or reject. If $\ell$ accepts, then she is committed to propose $x_t = y_t$ and $m_t$ transfers from $g$ to $\ell$. If $\ell$ rejects, then she can propose any $x_t \in X$ and $g$ keeps $m_t$. With probability $1 - \alpha$, $g$ cannot lobby in $t$. Then, $\ell$ simply proposes any $x_t \in X$ and $g$ does not make an offer.

In each case, all legislators observe the period-$t$ proposal, $x_t$. Next, the moderate legislator, $M$, chooses to accept or reject the proposal. If $M$ accepts, then the proposal passes and bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $M$ rejects, then the status quo $q \in \mathbb{R}$ is enacted in $t$ and bargaining proceeds to $t + 1$. This setup captures the spirit of a more general setting where all legislators vote and $M$ is a decisive median legislator.\(^{10}\)

If $i_t = \ell$, $\ell$ accepts $g$’s offer $(y, m)$, and $x_t$ is the enacted policy in $t$,\(^{11}\) then $g$’s stage payoff is $u_g(x_t) - m$ and $\ell$’s stage payoff is $u_\ell(x_t) + m$. All players have quadratic policy utility and discount streams of stage utility by the common discount factor $\delta \in (0, 1)$. See Appendix A for explicit expressions of dynamic payoffs. Figure 1 illustrates the within-period interaction and accumulation of payoffs for a period in which $\ell$ proposes and $g$ can lobby. For a period in which $\ell$ does not propose, or $g$ cannot lobby, the within-period interaction is analogous to Figure 1 following $\ell$ rejecting $g$’s offer.

\(^9\)Also see, e.g., Milbrath (1976); Hall and Wayman (1990); Hansen (1991); Grossman and Helpman (2002); Hall and Deardorff (2006); Gordon et al. (2007) and Powell (2014).

\(^{10}\)Under the maintained assumptions, $M$’s decision corresponds to the outcome of majority voting over policy lotteries (Banks and Duggan, 2006b; Duggan, 2014).

\(^{11}\)Notice $x_t = q$ if $y$ does not pass in period $t$. 
Model Commentary

I make several comments before proceeding to the analysis.

First, I model access, α, as g’s probability of being able to lobby when ℓ controls the agenda, but qualitatively similar results hold if access is binary or if access is modeled as ℓ’s marginal value of money. Also, α can equivalently be viewed as the proportion of legislators within a homogeneous bloc whom the group can lobby.

Second, there are multiple interpretations for access. One is personal connections possessed by g’s lobbyists affecting their chances of meeting with ℓ or being able to commit to a contract (Blanes i Vidal et al., 2012; Bertrand et al., 2014).12 Another is access gained from campaign contributions to ℓ during a preceding, yet unmodeled, election.13 A third is ℓ’s inclination to use policy proposals to appeal to constituents, which may affect her propensity to meet with lobbyists.

Third, g has access to only one legislator. I relax this assumption in the appendices, but it reflects the idea that groups are unable to access some legislators due to exogenous factors. For example, regional groups may not be able to access legislators without a geographic connection (Wright, 1989). Alternatively, voters in some districts

12Also see Cain and Drutman (2014); Kang and You (2015) and McCrain (2018).
13See, e.g., Langbein (1986); Romer and Snyder Jr. (1994); Kalla and Broockman (2015); Barber (2016); Grimmer and Powell (2016) and Fouirnaies and Hall (2017) for evidence that many interest groups use campaign contributions to get access.
may be strongly opposed to the group’s mission or tactics (Stratmann, 1992). Finally, the group simply may not be able to afford access to many different legislators.

Fourth, the lobbying technology has a substantive interpretation. In the model, groups offer binding contracts exchanging resources for policy. Groups spend substantial effort drafting legislation in practice (Schlozman and Tierney, 1986), and frequently present legislators with model bills (Levy and Razin, 2013; Kroeger, 2016). Formally, this corresponds to the policy offer, $y$. In return, legislators may gain an inside track on future employment opportunities (Diermeier et al., 2005). Moreover, legislators are freed to pursue other tasks such as constituent service and fundraising, in the spirit of Hall and Deardorff (2006). Groups may also provide valuable political intelligence, or write speeches to help sell policies to the legislator’s constituents and co-partisans (Schlozman and Tierney, 1983, 1986; Hall and Wayman, 1990; Wright, 1996). The group’s transfer, $m$, captures these benefits.

Finally, I abstract from lobbying to influence legislative votes. Ignoring this channel isolates considerations related to lobbying over policy details “in committee.” Furthermore, the median ideology is a robust statistic in large legislatures, and meaningful vote buying likely requires coordinating deals with several legislators.

**Equilibrium Policies and Lobbying Activity**

I study a selection of the model’s subgame perfect equilibria (SPE), applying standard refinements from the legislative bargaining literature. In particular, I focus on no-delay stationary legislative lobbying equilibria.\(^{14}\) Informally, a no-delay stationary legislative lobbying equilibrium requires four conditions: (i) $g$’s policy offer is socially acceptable and $g$ cannot profitably deviate to another offer; (ii) legislator $\ell$ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal; (iii) conditional on not receiving a payment from $g$, each legislator proposes socially acceptable policy and cannot profitably deviate to a different proposal; and (iv) $M$ supports a policy if and only if she weakly prefers it relative to rejecting and extending bargaining.\(^{15}\) Stationarity implies that $g$’s offers to $\ell$ are independent of previous play; $\ell$ accepts or rejects $g$’s offer based only on the terms of the current offer, and $\ell$’s policy proposals in lieu of acceptance are independent of the preceding history; legislators

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\(^{14}\)See Appendix A for a formal definition.

\(^{15}\)Specifically, $M$ uses stage-undominated voting strategies (Baron and Kalai, 1993).
other than \( \ell \) propose policy independent of preceding play; and \( M \)'s voting decision depends only on current proposal.

This equilibrium concept is less restrictive than it may appear. First, although players use straightforward behavioral rules, no player can profitably deviate to any other strategy. Second, \( g \) must make an offer in each period that \( \ell \) proposes and \( g \) can lobby, but this requirement is innocuous because \( g \) can effectively forgo lobbying by offering \( \ell \)'s default proposal along with no payment. Third, \( \ell \) always accepts \( g \)'s offer when indifferent, but this restriction is without loss of generality. Finally, I focus on no-delay strategy profiles for convenience, as this restriction is inconsequential.\(^{16}\)

Proposition 1 provides three results. First, a no-delay stationary legislative lobbying equilibrium exists. Along the way, I obtain a sharp characterization of equilibrium behavior. Second, a larger class of equilibria are equivalent in outcome distribution to the equilibria I analyze.\(^{17}\) Third, there is a unique equilibrium outcome distribution, capitalizing on Cho and Duggan (2003). This property ensures that subsequent analysis endogenizing access does not require additional equilibrium selection.

**Proposition 1.**

1. A no-delay stationary legislative lobbying equilibrium exists.

2. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium.

3. All stationary legislative lobbying equilibria have the same outcome distribution.

In light of Proposition 1, I simply refer to *equilibria* throughout the rest of the analysis. As is standard in the legislative bargaining literature, equilibria can be characterized by their *social acceptance set*, which is denoted \( A(\sigma) \) and corresponds to the set of policies that \( M \) accepts under the strategy profile \( \sigma \).\(^{18}\)

In an equilibrium \( \sigma \), the boundaries of \( A(\sigma) \) are the two policies that \( M \) is indifferent between approving and rejecting. Formally, the upper bound of \( A(\sigma) \), denoted \( x(\sigma) \), is the positive solution to

\[
 u_M(x) = (1 - \delta)u_M(q) + \delta V_M(\sigma),
\]

\(^{16}\)See Appendix B for more details.

\(^{17}\)In Appendix B, I define *stationary mixed strategy legislative lobbying equilibrium* and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium with deferential voting and deferential acceptance.

\(^{18}\)See, e.g., Banks and Duggan (2000) and Banks and Duggan (2006a).
where $V_M(\sigma)$ denotes $M$’s continuation value under $\sigma$.\textsuperscript{19} Thus, $A(\sigma) = [-\bar{\pi}(\sigma), \bar{\pi}(\sigma)]$, facilitating a sharp characterization of proposal strategies.

In the hypothetical legislature illustrated in Figure 2, $M$ proposes $\hat{x}_M = 0$ if recognized, legislator $L$ proposes $-\bar{\pi}(\sigma)$, and $R$ proposes $\bar{\pi}(\sigma)$. The partisans, $L$ and $R$, are thus constrained by $M$’s voting power because their respective ideal policies will not pass. In equilibrium, each compromises by proposing its favorite passable policy. If legislator $\ell$ is recognized and does not accept a lobby offer, either because $g$ cannot lobby or because $\ell$ rejects $g$’s offer, then $\ell$ proposes $z_\ell$, her favorite policy in $A(\sigma)$. In equilibrium, however, $g$ always makes an offer $\ell$ accepts. The group’s policy offer always passes and is skewed away from $\hat{x}_\ell$ towards $\hat{x}_g$. The group’s equilibrium transfer exactly satisfies $\ell$’s acceptance condition given the policy offer. To see why, note it is always feasible for $g$ to offer $\ell$’s independent proposal, $z_\ell$, with zero payment. Because $z_\ell \in A(\sigma)$, $g$ weakly prefers to make successful offers $\ell$ accepts. Of course, $g$ is strictly worse off giving $\ell$ a surplus transfer.

In general, $g$’s equilibrium offer $(y_g, m_g)$ consists of the policy

$$y_g = \arg \max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - u_\ell(z_\ell)$$

and transfer $m_g = u_\ell(z_\ell) - u_\ell(y_g)$. Because $u_\ell(z_\ell)$ does not depend on $g$’s offer,

$$y_g = \arg \max_{y \in A(\sigma)} u_g(y) + u_\ell(y),$$

which uniquely maximizes the joint surplus of $g$ and $\ell$, subject to the constraint that $y_g$ passes.\textsuperscript{20} For convenience, define $g$’s \textit{unconstrained policy offer} as

$$\hat{y} = \arg \max_{y \in X} u_g(y) + u_\ell(y).$$

Because $u_g$ and $u_\ell$ are quadratic, $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$. If $\hat{y} \in A(\sigma)$, then $y_g = \hat{y}$. Otherwise, strict concavity implies $y_g$ equals the boundary of $A(\sigma)$ closest to $\hat{y}$.

The model, although complicated by lobbying, can be reinterpreted as a one-dimensional, spatial bargaining environment with an additional legislator possessing $\alpha\rho_\ell$ recognition probability and an ideal point, $\hat{y}$, located between $\hat{x}_g$ and $\hat{x}_\ell$. After expanding the legislature to add this additional proposer representing the effect of $g$’s

\textsuperscript{19}See Appendix A for explicit expressions of continuation values.

\textsuperscript{20}Uniqueness follows because $u_g + u_\ell$ is strictly concave, and $A(\sigma)$ is compact, convex, and nonempty.
lobbying, legislators propose bills closest to their ideal point among those that pass. Uniqueness follows from applying Cho and Duggan (2003) to this fictitious enlarged legislature.

Figure 2 illustrates the equilibrium social acceptance set, $A(\sigma)$, for a hypothetical legislature, along with corresponding equilibrium proposals.

![Figure 2: Equilibrium characterization](image)

Figure 2 depicts equilibrium policy proposals. Arrows point from legislator ideal points to proposals. The bold interval is the acceptance set, $A(\sigma)$. If legislator $l$ is recognized, then she proposes the acceptable policy closest to $\hat{y} = \frac{\hat{x}_g + \hat{x}_l}{2}$ with probability $\alpha$ and otherwise proposes the acceptable policy closest to $\hat{x}_l$.

In general, the characterization implies that $M$’s equilibrium continuation value from rejecting a proposal is

$$V_M(\sigma) = \rho_M u_M(\hat{x}_M) + \alpha \rho_L u_M(y_g) + (1 - \alpha) \rho_L u_M(z_l) + \rho_L u_M(-\bar{x}(\sigma)) + \rho_R u_M(\bar{x}(\sigma)).$$

To characterize the upper bound of $A(\sigma)$, rearranging (1) and using $u_M(\hat{x}_M) = 0$ yields

$$\bar{x}(\sigma) = \left( -\frac{(1 - \delta) u_M(q) + \delta \left( \alpha \rho_L u_M(y_g) + (1 - \alpha) \rho_L u_M(z_l) \right)}{1 - \delta (\rho_L + \rho_R)} \right)^{\frac{1}{2}}. \tag{5}$$

Inspection of (5) shows that $\alpha$, along with several other legislative parameters, can affect the boundaries of $A(\sigma)$ and, consequently, proposals by the partisans $L$ and $R$. This indirect effect requires dynamic concerns, i.e. $\delta > 0$, and plays a key role in the analysis.

Before proceeding, I define terminology used to characterize the ideologies of $g$ and $l$.

**Definition 1.** In an equilibrium $\sigma$, legislator $l$ is **extremist** if $\hat{x}_l \notin \text{int}A(\sigma)$ and **centrist**...
otherwise. Analogous definitions apply to the interest group, $g$.

**Definition 2.** Legislator $\ell$ and the interest group, $g$, are *aligned* if their ideal points are on the same side of $\hat{x}_M = 0$, e.g. $\max\{\hat{x}_\ell, \hat{x}_g\} \leq 0$. Otherwise, $\ell$ and $g$ are *opposed.*

Two conditions are necessary for non-trivial lobbying in equilibrium. Of course, $g$ must have access, i.e. $\alpha_\ell > 0$. Second, $g$ and $\ell$ cannot be aligned extremists, as then $g$ cannot profitably lobby to improve upon $\ell$’s independent policy proposal.

**Who do Interest Groups Want to Access?**

To study where connections form, I now allow the group, $g$, to choose $\alpha$, its access to legislator $\ell$. By Proposition 1, $g$’s choice of $\alpha$ pins down equilibrium expected payoffs in the legislature. To isolate key tradeoffs of durable access, I focus on a one-time choice of perfectly persistent access. I discuss other possibilities later. Substantively, this setup reflects $g$ using campaign contributions or hiring connected lobbyists to form solid working relationships.

I abstract from the particular mapping that determines access by allowing $g$ to choose $\alpha$ freely. In practice, the cost of acquiring access almost certainly depends on idiosyncratic factors such as the connections of the group’s lobbyists (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kang and You, 2015), constituent interests within the legislator’s district (Stratmann, 1992), or the number of voters affiliated with the group (Bombardini and Trebbi, 2011).\(^{21}\) The following results are driven purely by policy considerations and hold for standard cost functions.

Propositions 2 and 3 fix $\hat{x}_g$ and study whether $g$ wants access, as a function of $\hat{x}_\ell$. As noted above, $\hat{x}_\ell$ and $\alpha$ can affect equilibrium legislative behavior by changing policy proposals directly and indirectly. Propositions 2 and 3 are distinguished by whether $g$ is centrist if $\hat{x}_g = \hat{x}_\ell$.\(^{22}\) Lemma 1 shows that this distinction has a simple partitional characterization. Define

\[ \tau = \left( \frac{1 - \delta)u_M(q)}{1 - \delta(\rho L + \rho R + \rho \ell)} \right)^{\frac{1}{2}}, \tag{6} \]

\(^{21}\)For example, La Raja and Schaffner (2015) emphasize that contributions do not translate into influence the same way for different pairs of interest groups and legislators.

\(^{22}\)Recall $g$ is *extremist* if $\hat{x}_g \notin \text{int}A(\sigma)$ and *centrist* otherwise, and similarly for $\ell$. 

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which satisfies \( \overline{x} \geq \overline{x}(\sigma) > 0 \). Also, let \( A(\hat{x}_g) \) denote \( A(\sigma) \) conditional on \( \hat{x}_\ell = \hat{x}_g \), suppressing \( \alpha \) because it is inconsequential.

**Lemma 1.** If the interest group, \( g \), satisfies \( \hat{x}_g \in (-\overline{x}, \overline{x}) \), then \( \hat{x}_g \in \text{int}A(\hat{x}_g) \). Otherwise, \( A(\hat{x}_g) = [-\overline{x}, \overline{x}] \).

Using Lemma 1, I define a notion of extremism pinned down by primitives.

**Definition 3.** The interest group, \( g \), is a non-ideologue if \( \hat{x}_g \in (-\overline{x}, \overline{x}) \). Otherwise, \( g \) is an ideologue.

Notably, \( g \)'s ideologue status does not depend on \( \alpha \) or \( \hat{x}_\ell \), even though these features can change whether \( g \) is extremist or centrist. For example, suppose \( g \) is a non-ideologue. Then, \( g \) can be extremist if \( \alpha \) is low and \( \ell \) is sufficiently centrist, but \( g \) is centrist if \( \ell \) is sufficiently extreme or \( \alpha \) is sufficiently large. If \( g \) is an ideologue, however, then it is always extremist.

**Non-ideologue Interest Groups**

I first consider non-ideologue groups, focusing on \( \hat{x}_g \in (0, \overline{x}) \), as illustrated in Figure 3. The symmetric case is analogous.

Let \( \overline{x}(\alpha; \hat{x}_\ell) \) denote the upper bound of the equilibrium acceptance set if \( g \) has \( \alpha \) access, given \( \hat{x}_\ell \). For non-ideologue \( g \), there exists \( \bar{x} < \hat{x}_g \) such that \( \hat{x}_\ell \geq \bar{x} \) implies \( \hat{x}_g \in (0, \overline{x}(\alpha; \hat{x}_\ell)) \) for all \( \alpha \in [0, 1] \). Thus, \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( y = \hat{y} \) and \( z_\ell = \hat{x}_\ell \) for all \( \alpha \in [0, 1] \). Then \( g \)'s ex ante expected utility from \( \alpha \) access is

\[
U^E_g(\alpha; \hat{x}_\ell) = \rho_{\ell} \left( \alpha \left[ u_g(\hat{y}) + u_\ell(\hat{y}) \right] + (1 - \alpha) u_g(\hat{x}_\ell) \right) + \rho_L u_g(-\overline{x}(\alpha; \hat{x}_\ell)) + \rho_R u_g(\overline{x}(\alpha; \hat{x}_\ell)) + \rho_M u_g(0),
\]

where \( u_g(\hat{y}) + u_\ell(\hat{y}) \) is \( g \)'s lobbying return from transferring \( m = u_\ell(\hat{y}) \) to \( \ell \) in exchange for proposing \( \hat{y} \). The second line in (7) is \( g \)'s expected policy utility if \( \ell \) does not propose.

Qualitatively, \( \alpha \) can affect \( U^E_g(\alpha; \hat{x}_\ell) \) in two ways. First, it directly changes the probability that \( g \) lobbies \( \ell \). Second, it can change the proposals of \( L \) and \( R \) by shifting \( \overline{x}(\alpha; \hat{x}_\ell) \). More explicitly, the marginal effect of increasing \( \alpha \) is

\[
\frac{\partial U^E_g(\alpha; \hat{x}_\ell)}{\partial \alpha} = \rho_{\ell} \left( u_g(\hat{y}) + u_\ell(\hat{y}) - u_g(\hat{x}_\ell) \right)
\]
\[ \partial x(\alpha; \hat{x}_\ell) \left( \frac{\partial u_g(-\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)} + \rho_R \frac{\partial u_g(\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)} \right). \] 

The first line in (8) is \( g \)'s direct benefit from lobbying more frequently, which is always positive.

The second line is the indirect effect. The term in parentheses is the effect of expanding the acceptance set, which allows \( L \) and \( R \) to pass more extreme policies. It is negative because \( \hat{x}_\ell \geq \hat{x} \) implies \( \hat{x}_g \in (0, \pi(\alpha; \hat{x}_\ell)) \). Crucially, however, the direction of \( \alpha \)'s overall indirect effect depends on whether the acceptance set shrinks or expands with \( \alpha \), i.e., the sign of \( \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} \). If \( \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} > 0 \), which holds if and only if \( g \) is more extreme than \( \ell \), then \( \alpha \)'s indirect effect is negative. Otherwise, it is positive.

Proposition 2 uses these observations to characterize whether non-ideologue groups want access to a range of aligned legislators.\(^{23}\) One key takeaway is that \( g \) optimally forgoes access to moderately more centrist legislators. Another takeaway is that \( g \) wants access to moderately more extreme legislators. Figure 3 depicts Proposition 2.

**Proposition 2.** Suppose the interest group, \( g \), satisfies \( \hat{x}_g \in (0, \pi) \). There exist \( x', x'' \) satisfying \( 0 < x' < \hat{x}_g < \pi < x'' \) such that:

(i) if legislator \( \ell \) satisfies \( \hat{x}_\ell \in (x', \hat{x}_g) \), then \( g \) forgoes access;

(ii) if \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( g \) acquires access;

(iii) if \( \hat{x}_\ell \geq x'' \), then \( g \) is indifferent over access.

An analogous result holds if \( \hat{x}_g \in (-\pi, 0) \).

Figure 3: Who do non-ideologue interest groups want to access?

\[ \begin{array}{ccccccc}
0 & x' & \hat{x}_g & \pi & x'' & \hat{x}_\ell \\
\text{Ambiguous} & \text{No} & \text{Yes} & \text{Indifferent}
\end{array} \]

Figure 3 illustrates Proposition 2 for a right-leaning interest group, \( g \). If \( \hat{x}_\ell \in (x', \hat{x}_g) \), then \( g \) forgoes access, i.e. \( \alpha = 0 \). If \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( \alpha > 0 \). If \( \hat{x}_\ell \geq x'' \), then \( g \) is indifferent. If \( \hat{x}_\ell \in [0, x'] \), then \( g \)'s preference is ambiguous.

\(^{23}\)Recall \( g \) and \( \ell \) are aligned if \( \hat{x}_g \) and \( \hat{x}_\ell \) are on the same side of \( \hat{x}_M \).
First, consider $\hat{x}_\ell \in [0, \hat{x}_g)$. Increasing $\alpha$ raises the probability that $\ell$ proposes $\hat{y}$, at the expense of $\hat{x}_\ell$. Because $M$ prefers $\hat{x}_\ell$ to $\hat{y}$, $M$’s continuation value from rejection thus decreases with $\alpha$. Therefore $\bar{\pi}(\alpha; \hat{x}_\ell)$ increases. Because $L$ and $R$ are always partisan, their legislative proposals are more extreme. Figure 4 illustrates these effects on the ex ante distribution of equilibrium policy.

Figure 4: Forgoing access to a more centrist legislator

Figure 4 illustrates why a non-ideologue group, $g$, forgoes access ($\alpha = 0$) to legislator $\ell$ if $\hat{x}_\ell \in (x', \hat{x}_g)$. Part (a) displays equilibrium behavior for $\alpha = 0$. Part (b) illustrates $\alpha > 0$. Increasing $\alpha$ has two immediate effects: (i) lobbying is more likely and (ii) $M$’s expectations worsen. Effect (ii) expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If $\hat{x}_g$ and $\hat{x}_\ell$ are close, then effect (ii) dominates and $g$ prefers $\alpha = 0$.

As $\hat{x}_\ell$ increases to $\hat{x}_g$, $g$’s lobbying surplus shrinks faster than the indirect loss from enabling more extreme policies. Proposition 2 simply shows existence of $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies the marginal indirect cost of increasing $\alpha$ outweighs $g$’s marginal direct benefit for all $\alpha \in [0, 1]$. To see this more concretely, $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$ yield a more explicit expression of (8) as

$$\frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} = \frac{\rho_L}{2}(\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_L (\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g)}{4\bar{\pi}(\alpha; \hat{x}_\ell)[1 - \delta(\rho_L + \rho_R)]} \left[\hat{x}_g(\rho_R - \rho_L) - \bar{\pi}(\alpha; \hat{x}_\ell)(\rho_L + \rho_R)\right].$$

(9)

which is proportional to

$$\frac{\hat{x}_g - \hat{x}_\ell}{2\bar{\pi}(\alpha; \hat{x}_\ell)[1 - \delta(\rho_L + \rho_R)]} \left[\hat{x}_g(\rho_R - \rho_L) - \bar{\pi}(\alpha; \hat{x}_\ell)(\rho_L + \rho_R)\right].$$

(10)

As $\hat{x}_\ell$ increases to $\hat{x}_g$, the first term is positive and goes to zero. The second term is
bounded away from zero and negative because \( \hat{x}_\ell \in (\hat{x}_L, \hat{x}_g) \) implies \( \hat{x}_L < \hat{x}_g \) for all \( \alpha \in [0, 1] \). Thus, the indirect cost dominates as \( \hat{x}_\ell \) approaches \( \hat{x}_g \) from below.

Proposition 2 does not preclude \( g \) forgoing access to sufficiently centrist \( \ell \), i.e. \( \hat{x}_\ell \in [0, x') \). In this case, \( g \) receives a larger benefit from lobbying and may want access.

Proposition 2 implies non-ideologue groups do not want access to some nearby \( \ell \), but does not imply they forgo access to all nearby \( \ell \). Instead, \( g \) wants access if \( \ell \) is moderately more extreme, i.e. \( \hat{x}_\ell \in (\hat{x}_g, x'') \). In this case, \( y = \hat{y} < z_\ell \) for all \( \alpha \in [0, 1] \).

Thus, \( \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} \) < 0, so the second term in (8) is positive. Intuitively, \( g \) strictly prefers \( \alpha > 0 \) because it increases \( g \)'s lobbying opportunities and further constrains partisan legislators to propose policies more favorable to \( g \). The direct and indirect effects both work in \( g \)'s favor. Figure 5 depicts these forces.

**Figure 5: Seeking access to a more extreme legislator**

![Diagram](image)

Figure 5 illustrates why a non-ideologue group, \( g \), prefers strictly positive access, \( \alpha > 0 \), if \( \hat{x}_\ell \in (\hat{x}_g, x'') \). Part (a) displays equilibrium behavior if \( \alpha = 0 \). Part (b) illustrates \( \alpha > 0 \). Access has two immediate effects: (i) \( g \)'s probability of lobbying increases and (ii) \( M \)'s expectations improve. Effect (ii) causes the acceptance set to shrink, as shown in (b). Partisans propose more centrist policy. Both effects improve \( g \)'s expected payoff.

Finally, if \( \hat{x}_\ell > x'' \), then \( y = z_\ell = \bar{x} \) for all \( \alpha \). Thus, \( g \) cannot profitably lobby to change \( \ell \)'s policy proposal. Consequently, \( \frac{\partial U^F_g(\alpha; \hat{x}_\ell)}{\partial \alpha} = 0 \) and \( g \) is indifferent over \( \alpha \). Obviously, any positive cost of access implies \( g \) prefers \( \alpha = 0 \).

**Ideologue Interest Groups**

Next, I study ideologue groups, i.e., \( \hat{x}_g \notin (-\bar{x}, \bar{x}) \). If \( \ell \) and \( g \) are aligned ideologues, then lobbying is inconsequential and \( g \) is indifferent over free access. Otherwise, if \( \ell \)
is a non-ideologue, or \( \ell \) and \( g \) are opposed ideologues, then lobbying is consequential. Yet, \( g \)'s preferences over access are ambiguous in general. Specifically, if \( \alpha \) changes the acceptance set, then one partisan's equilibrium proposal is worse for \( g \), but the other's is more favorable. The specific balance of partisan proposal power determines which change dominates. Thus, it is difficult to draw strong conclusions about an ideologue group's preference for access without restricting the balance of partisan proposal power.

Accordingly, I study a substantively motivated restriction on relative partisan power. In U.S. legislatures, majority parties typically exercise substantial control over committee assignments and leadership positions (Cox and McCubbins, 2005, 2007). To reflect this observation, I restrict proposal power to one side of \( M \). Formally, either \( \rho_L = 0 < \rho_R \) or \( \rho_R = 0 < \rho_L \).

**Definition 4.** The legislature exhibits *minority-party agenda exclusion* if one of the partisans, \( L \) or \( R \), has no agenda setting power, while the other, *majority*, partisan has positive recognition probability.

Definition 4 reflects the widespread belief that majority parties carefully allocate agenda setting power in the US. Yet, the model also aligns with empirical work suggesting that individual legislators possess some freedom from their party and also can be influenced by interest groups (Fouirnaies, 2017).

**Definition 5.** Legislator \( \ell \) is *majority-leaning* if aligned with the majority partisan and similarly for the interest group, \( g \).

Proposition 3 shows that a majority-leaning ideologue group wants access to any majority-leaning non-ideologue legislator under minority-party exclusion. Such groups are indifferent over free access to majority-leaning ideologue legislators. The result focuses on majority-leaning legislators because minority-leaning legislators do not have proposal power and thus \( g \) is indifferent.

**Proposition 3.** Assume there is minority-party agenda exclusion and the interest group, \( g \), is a majority-leaning ideologue.

(i) If legislator \( \ell \) is a majority-leaning ideologue, then \( g \) is indifferent over access.

(ii) If \( \ell \) is a majority-leaning non-ideologue, then \( g \) acquires access.
Part (i) follows because $g$ cannot profitably lobby to change $\ell$’s proposal. Part (ii) follows because access provides two benefits for $g$ under minority-party exclusion. First, lobbying is profitable and greater $\alpha$ increases $g$’s chances of enjoying that profit. Second, greater $\alpha$ diminishes $M$’s expectations about future policy and expands the acceptance set. Partisans can thus pass more extreme policy. Because minority partisans are unable to propose policy under minority-party agenda exclusion, $g$ benefits from emboldening aligned partisan legislators without risking more extreme proposals by opposing partisans.

Access and the Welfare of Legislators and Society

**Legislator Welfare:** Ex ante, access can be good or bad from legislator $\ell$’s perspective. These effects arise entirely from changes in expected extremism because access affects $\ell$ only through the indirect effect on partisan proposals. Whenever group $g$ lobbies $\ell$, it compensates $\ell$ for any policy loss she suffers. The relative extremism of $\ell$ and $g$ determine whether $\ell$ is better off. For example, $\ell$ may improve her expected welfare by giving access if $g$ is slightly more centrist. Here, access is mutually desirable because it acts as a commitment device on $\ell$’s proposals that indirectly constrains partisan proposals. In contrast, $\ell$ is always weakly worse off giving access to a more extreme aligned group because extremism increases. Although not modeled, these observations suggest legislators may price discriminate based on group ideology when selling access.

**Social Welfare:** To measure social welfare, I use $M$’s expected payoff. This approach is appropriate if the median citizen is close to $\hat{x}_M$, so that the legislature suitably represents the ideological distribution of the unmodeled citizenry. Then $M$’s expected payoff corresponds to majority welfare in this ordered setting (Banks and Duggan, 2006b).

Connections shifting policy towards $\hat{x}_M$ also reduce expected extremism, and vice versa. Social welfare thus improves if groups access more extreme legislators and decreases if they access relative centrists. Therefore Propositions 2 and 3 have immediate welfare implications for a wide range of aligned group-legislator pairs. In some cases, groups do not acquire access and thus do not effect welfare. Highlighting this possibility, and the conditions producing it, is a key benefit of the formal analysis.

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24This is also true in, e.g., Schnakenberg (2017).
Willingness to Acquire Access

Thus far, the analysis considers whether groups want access. But what about contribution *amounts*? I provide two results in this direction. First, and consistent with a large body of empirical work, I demonstrate that groups are willing to pay more for access to legislators with greater proposal power. Second, under broad conditions I show that ideologically distant groups are willing to pay more for access to a given legislator.

The results study *g*’s willingness to pay (WTP) for access. Alternatively, we could characterize *g*’s optimal amount of access and compare the cost of that access under different conditions. But this task requires specifying a cost function for access. Instead of restricting this class of functions, I study *g*’s WTP for access.25

**Proposition 4.** All else equal, an interest group’s willingness to pay for $\alpha$ access weakly increases with the targeted legislator’s proposal power.

Proposition 4 does not depend on the respective ideologies of the legislator and interest group. Proposal power amplifies the marginal benefit of access by increasing the probability that *g* can extract surplus via lobbying. This increases the value of additional access. On the other hand, greater proposal power also increases how sensitive the acceptance set is to $\alpha$, which may help or harm the interest group. Whenever *g*’s WTP is strictly positive, the overall effect is proportional to the legislator’s recognition probability. Thus, if the group is willing to pay for access, then greater proposal power amplifies this desire.

Proposition 4 suggests groups will pay a higher price to access powerful legislators. This implication fits the empirical regularity that legislators on important and relevant committees, especially committee chairmen, attract more contributions (Ainsworth, 2002; Grimmer and Powell, 2016; Berry and Fowler, 2018; Fournaies, 2017).

Next, I analyze how *g*’s ideology affects its willingness to buy access to a majority-leaning legislator under minority-party exclusion. **Proposition 5** fixes $\hat{x}_\ell$ and analyzes *g*’s willingness to pay to increase $\alpha$ at zero, which I refer to as *g*’s *willingness to acquire access* (WTA). Under broad conditions, *g*’s WTA weakly increases as $\hat{x}_g$ diverges from $\hat{x}_\ell$ in either direction.

**Proposition 5.** Suppose there is minority-party agenda exclusion and legislator $\ell$ is

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25See, e.g., Denzau and Munger (1986) and Hall and Deardorff (2006) for previous work on access-seeking campaign contributions studying willingness to pay.
majority-leaning. If either (i) the interest group, \( g \), is more centrist than \( \ell \), or (ii) \( g \) is majority-leaning and more extreme than \( \ell \), then \( g \)'s willingness to acquire access weakly increases as \( g \) becomes less ideologically similar to \( \ell \).

I discuss the logic using the case of right-party control, depicted in Figure 6. Part (i) assumes \( g \) is more centrist than \( \ell \). Two forces increase \( g \)'s WTA as \( |\hat{x}_g - \hat{x}_\ell| \) increases. First, \( g \)'s lobbying surplus grows, so it has more to gain from additional access. Second, \( g \)'s access forces majority-party partisans to moderate their policy proposals further because \( g \)'s policy offer gets better for \( M \). Thus, \( g \) gains more from inducing partisan moderation. These effects increase \( g \)'s WTA as \( \hat{x}_g \) decreases away from \( \hat{x}_\ell \).

Figure 6: Willingness to acquire access

![Figure 6: Willingness to acquire access](image)

Figure 6 illustrates Proposition 5 for a majority-centrist legislator, \( \ell \), under left-party agenda exclusion. The group’s willingness to acquire access decreases as its ideal point approaches \( \hat{x}_\ell \).

The logic for part (ii) of Proposition 5 is best described in two cases.

First, suppose \( g \) is partisan when \( \alpha = 0 \). In Figure 6, this corresponds to \( \hat{x}_g \geq \pi_0 \). If \( \ell \) is centrist, as pictured in Figure 6, then \( g \)'s WTA decreases as \( \hat{x}_g \) shifts towards \( \hat{x}_\ell \) for reasons symmetric to those described above: (i) \( g \)'s lobbying surplus decreases and (ii) \( g \)'s benefit from inciting more extreme partisan proposals also decreases. If \( \ell \) is partisan, \( \hat{x}_\ell \geq \pi_0 \) in Figure 6, then \( g \)'s lobbying is inconsequential. Therefore \( g \)'s WTA is zero and thus constant as \( \hat{x}_g \) approaches \( \hat{x}_\ell \).

Second, suppose \( g \) is centrist when \( \alpha = 0 \), e.g. \( \hat{x}_g \in (\hat{x}_\ell, \pi_0) \) in Figure 6. Access now has competing effects. By logic similar to Proposition 2, \( g \) forgoes access if \( \hat{x}_g \) is close to \( \hat{x}_\ell \). But \( g \)'s WTA increases as \( \hat{x}_g \) shifts away from \( \hat{x}_\ell \). Specifically, whenever
$g$’s WTA is positive, lobbying surplus grows faster than the loss from inciting more extreme partisan proposals.

An additional observation is that if $g$’s WTA is zero, then $g$ is not willing to pay for any positive amount of access. Proposition 5 thus implies that a majority-leaning group forgoes access if it is slightly more extreme than $\ell$, mirroring Proposition 2.\footnote{See Lemma 7 in Appendix A for more details.}

**Persistent vs. Short-term Access**

Thus far, I have studied persistent access, which has direct and indirect effects on the group’s ex ante welfare. The direct effect, opportunities to lobby, always benefits the group. The spillover effect, changing partisan proposals, can be good or bad. Any amount of persistence can produce these spillovers.

Temporary access can avoid spillovers and produce universal access seeking. First, suppose groups choose access only once and it lasts one period. Then access today does not affect expectations about future policymaking and the acceptance set, along with partisan proposals, do not change. Groups always want immediate, one-shot access because it provides only a direct benefit. Alternatively, if the group can set access freely each period, then it chooses full access every period of every stationary equilibrium.\footnote{In a stationary strategy profile, a one-shot access deviation does not change expectations about future policymaking. Thus, the group always has a profitable deviation if it is not choosing full access.}

In some cases, a group’s optimal access contract is one period of immediate access followed by no future access. Consider the stationary equilibria mentioned above. Groups choose full access every period. Expected payoffs are thus equal to those from persistent full access. The main analysis implies some groups would rather commit to forgo access. But these groups most prefer immediate, one-shot access with no chance for later access. This arrangement has a direct benefit without any indirect cost because the acceptance set is unchanged. Thus, one of two possibilities must hold for these groups to pursue access. Either (i) access is temporary and commitment is impossible, or (ii) groups can contract against future access.

The preceding discussion has focused on group-legislator pairs in which legislative considerations discourage access. But recall that some groups enjoy the indirect effect of persistent access. These groups always want access, regardless of its durability or contractability. In fact, legislative considerations increase their desire for persistent

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access. Thus, the main analysis distinguishes groups that covet access from those less inclined.

**Multiple Groups and Conceding Access**

The main analysis abstracts from competition over access to isolate how legislative considerations affect access-seeking. Interest group competition has been studied elsewhere (e.g. Chamon and Kaplan, 2013; Levy and Razin, 2013) and a full analysis is outside the scope of this paper. Yet, the analysis offers several insights about when competition is unlikely.

In many policy areas, interest groups frequently lobby unopposed (Baumgartner and Leech, 2001; Leaver and Makris, 2006; Dal Bó, 2007). Where are competing interests? Existing explanations include collective action problems, free-riding incentives, and entry costs. I shed further light on when these competitive voids can arise, as legislative considerations can produce strong anti-competitive incentives.

To fix ideas, say that group $g$ concedes access to group $g'$ if $g$ strictly prefers letting $g'$ optimally choose access. In the model, some groups concede durable access purely for policy reasons. Specifically, some groups prefer conceding to more centrist groups even if both access and lobbying are free. Moreover, this preference can arise regardless of whether the group wants access in isolation.

For example, suppose $g$ is a non-ideologue contemplating access to a slightly more centrist legislator, $\ell$. By Proposition 2, $g$ does not want access to $\ell$. Furthermore, if there is another group $g'$ slightly more centrist than $\ell$, then $g$ strictly prefers to concede access to $g'$. Why? Group $g$ forgoes access to avoid increasing expected extremism, but is even better off reducing expected extremism. Because $g'$ is more centrist than $\ell$, it seeks access by Proposition 2, thereby reducing expected extremism. Thus, if $g'$ is not too far from $\ell$, then $g$ concedes access to $g'$. In this case, conceding access makes $g$ better off than forgoing access in isolation.

This illustration reveals how legislative forces can discourage competition. Conceding to more centrist groups may be attractive because it allows otherwise unobtainable moderation. As discussed, groups may even concede access to groups on the opposite side of a given legislator. Substantively, this could partially explain why some industries lobby through trade associations funded by diverse interests.
Legislative Conditions and Lobbying Expenditures

Having studied access acquisition, I now fix access and characterize how equilibrium lobbying expenditures vary with legislative features. In general, lobbying expenditures weakly increase as the acceptance set expands. Thus, the characterization of $\pi(\sigma)$ in (5) has direct implications for expenditures. I begin by cataloging the relevant legislative features and describing their effects on the acceptance set.

The first legislative feature I study is a function of the distribution of agenda power, $\rho$, and access, $\alpha$. Given $\rho$ and access $\alpha$, let the moderate legislator’s unconstrained extremism lottery be the lottery putting probability $\alpha\rho_\ell$ on $|\hat{y}|$, probability $\rho_\ell(1 - \alpha)$ on $|\hat{x}_\ell|$, and probability $\rho_j$ on $|\hat{x}_j|$ for each legislator $j \neq \ell$. Thus, the outcomes of an unconstrained extremism lottery are measured in terms of absolute distance between each player’s ideal proposal and $\hat{x}_M = 0$. Say that legislative extremism under $(\rho', \alpha')$ is higher than $(\rho, \alpha)$ if $M$’s unconstrained extremism lottery induced by $(\rho', \alpha')$ first order stochastically dominates the lottery induced by $(\rho, \alpha)$.\footnote{In this context, the unconstrained extremism lottery $(\rho', \alpha')$ first order stochastically dominates another unconstrained extremism lottery $(\rho, \alpha)$ if: (i) for all $x \in X$, $(\rho', \alpha')$ puts weakly greater probability on $x'$ such that $|x'| \geq |x|$ and (ii) for some $x \in X$, $(\rho', \alpha')$ puts strictly greater probability on $x'$ such that $|x'| \geq |x|$.} For example, legislative extremism increases if proposal power shifts away from $M$ to other legislators. This change lowers $M$’s reservation value because extreme policy proposals become more likely, without an offsetting increase in the chance of moderate policy proposals. Thus, the acceptance set expands.

Second, I vary the location of the status quo, $q$. More extreme status quo lower $M$’s reservation value because she is more averse to waiting until a new policy passes. Thus, the acceptance set expands.

Third is legislator patience, $\delta$. As $\delta$ increases, $M$ is less bothered by enduring the status quo and places more weight on policies that eventually pass. Thus, greater patience shrinks the acceptance set.

As noted above, expanding the acceptance set weakly increases ex post lobbying payments. The preceding observations thus characterize how expenditures vary.

**Proposition 6.** The interest group’s equilibrium lobbying expenditures weakly increase as either (i) legislative extremism increases, holding constant $\hat{x}_g$ and $\hat{x}_\ell$; (ii) the status quo policy becomes more extreme; or (iii) legislator patience decreases.

To see the logic for Proposition 6, recall $g$’s equilibrium transfer to $\ell$ is $m = u_\ell(z_\ell) -$
Therefore equilibrium lobbying expenditures increase if either (i) $g$’s policy offer becomes worse for $\ell$ or (ii) $\ell$ can pass more favorable policy after rejecting $g$’s overtures. Thus, there are two ways that a larger acceptance set can increase expenditures: (i) more slack for $g$ to shift $\ell$’s proposal, or (ii) a better outside option for $\ell$.

Figure 7: Increasing lobbying expenditures – extreme group

![Diagram](image)

Figure 7(a) displays legislator $\ell$’s proposals for a baseline acceptance set where $g$’s offer is constrained and $\ell$’s proposal is unconstrained. Figure 7(b) displays $\ell$’s proposals if the acceptance set expands. Expenditures increase because $g$ pays more for a better offer.

First, if $g$ is extremist and $M$ constrains $g$’s equilibrium policy offer, then greater legislative extremism gives $g$ more slack to lobby $\ell$ to more extreme policy. Consequently, lobbying expenditures increase because $g$’s policy offer is worse for $\ell$. Figure 7 displays this case. If $g$ and $\ell$ are aligned extremists, however, then $z_\ell = y$ and lobbying expenditures are constant for small enough changes in legislative extremism.

Second, if $\ell$ is extremist, then increasing legislative extremism improves $\ell$’s outside option because $z_\ell$ equals the boundary of $A(\sigma)$ closest to $\hat{x}_\ell$. This boundary shifts towards $\hat{x}_\ell$ as legislative extremism increases, improving $\ell$’s outside option and forcing $g$ to transfer more to lobby $\ell$ away from $z_\ell$. If $\ell$ is not too extreme, and $g$ is aligned with $\ell$ but not extremist, then $y \neq z_\ell$. Lobbying grows more expensive even though the policy offer does not change. Figure 8 illustrates.

Next, I state three corollaries of Proposition 6 showing how substantively meaningful features of the model affect extremism and, in turn, lobbying expenditures.

First, an important special case of changing legislative extremism is varying $g$’s access, $\alpha$. Greater access causes $M$ to anticipate more frequent lobbying by $g$. Thus, $\alpha$’s effect on the acceptance set depends on $g$ and $\ell$’s relative ideology. If $g$ is more extreme,
then increasing $\alpha$ raises legislative extremism and the acceptance set expands. This relationship flips if $g$ is more centrist. Given a group-legislator pair, Proposition 6 yields an immediate corollary on the relationship between access and lobbying expenditures.

**Corollary 1.** Suppose the interest group, $g$, is aligned with legislator $\ell$. If $g$ is more extreme than $\ell$, then equilibrium lobbying expenditures weakly increase with access. Otherwise, they weakly decrease with access.

Next, Corollary 2 establishes that lobbying expenditures grow if $M$ loses proposal power, which weakly increases legislative extremism. Substantively, this result suggests that weakening centrist agenda setting power encourages more vigorous lobbying.

**Corollary 2.** If proposal power transfers away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.

Corollary 3 states that lobbying expenditures grow weakly as $\ell$ shifts away from $M$, weakly increasing legislative extremism. This result suggests that groups spend more on lobbying in more polarized legislatures, in the colloquial sense of having greater ideological spread among legislators.

**Corollary 3.** If legislator $\ell$ shifts farther away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.
Conclusion

I study which legislators and interest groups form connections to facilitate lobbying. To do so, I analyze a model where groups choose access before policymaking. Access provides groups with opportunities to influence policy proposals by lobbying. The model provides a tractable framework to explore how access-seeking depends on the larger legislative context.

Interest groups weigh various institutional and political factors when deciding whether to pursue access. Does greater access increase or decrease policy extremism in the legislature? Is the targeted legislator likely to have much control over policymaking? Are partisan legislators likely to draft policy?

I refine our understanding of how groups weigh these questions when evaluating who they want to access. First, I highlight conditions under which lobbying increases policy extremism and those where it decreases. Then, I show that groups avoid access to particular legislators under broad conditions. Specifically, if groups are not too extreme, they forego access to a range of more centrist legislators. Policy considerations drive this behavior, as access to these legislators generates increased policy polarization that counteracts better lobbying prospects. More broadly, the analysis unpacks a neglected consequence of access: the prospect of lobbying can spill over and affect policies proposed by other legislators.

The analysis has implications for campaign finance, revolving-door hiring, and lobbying expenditures. First, which legislators do access-seeking interest groups direct campaign contributions towards? And whose associates do they hire through the revolving door? Second, which groups lobby which legislators? Third, what can lobbying expenditures tell us about access. Finally, why do many groups contribute so little (Tullock, 1972; Ansolabehere et al., 2003)? Distinguishing between access and lobbying is key for these implications. I now elaborate on each.

First, analyzing endogenous access suggests which connections are likely to form. In practice, campaign contributions and revolving-door hiring are prominent channels for access. Propositions 2 and 3 thus have implications for both (i) whose staffers interest groups hire and (ii) to whom they contribute. Data exist for both, but using revolving-door hiring data may be a better starting point because most measures of group and legislator ideology use contributions to locate the actors on a common scale. Given group ideology, the model suggests a curvilinear, and possibly multimodal, relationship between hiring/contributions and legislator ideology. A robust prediction is that groups
do not access very extreme legislators.

Second, and reflecting the widespread view, access is necessary for lobbying in the model. The analysis thus suggests immediate qualitative predictions about who lobbies whom. In some instances, detailed lobbying data specify targeted legislators. Another possibility is data on “points of contact,” which may provide information about which legislators groups target.

Third, the relationship between observed expenditures and access is conditional on relative ideology. Thus, empirical work should control for relative ideology when evaluating the connection between access and lobbying expenditures. Otherwise, offsetting observations may obscure a meaningful effect. Related, recovering a negative relationship between access and lobbying expenditures for centrist groups targeting extreme legislators need not imply that these groups lobby less or, alternatively, that legislators value their dollars more. Instead, expenditures may decline because the targeted legislator’s outside option is worse, making the group’s desired policy cheaper. Another implication is that ceteris paribus changes in lobbying expenditures can indicate changes in access amounts. With information about relative ideology, we can infer the direction of the change.

Finally, the analysis speaks to Tullock’s puzzle, that many groups do not contribute at all and those that do rarely reach legal limits (Tullock, 1972). Some view this empirical regularity as evidence that either contributions are not valuable, or donors are unsophisticated (Ansolabehere et al., 2003). Previous work has shown that interest group competition can lead sophisticated groups to contribute small amounts (Chamon and Kaplan, 2013). I provide a new strategic mechanism for such behavior, legislative considerations, which can reduce contributions by sophisticated groups precisely because they are valuable for gaining access and generate adverse spillover effects. Additionally, I highlight when we should expect low contributions, as these considerations discourage groups from contributing to more centrist legislators, but encourage access-seeking contributions to more extreme legislators.
Appendix A

Model

I prove the main results in a more general version of the model, relaxing restrictions on the number of legislators and interest groups. There are three disjoint sets of players: $n^V$ (finite and odd) voting legislators in $N^V$; $n^L \geq 3$ committee members in $N^L$; and $n^G \leq n^L$ interest groups in $N^G$. Let $N = N^V \cup N^L \cup N^G$.

Throughout, voting legislators are called voters and denoted by $i$. To align with the main text, $M$ denotes the median voter. I denote committee members by $\ell$ and interest groups by $g$. Each $\ell \in N^L$ is associated with only one group, $g_{\ell}$. Each $g \in N^G$ can have access to multiple $\ell \in N^L$ and this set is $N^L_g \subseteq N^L$. Let $\alpha_{\ell} \in [0, 1]$ denote $g_{\ell}$’s access to $\ell$.\(^{29}\)

Legislative bargaining occurs over an infinite number of periods $t \in \{1, 2, \ldots\}$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Let $\rho = (\rho_1, \ldots, \rho_{n^L}) \in \Delta([0, 1])^{n^L}$ be the distribution of recognition probability among $\ell \in N^L$.\(^{30}\) In each period $t$, bargaining proceeds as follows. If no policy has passed before $t$, then $\ell$ proposes with probability $\rho_{\ell} > 0$. All players observe the period-$t$ proposer, $\ell_t$. With probability $1 - \alpha_{\ell_t}$, $g_{\ell_t}$ cannot lobby and $\ell_t$ freely proposes any $x_t \in X$. With probability $\alpha_{\ell_t}$, $g_{\ell_t}$ can lobby and offers $\ell_t$ a binding contract $(y_{t}, m_{t}) \in X \times \mathbb{R}_+$. Next, $\ell_t$ accepts or rejects. Let $a_t \in \{0, 1\}$ denote $\ell_t$’s period-$t$ acceptance decision, where $a_t = 1$ indicates acceptance and $a_t = 0$ if either $\ell_t$ rejects or $g_{\ell_t}$ is unable to lobby in $t$. If $\ell_t$ accepts, then $\ell_t$ is committed to propose $x_t = y_{t}$ in $t$ and $g_{\ell_t}$ transfers $m_t$ to $\ell_t$. If $\ell_t$ rejects, then she can propose any $x_t \in X$ and $g_{\ell_t}$ keeps $m_t$. All players observe $x_t$. There is a simultaneous vote by $i \in N^V$ using simple majority rule. If $x_t$ passes, then bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $x_t$ fails, then $q$ is enacted in $t$ and bargaining proceeds to $t + 1$.

Each player $j \in N$ has quadratic policy utility with ideal point $\hat{x}_j \in X$. As in the main text, I normalize $\hat{x}_M = 0$ and assume $q \neq 0$. Additionally, I assume there exists $\ell \in N^L$ on the same side of $q$ as $M$ such that $\alpha_{\ell} < 1$ or $g_{\ell}$ is on the same side of $q$. For example, assume $q > 0$. Then some $\ell \in N^L$ satisfies $\hat{x}_\ell < q$ and at least one of the following holds: $\hat{x}_{g_{\ell}} \leq q$ or $\alpha_{\ell} < 1$.

Players discount streams of stage utility by common discount factor $\delta \in (0, 1)$.

\(^{29}\)An independent legislator is accommodated by setting $\alpha_{\ell} = 0$.

\(^{30}\)Where $\Delta([0, 1])^{n^L}$ denotes the $n^L$-dimensional unit simplex.
For convenience, I normalize per-period payoffs by \((1 - \delta)\). Let \(I^\ell_t \in \{0, 1\}\) equal one iff \(\ell\) is the period-\(t\) proposer and \(g^\ell\) can lobby in \(t\). Given a sequence of offers \((y_1, m_1), (y_2, m_2), \ldots\), a sequence of proposers \(\ell_1, \ell_2, \ldots\) a sequence of acceptance decisions \(a_1, a_2, \ldots\), and a sequence of independent policy proposals \(x_1, x_2, \ldots\) such that bargaining continues until \(t\), the discounted sum of per-period payoffs for \(i \in N^V\) is

\[
(1 - \delta^{t-1})u_i(q) + \delta^{t-1}\left[(1 - a_t)u_i(x_t) + a_t u_i(y_t)\right];
\]

for \(\ell \in N^\ell\),

\[
(1 - \delta) \sum_{t' = 1}^{t-1} \delta^{t'-1}\left[u_{\ell'}(q) + I^\ell_{t'}a_{\ell'}m_{t'}\right] + \delta^{t-1}\left[(1 - a_t)u_{\ell}(x_t) + a_t \left(u_{\ell}(y_t) + I^\ell_t m_t\right)\right];
\]

and for \(g \in N^g\),

\[
(1 - \delta) \sum_{t' = 1}^{t-1} \delta^{t'-1}\left[u_g(q) - a_{\ell'}m_{t'} \sum_{\ell \in N^g_{t'}} I^\ell_{t'}\right] + \delta^{t-1}\left[(1 - a_t)u_g(x_t) + a_t \left(u_g(y_t) - m_t \sum_{\ell \in N^g_{t'}} I^\ell_{t'}\right)\right].
\]

Unless noted otherwise, results are proved for this more general setting. The model in the main text is a special case featuring one voter with ideal point \(\hat{x}_M\); four committee members with ideal points \(\hat{x}_L, \hat{x}_M, \hat{x}_R\); and one group at \(\hat{x}_g\) with access \(\alpha_\ell \geq 0\) and \(\alpha_j = 0\) for all \(j \neq \ell\).

**Strategies**

I study a class of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.\(^{31}\)

Let \(\Delta(X)\) be the set of probability measures on \(X\). Let \(W = X \times \mathbb{R}_+\) denote the lobby offer space and \(\Delta(W)\) denote the set of probability measures on \(W\). A stationary mixed strategy for \(g \in N^G\) is a probability measure \(\lambda_g \in \Delta(W)^{|N^L_g|}\) over \(g\)'s offers \((y, m) \in W\) to each \(\ell \in N^L_g\). A stationary mixed legislative strategy for \(\ell \in N^L_g\)

\(^{31}\)In Appendix B, I define stationary mixed strategy legislative lobbying equilibria and show that they are all equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium with deferential voting and deferential acceptance.
is a pair \((\pi_\ell, \varphi_\ell)\); where \(\pi_\ell \in \Delta(X)\) specifies a probability measure over \(\ell\)'s independent proposals and \(\varphi_\ell : W \to [0, 1]\) is the probability \(\ell\) accepts each \((y, m) \in W\). Finally, voter \(i\)'s stationary mixed strategy \(\nu_i : X \to [0, 1]\) specifies the probability \(i\) votes for each \(x \in X\).

Let \(\lambda\) denote a profile of interest group strategies, \((\pi, \varphi)\) a profile of committee member strategies, and \(\nu\) a profile of voter strategies. A stationary strategy profile is \(\sigma = (\lambda, \pi, \varphi, \nu)\). Under \(\sigma\), let \(\nu_\sigma(x)\) represent be the probability that \(x\) passes in a given period.

**Continuation Values**

Let \(w = (y, m) \in W\) denote an arbitrary lobby offer. For convenience, define

\[
\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda^\ell_y(dw), \tag{11}
\]

which is the probability under \(\sigma\) that \(\ell\) makes an independent policy proposal in any period she is recognized. Given \(\sigma\), \(i \in N^V\) has continuation value

\[
V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y) u_i(y) + [1 - \nu_\sigma(y)] \left[ (1 - \delta) u_i(q) + \delta V_i(\sigma) \right] \right] \lambda^\ell_y(dw) 
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x) u_i(x) + [1 - \nu_\sigma(x)] \left[ (1 - \delta) u_i(q) + \delta V_i(\sigma) \right] \right] \pi_\ell(dx) \right), \tag{12}
\]

the continuation value of \(\ell \in N^L\) is

\[
\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \int_W \varphi_j(y, m) \left[ \nu_\sigma(y) u_\ell(y) + [1 - \nu_\sigma(y)] \left[ (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \right] \lambda^j_y(dw) 
+ \xi_j(\alpha, \sigma) \int_X \left[ \nu_\sigma(x) u_\ell(x) + [1 - \nu_\sigma(x)] \left[ (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \right] \pi_j(dx) \right), 
\]

\[
+ \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y) u_\ell(y) + [1 - \nu_\sigma(y)] \left[ (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \right] \lambda^\ell_y(dw) 
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x) u_\ell(x) + [1 - \nu_\sigma(x)] \left[ (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \right] \pi_\ell(dx) \right) + m \lambda^\ell_x(dw). \]
and the continuation value of $g \in N^G$ is

$$
\hat{V}_g(\sigma) = \sum_{\ell \in N^L_g} \rho_\ell \left( \alpha_\ell \int_{W} \varphi_\ell(y, m) \left[ \nabla_{\sigma}(y)u_g(y) + [1 - \nabla_{\sigma}(y)][(1 - \delta)u_g(q) + \delta \hat{V}_g(\sigma)] \right] \lambda_\ell(dw) 
+ \xi_\ell(\alpha, \sigma) \int_{X} \left[ \nabla_{\sigma}(x)u_g(x) + [1 - \nabla_{\sigma}(x)][(1 - \delta)u_g(q) + \delta \hat{V}_g(\sigma)] \right] \pi_\ell(dx) \right),
$$

(14)

Stationary Legislative Lobbying Equilibrium

A stationary pure strategy for $g \in N^G$ is $(y_g, m_g) \in X^{|N^L_g|} \times \mathbb{R}^{|N^L_g|}$, where $y_g$ is $g$’s profile of policy offers and $m_g$ is $g$’s profile of monetary offers. A pure stationary strategy for $\ell \in N^L$ is $(z_\ell, a_\ell)$; where $z_\ell \in X$ specifies $\ell$’s independent proposals, and $a_\ell : X \times \mathbb{R} \to \{0, 1\}$ equals one iff $\ell$ accepts $g_\ell$’s offer. Finally, for each $i \in N^V$, $v_i : X \to \{0, 1\}$ equals one iff $i$ supports the proposal.

Given $\sigma$, the set of policies that pass is constant across periods by stationarity and denoted $A(\sigma) \subset X$. For $\ell \in N^L$, define

$$
\tilde{U}_\ell(x; \sigma) = \begin{cases} 
    u_\ell(x) & \text{if } x \in A(\sigma) \\
    (1 - \delta)u_\ell(q) + \delta \hat{V}_\ell(\sigma) & \text{else.}
\end{cases}
$$

(15)

Formally, $\sigma = (y, m, z, a, v)$ is a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance if it satisfies five conditions. First, for all $g \in N^G$ and $\ell \in N^L_g$, $(y^\ell_g, m^\ell_g)$ satisfies

$$
y^\ell_g = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell)
$$

(16)
and

\[ m^\ell_g = u^\ell(z^\ell) - u^\ell(y^\ell_g). \]  

(17)

Second, for all \( \ell \in N^L \) and \( (y, m) \in W \), \( a^\ell(y, m) = 1 \) iff

\[ \tilde{U}^\ell(y; \sigma) + m \geq \tilde{U}^\ell(z^\ell; \sigma). \]  

(18)

Third, for each \( \ell \in N^L \), \( z^\ell \) solves

\[ \max_{x \in A(\sigma)} u^\ell(x). \]  

(19)

Finally, for each \( i \in N^V \), \( v_i(x) = 1 \) iff

\[ u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma). \]  

(20)

Appendix B shows that all stationary mixed strategy legislative lobbying equilibria are equivalent in outcome distribution to strategy profiles satisfying (16)-(20).

**Existence**

I now prove part 1 of Proposition 1 from the main text.

**Proposition 1.1.** There exists a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.

**Proof.** There are three parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for \( g \in N^G \) and \( \ell \in N^L \). Part 2 uses the fixed point to construct a strategy profile \( \sigma \). Part 3 verifies \( \sigma \) satisfies (16) - (20).

**Part 1:** Let \( (y, z) = (y_1, \ldots, y_{n^L}, z_1, \ldots, z_{n^L}) \in X^{2n^L} \) and for each \( j \in N \) define

\[ r_j(y, z) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell u_j(y_\ell) + (1 - \alpha_\ell)u_j(z_\ell) \right). \]  

(21)

Set \( A(r(y, z)) = \{ x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta r_M(y, z) \} \), which is non-empty, compact, and convex because \( u_M \) is strictly concave, \( q = 0 \), and \( \delta \in (0,1) \). Moreover,
Define \( A(r(y, z)) \) is continuous in \((y, z)\).

For each \( \ell \in N^L \), define

\[
\tilde{\phi}_\ell(y, z) = \arg \max_{y \in A(r(y, z))} u_{y^*}(y_\ell) + u_{\ell}(y_\ell),
\]

which is unique for all \((y, z)\) because the objective function is strictly concave and continuous and \( A(r(y, z)) \) is non-empty, compact and convex. Because \( A(r(y, z)) \) is continuous, the Theorem of the Maximum implies continuity of \( \tilde{\phi}_\ell(y, z) \). Next, define

\[
\phi_\ell(y, z) = \arg \max_{z_\ell \in A(r(y, z))} u_\ell(z_\ell),
\]

which is unique for all \((y, z)\) and continuous by the Theorem of the Maximum.

Define the mapping \( \Phi : X^{2n_L} \rightarrow X^{2n_L} \) as \( \Phi(y, z) = \prod_{\ell \in N^L} \Phi_\ell(y, z) \times \prod_{\ell \in N^L} \Phi_\ell(y, z) \), which is a product of continuous functions and thus continuous in \((y, z)\). By Brouwer’s theorem, a fixed point \((y^*, z^*) = \Phi(y^*, z^*)\) exists because \( \Phi \) is a continuous function mapping a non-empty, compact, and convex set into itself.

**Part 2:** Define a stationary pure strategy profile \( \sigma \) as follows. First, for all \( g \in N^G \) and \( \ell \in N^L \), set \( y^*_g = y^*_\ell \) and \( m^L_g = u_\ell(z^*_\ell) - u_\ell(y^*_\ell) \). Next, for \( \ell \in N^L \), set \( z_\ell = z^*_\ell \) and define

\[
a_\ell(y, m) = \begin{cases} 
1 & \text{if } u_\ell(y) + m \geq u_\ell(z^*_\ell), \text{ for } y \in A(r(y^*, z^*)) \\
1 & \text{if } (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^L_g) + m \geq u_\ell(z^*_\ell), \text{ for } y \notin A(r(y^*, z^*)) \\
0 & \text{else.}
\end{cases}
\]

Finally, for each \( i \in N^V \) define \( v_i \) so that \( v_i(x) = 1 \) if \( u_i(x) \geq (1 - \delta)u_i(q) + \delta r_\ell(y^*, z^*) \) and \( v_i(x) = 0 \) otherwise.

**Part 3:** I check that \( \sigma \) satisfies \((16)-(20)\).

First, I verify \((20)\) to show \( A(\sigma) = A(r(y^*, z^*)) \). Note that for each \( g \in N^G \) and all \( \ell \in N^L_g \), \( y^*_g \in A(r(y^*, z^*)) \) and \( a_\ell(y^*_g, m^L_g) = 1 \). Moreover, \( z_\ell \in A(r(y^*, z^*)) \) for all \( \ell \in N^L \). Thus, voter \( i \)'s continuation value under \( \sigma \) is \( V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell [\alpha_\ell u_i(y^*_\ell) + (1 - \alpha_\ell)u_i(z^*_\ell)] = r_i(y^*, z^*) \). Thus, each voter \( i \)'s strategy satisfies \((20)\). Banks and Duggan (2006b) and Duggan (2014) apply, so \( M \) is decisive over lotteries and \( A(\sigma) = A(r(y^*, z^*)) \).
To check (16), consider \( g \in N^G \) and \( \ell \in N^L_g \). Focusing on acceptable offers is without loss of generality because \( a_\ell(z_\ell, 0) = 1 \). Because \( A(\sigma) = A(r(y^*, z^*)) \), (22) implies \( \tilde{\phi}_\ell(y^*, z^*) = \arg \max_{y_\ell \in A(\sigma)} u_{g_\ell}(y_\ell) + u_\ell(y_\ell) - u_\ell(z^*_\ell) \). Thus, (16) holds because \( \tilde{\phi}_\ell(y^*, z^*) = y^*_\ell = y^*_\ell \). Lemma B.6 in Appendix B implies \( y \not\in A(\sigma) \) is not a profitable deviation for any \( g \in N^G \).

It is immediate that \( m^\ell_g \) satisfies (17).

To check (18), note that \( \ell \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( \tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z^*_\ell) \). Thus, \( \ell \) weakly prefers to accept any \((y, m)\) satisfying \( y \in A(r(y^*, z^*)) \) iff \( u_\ell(y) + m \geq u_\ell(z^*_\ell) \). If \( y \not\in A(r(y^*, z^*)) \), then \( \ell \) weakly prefers to accept \((y, m)\) iff \((1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell) \). Thus, \( a_\ell \) satisfies (18).

To check (19), note that (23) implies \( \phi_\ell(y^*, z^*) = \arg \max_{x \in A(\sigma)} u_\ell(x) \) because \( A(\sigma) = A(r(y^*, z^*)) \). Thus, (19) holds because \( \phi_\ell(y^*, z^*) = z^*_\ell = z_\ell \) for each \( \ell \in N^L \). By Lemma B.6 in Appendix B, \( x \not\in A(\sigma) \) is not a profitable deviation for any \( \ell \in N^L \).

Equilibrium Analysis

Appendix B shows that every stationary mixed strategy legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance. The rest of the analysis omits qualifiers, simply referring to equilibria.

Define

\[
\hat{y}_\ell = \arg \max_{y \in X} u_{g_\ell}(y) + u_\ell(y) = \frac{\hat{x}_{g_\ell} + \hat{x}_\ell}{2}.
\]

Recall \( u_\ell(z_\ell) \) is \( \ell \)'s expected dynamic payoff in equilibrium, conditional on rejecting \( g_\ell \)'s offer. By (16), in equilibrium

\[
y^\ell_g = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y).
\]

If \( \hat{y}_\ell \in A(\sigma) \), then \( y^\ell_g = \hat{y}_\ell \). Otherwise, strict concavity implies \( y^\ell_g \) equals the boundary of \( A(\sigma) \) closest to \( \hat{y}_\ell \). As this characterization applies to every equilibrium, there is a clear connection to the characterization in Cho and Duggan (2003), where lobbying is absent.

Proposition 1.3 establishes Part 3 of Proposition 1 from the main text.
Proposition 1.3. Every stationary legislative lobbying equilibrium has the same outcome distribution.

Proof. Let \( \sigma \) and \( \sigma' \) be stationary legislative lobbying equilibria. It suffices to show \((y_g, m_g) = (y_g', m_g')\) for all \( g \in N^C \) and \( z_\ell = z'_\ell \) for all \( \ell \in N^L \). Assume \( y_{g\ell} \neq y'_{g\ell} \) or \( z_\ell \neq z'_\ell \) for some \( \ell \in N^L \). Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, \( A(\sigma) = A(\sigma') \). Because \( \sigma \) and \( \sigma' \) are no-delay by Lemma 2, \( \ell \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( u_\ell(z_\ell) \) under both \( \sigma \) and \( \sigma' \). Lemma B.1 implies \( m_g = u_\ell(y_g') - u_\ell(z_\ell) \). Therefore \((y_g, m_g) = (y_g', m_g')\) and \( z_\ell = z'_\ell \).

Comparative Statics on Lobbying Expenditures

To facilitate the analysis of endogenous access, it is useful to first prove Proposition 6. Set \( \theta = (\hat{x}, \rho, \alpha) \). Let \( \mu_\theta \) denote the unconstrained extremism lottery, which puts probability \( \rho_\ell \alpha_\ell \) on \( |\hat{y}_\ell| \) and probability \( \rho_\ell (1 - \alpha_\ell) \) on \( |\hat{x}_\ell| \) for each \( \ell \in N^L \). Given \( \theta \) and \( \theta' \), legislative extremism is greater under \( \theta' \) if \( \mu_{\theta'} \) first order stochastically dominates \( \mu_\theta \).

Lemma 2. The equilibrium acceptance set weakly expands with legislative extremism.

Proof. Consider \( \theta \) and \( \theta' \), with legislative extremism greater under \( \theta' \). By Proposition 1.3, \( \theta \) and \( \theta' \) each induce a unique equilibrium acceptance set. Let \( \bar{x}_\theta \) and \( \bar{x}_{\theta'} \) denote the respective upper bounds of these sets. I show \( \bar{x}_{\theta'} \geq \bar{x}_\theta \).

For \( b \geq 0 \), let \( C^b_j = \mathbb{I}\{\hat{x}_j \in (-b, b)\} \) and \( \tilde{C}^b_j = \mathbb{I}\{\hat{y}_j \in (-b, b)\} \). Define \( C_j^b \) and \( \tilde{C}_j^b \) analogously for \( \hat{x}_j \) and \( \hat{y}_j \). For all \( b \geq 0 \),

\[
(1 - \delta) u_M(q) + \delta \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)C^b_j u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^b u_M(\hat{y}_j) \right) \\
+ \delta u_M(b) \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)(1 - C^b_j) + \alpha_j (1 - \tilde{C}_j^b) \right) \\
\geq (1 - \delta) u_M(q) + \delta \sum_{j \in N^L} \rho'_j \left( (1 - \alpha'_j)C^b_j u_M(\hat{x}'_j) + \alpha'_j \tilde{C}_j^b u_M(\hat{y}'_j) \right) \\
+ \delta u_M(b) \sum_{j \in N^L} \rho'_j \left( (1 - \alpha'_j)(1 - C^b_j) + \alpha'_j (1 - \tilde{C}_j^b) \right),
\]

(27)

where (28) follows because \( \mu_{\theta'} \) FOSD \( \mu_\theta \) and \( u_M \) is negative quadratic. The equilibrium characterization, and construction of \( C_j \) and \( \tilde{C}_j \), implies \( \bar{x}_\theta \) is the unique \( b \geq 0 \) such
that \( u_M(b) \) equals (27). Analogously, \( \overline{x}_\theta \) is the unique \( b \geq 0 \) such that \( u_M(b) \) equals (28). Thus, \( \overline{x}_\theta \geq \underline{x}_\theta \).

**Proposition 6.** For all \( \ell \in N^L \), \( g_\ell \)'s equilibrium lobbying expenditures increase as either (i) legislative extremism increases, fixing \( \hat{x}_\ell \) and \( \hat{x}_{g_\ell} \); (ii) \( |q| \) increases; or (iii) \( \delta \) decreases.

**Proof.** (i) Increase legislative extremism. Let \( \sigma \) denote an equilibrium, suppressing dependence on legislative extremism. By Lemma 2, \( \overline{x}(\sigma) \) weakly increases with legislative extremism. There are two cases.

- **Case 1.** Suppose \( \hat{x}_\ell \in A(\sigma) \). Then \( z_\ell = \hat{x}_\ell \). There are two subcases.

  First, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y_\ell = \hat{y}_\ell \). Lemma B.1 and (16) imply \( m_\ell^g = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell) \). Lemma 2 implies \( z_\ell = \hat{x}_\ell \) and \( y_\ell = \hat{y}_\ell \) as legislative extremism increases, so \( m_\ell^g \) is constant.

  Second, assume \( \hat{y}_\ell \notin A(\sigma) \). Since \( \hat{x}_\ell \in A(\sigma) \), this requires \( \hat{x}_{g_\ell} \notin [-\overline{x}(\sigma), \overline{x}(\sigma)] \).

  Without loss of generality, assume \( \hat{x}_{g_\ell} > \overline{x}(\sigma) \). Thus, \( z_\ell = \hat{x}_\ell \) and \( y_\ell = \hat{y}_\ell \).

  Lemma B.1 and (16) imply \( m_\ell^g = u_\ell(\hat{x}_\ell) - u_\ell(\overline{x}(\sigma)) \). By Lemma 2, \( \overline{x}(\sigma) \) increases in legislative extremism. Therefore \( m_\ell^g \) increases.

- **Case 2.** Suppose \( \hat{x}_\ell \notin A(\sigma) \). Without loss of generality, assume \( \hat{x}_\ell > z_\ell = \overline{x}(\sigma) \). There are three subcases.

  First, assume \( \hat{y}_\ell < -\overline{x}(\sigma) \). Then \( y_\ell = -\overline{x}(\sigma) \). By Lemma B.1 and (16), \( m_\ell^g = u_\ell(\overline{x}(\sigma)) - u_\ell(-\overline{x}(\sigma)) \). By Lemma 2, increasing legislative extremism increases \( \overline{x}(\sigma) \) and decreases \( -\overline{x}(\sigma) \). Thus, \( m_\ell^g \) increases because \( -\overline{x}(\sigma) < \overline{x}(\sigma) < \hat{x}_\ell \).

  Second, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y_\ell = \hat{y}_\ell \) and \( y_\ell \) is constant as legislative extremism increases. Arguments similar to subcase 2 of Case 1 imply \( m_\ell^g \) increases.

  Third, assume \( \hat{y}_\ell \geq \overline{x}(\sigma) \), which implies \( y_\ell = \overline{x}(\sigma) \). By Lemma B.1 and (16), \( m_\ell^g = u_\ell(\overline{x}(\sigma)) - u_\ell(\overline{x}(\sigma)) = 0 \), which is constant in legislative extremism.

  Altogether, \( m_\ell^g \) weakly increases in legislative extremism.

(ii) Increase \( |q| \). First, let \( C_{\ell}\{\hat{x}_j \in \text{int}A(\sigma)\} \). Similarly, let \( \tilde{C}_{\ell}\{\hat{y}_j \in \text{int}A(\sigma)\} \).
Then
\[
\pi(\sigma) = \left( \frac{(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left[ C_j(\hat{x}_\ell)(1 - \alpha_j)u_M(\hat{x}_j) + \tilde{C}_j(\hat{y}_j)\alpha_j u_M(\hat{y}_j) \right]}{1 - \delta \sum_{j \in N^L} \rho_j \left[(1 - C_j(\hat{x}_\ell))(1 - \alpha_j) + (1 - \tilde{C}_j(\hat{y}_j))\alpha_j \right]} \right)^{\frac{1}{2}},
\]
(29)

Inspection of (29) shows \(\pi(\sigma)\) strictly increases in \(|q|\) and thus \(A(\sigma)\) expands. Arguments analogous to Part (i) imply \(m^f_g\) weakly increases in \(|q|\).

(iii) Decrease \(\delta\). Inspection of (29) shows \(\pi(\sigma)\) strictly decreases as \(\delta\) increases and thus \(A(\sigma)\) shrinks. Arguments analogous to Part (i) imply \(m^f_g\) weakly increases as \(\delta\) decreases. \(\square\)

### Endogenous Access

Fix \(\ell \in N^L\) and recall \(\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}\). For convenience, refer to \(g_\ell\) as \(g\). The results fix \(\hat{x}_g\) and vary \(\hat{x}_\ell\). Let \(\sigma(\alpha_\ell; \hat{x}_\ell)\) denote an equilibrium, given \(\hat{x}_\ell\) and \(\alpha_\ell\). Denote the corresponding social acceptance set as \(A(\alpha_\ell; \hat{x}_\ell)\), with upper bound \(\pi(\alpha_\ell; \hat{x}_\ell)\). Finally, let \(A(\hat{x}_g)\) denote the equilibrium acceptance set if \(\hat{x}_\ell = \hat{x}_g\), suppressing \(\alpha_\ell\) because it is inconsequential.

First, I establish properties used to state analogues of Propositions 2 and 3.

Building upon Lemmas C.1–C.6 in Appendix C, Lemma 1 partitions whether \(\hat{x}_g \in \text{int}A(\hat{x}_g)\) as a function of primitives. See Appendix C for the proof. I state the result here for reference when proving Lemma 3.

**Lemma 1.** For all \(\ell \in N^L\), there exists \(\pi_\ell \in (0, q]\) such that \(\hat{x}_g \in (\pi_\ell, \pi_\ell)\) implies \(\hat{x}_g \in \text{int}A(\hat{x}_g)\). Otherwise, \(A(\hat{x}_g) = [\pi_\ell, \pi_\ell]\).

**Lemma 3.** Suppose \(\hat{x}_g \in (0, \pi_\ell)\). There exists \(\hat{x} \in [0, \hat{x}_g]\) such that \(\hat{x}_\ell \in (\hat{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). A symmetric result holds if \(\hat{x}_g \in (-\pi_\ell, 0)\).

**Proof.** Consider \(\hat{x}_g \in (0, \pi_\ell)\). By Lemma 1, \(\hat{x}_\ell = \hat{x}_g\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies \(A(0; \hat{x}_\ell)\) is continuous in \(\hat{x}_\ell\). Thus, there exists \(\hat{x} \in [0, \hat{x}_g]\) such that \(\hat{x}_\ell \in (\hat{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). By Lemma 2, \(\hat{x}_\ell \in (\hat{x}, \hat{x}_g)\) thus implies \(A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). \(\square\)
For each $j \in N^L \setminus \{\ell\}$, define $E_j^{\text{LB}}(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \leq -\bar{y}(\alpha; \hat{x}_\ell)\}$, $E_j^{\text{UB}}(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \geq \bar{y}(\alpha; \hat{x}_\ell)\}$, and $C_j(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \in \text{int}A(\alpha; \hat{x}_\ell)\}$. Define $\tilde{E}_j^{\text{LB}}(\alpha; \hat{x}_\ell)$, $\tilde{E}_j^{\text{UB}}(\alpha; \hat{x}_\ell)$, and $\tilde{C}_j(\alpha; \hat{x}_\ell)$ analogously for $\hat{y}_j$. Let $I_{\hat{g}} \in \{0, 1\}$ indicate whether $j \in N^L_{\hat{g}}$.

**Assumption A.1.** There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j < 1$ and $\hat{x}_j \notin A(\sigma(\hat{x}_g))$.

**Assumption A.2.** There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j > 0$ and $\hat{y}_j \notin A(\sigma(\hat{x}_g))$.

Next, define
\[
v_1^g(\alpha; \hat{x}_\ell) = \rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_\ell(\hat{x}_\ell) \right)
\] (30)
and
\[
v_2^g(\alpha; \hat{x}_\ell) = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_j^{\text{LB}}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{\text{LB}}(\alpha; \hat{x}_\ell) \right] u_g(-\bar{y}(\alpha; \hat{x}_\ell))
\]
\[
+ \left[ \alpha_j \tilde{E}_j^{\text{UB}}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{\text{UB}}(\alpha; \hat{x}_\ell) \right] u_g(\bar{y}(\alpha; \hat{x}_\ell))
\]
\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) - I_{\hat{g}} j m_{\hat{g}}^j(\alpha; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j).
\]
\] (31)

**Lemma 4.** If $\hat{x}_\ell \neq \hat{x}_g$, then $\frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$.

**Proof.** Suppose $\hat{x}_\ell \neq \hat{x}_g$. From (30) and $\hat{y}_\ell = \frac{x_\ell + x_g}{2}$, $\frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial y}{\partial x_\ell} \left( \hat{x}_g - \hat{x}_\ell \right)^2 > 0$. □

**Lemma 5.** Suppose $0 \leq \hat{x}_\ell < \hat{x}_g < \bar{y}_\ell$ and at least one of Assumption A.1 or A.2 holds. Then $v_2^g(\alpha; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$. A symmetric result holds for $\hat{x}_g < 0$.

**Proof.** Assume $0 \leq \hat{x}_\ell < \hat{x}_g < \bar{y}_\ell$ and at least one of Assumption A.1 or A.2 holds. It suffices to show that
\[
\left[ \alpha_j \tilde{E}_j^{\text{LB}}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{\text{LB}}(\alpha; \hat{x}_\ell) \right] u_g(-\bar{y}(\alpha; \hat{x}_\ell))
\]
\[
+ \left[ \alpha_j \tilde{E}_j^{\text{UB}}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{\text{UB}}(\alpha; \hat{x}_\ell) \right] u_g(\bar{y}(\alpha; \hat{x}_\ell))
\]
\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) - I_{\hat{g}} j m_{\hat{g}}^j(\alpha; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j)
\]
\] (32)
decreases in $\alpha_\ell$ for all $j \in N^L \setminus \{\ell\}$ and strictly decreases for some $j$. 39
Thus, both \( x \in \mathbb{R} \) for all \( \alpha \). \( x \) implies \( \overline{a} \). Second, either: \( E_j^{UB}(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha \); \( C_j(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha \); or there is a unique \( \overline{a}_\ell \in [0, 1] \) such that \( \alpha \in [0, \overline{a}_\ell] \) implies \( E_j^{UB}(\alpha; \hat{x}_\ell) = 1 \), and \( \alpha \in (\overline{a}_\ell, 1] \) implies \( C_j(\alpha; \hat{x}_\ell) = 1 \). An analogous observation holds for \( E_j^{UB}(\alpha; \hat{x}_\ell) \) and \( C_j(\alpha; \hat{x}_\ell) \).

Thus, both

\[
E_j^{LB}(\alpha; \hat{x}_\ell) u_g(-\overline{a}(\alpha; \hat{x}_\ell)) + E_j^{UB}(\alpha; \hat{x}_\ell) u_g(\overline{a}(\alpha; \hat{x}_\ell)) + C_j(\alpha; \hat{x}_\ell) u_g(\hat{x}_\ell) \tag{33}
\]

and

\[
\tilde{E}_j^{LB}(\alpha; \hat{x}_\ell) u_g(-\overline{a}(\alpha; \hat{x}_\ell)) + \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) u_g(\overline{a}(\alpha; \hat{x}_\ell)) + \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) \tag{34}
\]

decrease in \( \alpha \). Furthermore, at least one of (33) and (34) strictly decreases for some \( j \in N_0 \setminus \{\ell\} \) because at least one of Assumptions A.1 or A.2 holds. Proposition 6 implies \( m_j^a(\alpha; \hat{x}_\ell) \) weakly increases in \( \alpha \) for all \( j \in N_0 \). Altogether, (32) decreases in \( \alpha \) for all \( j \in N_0 \setminus \{\ell\} \) and strictly decreases for some \( j \), as desired.

For \( g \in N_G \), define

\[
U_g^E(\alpha; \hat{x}_\ell) = \psi_1(\alpha; \hat{x}_\ell) + \psi_2(\alpha; \hat{x}_\ell). \tag{35}
\]

**Lemma 6.** Assume \( \hat{x}_g \in (0, \overline{a}) \) and at least one of Assumption A.1 or A.2 holds. There exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U_g^E(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \).

**Proof.** Consider \( \ell \in N_0 \) with associated \( g \in N_G \). Assume \( \hat{x}_g \in (0, \overline{a}) \) and at least one of Assumption A.1 or A.2 holds. I show \( \frac{\partial \psi_1^a(\alpha; \hat{x}_\ell)}{\partial \alpha} \bigg|_{\hat{x}_\ell} \) for \( \hat{x}_\ell \) sufficiently close to \( \hat{x}_g \).

By Lemma 3, there exists \( \overline{x} \) such that \( \hat{x}_\ell \in (\overline{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Fix \( \hat{x}_\ell \in (\overline{x}, \hat{x}_g) \) and \( \alpha \in [0, 1] \).

First, I characterize a lower bound on \( \frac{\partial \psi_1^a(\alpha; \hat{x}_\ell)}{\partial \alpha} \). Define

\[
\Gamma = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_j^{LB}(\hat{x}_g) + (1 - \alpha_j) E_j^{LB}(\hat{x}_g) \right] \frac{\partial u_g(-\overline{a}(\overline{x}))}{\partial \overline{a}(\overline{x})} + \left[ \alpha_j \tilde{E}_j^{UB}(\hat{x}_g) + (1 - \alpha_j) E_j^{UB}(\hat{x}_g) \right] \frac{\partial u_g(\overline{a}(\overline{x}))}{\partial \overline{a}(\overline{x})}, \tag{36}
\]
Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\bar{x}(\hat{x}), \bar{x}(\hat{x}))$ implies $\frac{\partial u_g(\bar{x}(\hat{x}))}{\partial \alpha} < 0$ and $\frac{\partial u_g(-\bar{x}(\hat{x}))}{\partial \alpha} < 0$, and (ii) at least one of Assumptions A.1 and A.2 hold.

I claim $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} < \Gamma$, where

$$
\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_j^{LB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(\alpha; \hat{x}_\ell) \right] \frac{\partial u_g(-\bar{x}(\alpha; \hat{x}_\ell))}{\partial \alpha} + \left[ \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(\alpha; \hat{x}_\ell) \right] \frac{\partial u_g(\bar{x}(\alpha; \hat{x}_\ell))}{\partial \alpha} - I_j^g \alpha_j \frac{\partial m_j^g(\alpha; \hat{x}_\ell)}{\partial \alpha}.
$$

(37)

Three steps show the claim. First, note $\hat{x}_\ell \in (\hat{x}, \hat{x}_g)$ implies $\bar{x}(\hat{x}_\ell) \geq \bar{x}(\alpha; \hat{x}_\ell)$ by Lemma 2. Thus, we have $\tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \leq E_j^{UB}(\alpha; \hat{x}_\ell), E_j^{UB}(\hat{x}_\ell), \tilde{E}_j^{LB}(\alpha; \hat{x}_\ell), E_j^{LB}(\hat{x}_\ell)$ for all $j \neq \ell$. Next, $\hat{x}_g > \bar{x}(\hat{x}_\ell) > \bar{x}(\alpha; \hat{x}_\ell)$ implies $\frac{\partial u_g(\bar{x}(\alpha; \hat{x}_\ell))}{\partial \alpha} < 0$ and symmetrically $\frac{\partial u_g(-\bar{x}(\alpha; \hat{x}_\ell))}{\partial \alpha} < 0$. Finally, $\frac{\partial m_j^g(\alpha; \hat{x}_\ell)}{\partial \alpha} \geq 0$ for all $j \in N^L_g$ by Proposition 6.

For almost all $\alpha \in [0, 1]$, $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} = \frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} \frac{\partial x(\alpha; \hat{x}_\ell)}{\partial \alpha}$. Define $C_j(\alpha; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_j(\alpha; \hat{x}_\ell)) + \alpha_j(1 - C_j(\alpha; \hat{x}_\ell))]$. Then,

$$
\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} < \Gamma \frac{\partial x(\alpha; \hat{x}_\ell)}{\partial \alpha} \left[ u_M(\hat{x}_\ell) - u_M(\hat{x}_\ell) \right] - \delta \rho \Gamma \left[ u_M(\hat{x}_\ell) - u_M(\hat{x}_\ell) \right] \left[ 1 - \delta \left( \sum_{j \in N^L} \rho_j C_j(\alpha; \hat{x}_\ell) \right) \right]
$$

(39)

(40)

where (38) follows from $\frac{\partial x(\alpha; \hat{x}_\ell)}{\partial \alpha} > 0$ and $0 > \Gamma > \frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha}$. (39) from applying the implicit function theorem to $x(\alpha; \hat{x}_\ell)$, which is possible for almost all $\alpha \in [0, 1]$; (40) because $\Gamma[u_M(\hat{x}_\ell) - u_M(\hat{x}_\ell)] < 0, \bar{x}(\hat{x}_\ell) > \bar{x}(\alpha; \hat{x}_\ell) > 0$, and $\delta \sum_{j \in N^L} \rho_j C_j(\alpha; \hat{x}_\ell) \in (0, 1)$; and (41) from using $\hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ and simplifying.

By Lemma 4, $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} = \frac{\alpha_j}{2} (\hat{x}_g - \hat{x}_\ell)^2$. By (41), $\frac{\partial U^j(\alpha; \hat{x}_\ell)}{\partial \alpha} < \frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha} + \Gamma \frac{\partial x(\alpha; \hat{x}_\ell)}{\partial \alpha}$

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for almost all \( \alpha \in [0, 1] \). Thus, \( \frac{\partial \bar{U}_E^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < 0 \) if

\[
\frac{\rho}{2}(\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho \Gamma}{2 \bar{\tau}_\ell} \left[ \frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g) \right] < 0,
\]

which holds for \( \hat{x}_\ell > \hat{x}_g \left( \frac{4\bar{\tau}_\ell + \delta \Gamma}{4\bar{\tau}_\ell - 3\delta \Gamma} \right) \). Define \( x' = \max\{\tilde{x}, \hat{x}_g \left( \frac{4\bar{\tau}_\ell + \delta \Gamma}{4\bar{\tau}_\ell - 3\delta \Gamma} \right) \} \). Then \( x' < \hat{x}_g \) because (i) \( \tilde{x} < \hat{x}_g \) and (ii) \( \delta \Gamma < 0 \) implies \( \frac{4\bar{\tau}_\ell + \delta \Gamma}{4\bar{\tau}_\ell - 3\delta \Gamma} < 1 \). Thus, \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( \frac{\partial \bar{U}_E^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < 0 \) for almost all \( \alpha \in [0, 1] \). Continuity implies \( U_E^E(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \) for such \( \hat{x}_\ell \).

Next, I prove the analogue of Proposition 2.

**Proposition A.2** Assume \( \hat{x}_g \in (-\bar{\tau}_\ell, \bar{\tau}_\ell) \) and at least one of Assumptions A.1 and A.2 holds. If \( \hat{x}_g > 0 \), then there exist \( x' \) and \( x'' \) satisfying \( x' < \hat{x}_g < \bar{\tau}_\ell < x'' \) such that

1. if \( \hat{x}_\ell \in (x', \hat{x}_g) \), then \( \alpha = 0 \) is uniquely optimal;

2. if \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( \alpha = 0 \) is not optimal; and

3. if \( \hat{x}_\ell \geq x'' \), then \( g \) is indifferent over \( \alpha \).

A symmetric result holds for \( \hat{x}_g < 0 \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_g \in (0, \bar{\tau}_\ell) \) and at least one of Assumptions A.1 and A.2 hold.

1. By Lemma 3, there exists \( \tilde{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\tilde{x}, \hat{x}_g) \) implies \( \hat{x}_g \in A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). By Lemma 6, there exists \( \tilde{x} < \hat{x}_g \) such that \( \hat{x}_\ell \in (\tilde{x}, \hat{x}_g) \) implies \( U_E^E(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \). Consider \( \hat{x}_\ell \in (\tilde{x}, \hat{x}_g) \). Then \( z_\ell = \hat{x}_\ell \in A(\alpha; \hat{x}_\ell) \) and \( y_\ell = \bar{y}_\ell \in A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Thus, \( g \)'s ex ante expected utility equals \( U_E^E(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). It follows that \( g \) strictly prefers \( \alpha = 0 \).

2. Assume \( \hat{x}_\ell \in (\hat{x}_g, x'') \), where \( x'' = 2\bar{\tau}_\ell - \hat{x}_g \). It suffices to show \( g \)'s ex ante expected utility strictly increases at \( \alpha = 0 \). There are two cases.

- **Case 1:** If \( \hat{x}_\ell < \bar{\tau}_\ell \), then \( g \)'s ex ante expected payoff equals \( U_E^E(\alpha; \hat{x}_\ell) \) for sufficiently small \( \alpha \). By Lemma 4, \( \frac{\partial \bar{U}_E^E(\alpha; \hat{x}_\ell)}{\partial \alpha} > 0 \). To complete this case, I argue that \( \bar{U}_E^E(\alpha; \hat{x}_\ell) \) increases for sufficiently small \( \alpha \). Under the maintained assumptions, \( \hat{x}_g \in (-\bar{\tau}(0; \hat{x}_\ell), x(0; \hat{x}_\ell)) \) and \( y_\ell = \bar{y}_\ell \in (\hat{x}_g, \bar{\tau}(0; \hat{x}_\ell)) \). Thus, \( \bar{v}_E(\alpha; \hat{x}_\ell) \) strictly decreases
for sufficiently small $\alpha_\ell$. Therefore $u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell))$ are strictly increasing for such $\alpha_\ell$. Proposition 6 implies $m_j^g(\alpha_\ell; \hat{x}_\ell)$ weakly decreases in $\alpha_\ell$ for all $j \in N^L_g \setminus \{\ell\}$. Thus, $u_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly increases over sufficiently small $\alpha_\ell$.

- **Case 2:** If $\hat{x}_\ell > \overline{x}_\ell$, then $\overline{x}(0; \hat{x}_\ell) = \overline{x}_\ell$. Thus, $g$’s ex ante expected payoff from $\alpha_\ell = 0$ is

$$
\rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\overline{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\overline{x}_\ell) \right) + \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \overline{E}^{LB}_j(0; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(0; \hat{x}_\ell) \right] u_g(-\overline{x}_\ell) \right. \\
+ \left[ \alpha_j \overline{E}^{UB}_j(0; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(0; \hat{x}_\ell) \right] u_g(\overline{x}_\ell) \\
+ \alpha_j \overline{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j) \\
- \left. I^j_g \alpha_j m_j^g(0; \hat{x}_\ell) \right). 
$$

Arguments analogous to Case 1 show (42) strictly increases in $\alpha_\ell$ at $\alpha_\ell = 0$.

3. Assume $\hat{x}_\ell \geq x''$, where $x''$ is defined as in Case 2 of Part 2. Then $z_\ell = y_\ell^\ell = \overline{x}(\alpha_\ell; \hat{x}_\ell) = \overline{x}_\ell$ for all $\alpha_\ell \in [0, 1]$ and $g$’s ex ante expected payoff is constant. □

I prove the analogue of Proposition 3 from the main text.

**Proposition A.3** Assume $\hat{x}_g \geq \overline{x}_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\overline{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_\ell \geq \overline{x}_\ell$, then $g$ is indifferent over $\alpha_\ell$.

2. If $\hat{x}_\ell \in [0, \overline{x}_\ell)$, then $\alpha_\ell = 0$ is not optimal.

A symmetric result holds if $\hat{x}_g \leq -\overline{x}_\ell$ and $\max\{\hat{x}_j, \hat{y}_j\} < \overline{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

**Proof.** Suppose $\hat{x}_g \geq \overline{x}_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\overline{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_\ell \geq \overline{x}_\ell$, then apply arguments analogous to Part 3 of Proposition A.2.

2. Assume $\hat{x}_\ell \in [0, \overline{x}_\ell)$. I show $g$’s ex ante expected utility strictly increases at $\alpha_\ell = 0$. 43
We have \( \hat{x}_\ell \in [0, \pi(0; \hat{x}_\ell)) \) and \( \hat{y}_\ell > \hat{x}_\ell \). Therefore \( 0 \leq z_\ell(0; \hat{x}_\ell) = \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \). Furthermore, no \( j \in N^L \) proposes \(-\pi(0; \hat{x}_\ell)\) because \( \min\{\hat{x}_j, \hat{y}_j\} > -\pi(0; \hat{x}_\ell) \). Thus, \( g \)'s ex ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right) + \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \tilde{E}_j^UB(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^UB(0; \hat{x}_\ell) \right] u_g(\pi(0; \hat{x}_\ell)) \right.
\]

\[
+ \alpha_j \left[ \tilde{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) - I_j^i m_j^i(0; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j) \right) . \quad (43)
\]

Three steps show (43) strictly increases at \( \alpha_\ell = 0 \).

- First, \( 0 \leq \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \) implies \( y_g^\ell(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Therefore \( u_g(y_g^\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly increases and \( u_\ell(y_g^\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly decreases. Because \( u \) is quadratic and \( \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell = \frac{x_g + \hat{x}_\ell}{2} < \hat{x}_g \), it follows that \( u_g(y_g^\ell(\alpha_\ell; \hat{x}_\ell)) \) increases weakly faster than \( u_\ell(y_g^\ell(\alpha_\ell; \hat{x}_\ell)) \) decreases. Therefore \( u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Furthermore, \( \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell < \hat{x}_g \) also implies \( u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0 \). It follows that \( \alpha_\ell \left[ u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \) weakly increases at \( \alpha_\ell = 0 \).

- Second, \( 0 \leq z_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \pi(0; \hat{x}_\ell) \) implies \( \pi(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \) by Lemma 2. Since \( \pi(0; \hat{x}_\ell) < \hat{x}_g \), it follows that \( u_g(\pi(0; \hat{x}_\ell)) \) increases at \( \alpha_\ell = 0 \).

- Third, Proposition 6 implies \( m_j^g(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N^L_g \). However, \( \hat{y}_j > \pi(0; \hat{x}_\ell) \) for all \( j \in N^L_g \) such that \( m_j^g(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \), which implies \( g \)'s lobbying surplus weakly increases in \( \alpha_\ell \) for any such \( j \in N^L_g \).

\[\square\]

**Willingness to Pay for Access**

The following results apply to the model in the main text. Define \( \theta = (\hat{x}, \rho, \alpha) \). Let \( U_g^E(\theta) \) be \( g \)'s ex ante expected utility. Additionally, let \( \pi_\alpha = \pi(\alpha; \hat{x}_\ell) \) denote the upper bound of \( A(\alpha; \hat{x}_\ell) \). Define \( \frac{\partial \pi_\alpha}{\partial \alpha} \bigg|_{\alpha=0} = \frac{\partial \pi_\alpha}{\partial \alpha} \bigg|_{\alpha=0} = \frac{\partial \pi_\alpha}{\partial \alpha} \bigg|_{\alpha=0} = \frac{\partial \pi_\alpha}{\partial \alpha} \bigg|_{\alpha=0} \).
To state Proposition 4, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, consider the baseline model, but replace $\ell$ with two legislators, $\ell_1$ and $\ell_2$, and replace $g$ with two groups, $g_1$ and $g_2$. To isolate differences in proposal power, assume $\hat{x}_{g_1} = \hat{x}_{g_2}$ and $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, but $\rho_{\ell_1} \neq \rho_{\ell_2}$. These modifications do not qualitatively change the equilibrium characterization. I use two identical groups to avoid complications arising if one group has access to two legislators, because the group accounts for how access to one legislator affects its lobby offer to the other.

**Proposition 4.** Consider the modified baseline model with: $\ell_1$ and $\ell_2$ such that $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, and $g_1$ and $g_2$ satsifying $\hat{x}_{g_1} = \hat{x}_{g_2}$. For all $\alpha \in [0, 1]$, $\rho_{\ell_2} > \rho_{\ell_1}$ implies

$$\frac{\partial U_{g_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha}.$$

**Proof.** Assume $\rho_{\ell_2} > \rho_{\ell_1}$ and fix $\alpha \in [0, 1]$. It suffices to show $\frac{\partial U_{g_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha}$ for $\alpha \in [0, 1]$.

Because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$ and $\hat{x}_{g_1} = \hat{x}_{g_2}$, we have $y_{g_1} = y_{g_2}$ and $z_{\ell_1} = z_{\ell_2}$. Thus, $m_{g_1} = m_{g_2}$.

For convenience, let $y = y_{g_1}$, $z = z_{\ell_1}$, and $m = m_{g_1}$. Assume $\frac{\partial U_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq 0$. There are five cases.

- **Case 1:** Suppose $z = \hat{x}_{\ell}$ and $y = \hat{y}$. Then,

$$\frac{\partial U_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} = \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_{\ell}) - u_{\ell_1}(\hat{x}_{\ell}) \right) - \frac{\partial \pi_{\alpha}}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\pi_{\alpha})}{\partial \pi_{\alpha}} - \rho_R \frac{\partial u_{g_1}(\pi_{\alpha})}{\partial \pi_{\alpha}} \right)
$$

$$= \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_{\ell}) - u_{\ell_1}(\hat{x}_{\ell}) \right)
$$

$$+ \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_{\ell})]}{\partial u_M(\pi_{\alpha})} \left[ 1 - \delta(\rho_L + \rho_R) \right] \left( \rho_L \frac{\partial u_{g_1}(-\pi_{\alpha})}{\partial \pi_{\alpha}} + \rho_R \frac{\partial u_{g_1}(\pi_{\alpha})}{\partial \pi_{\alpha}} \right)$$

$$\leq \rho_{\ell_2} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_{\ell}) - u_{\ell_1}(\hat{x}_{\ell}) \right)
$$

$$+ \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_{\ell})]}{\partial u_M(\pi_{\alpha})} \left[ 1 - \delta(\rho_L + \rho_R) \right] \left( \rho_L \frac{\partial u_{g_1}(-\pi_{\alpha})}{\partial \pi_{\alpha}} + \rho_R \frac{\partial u_{g_1}(\pi_{\alpha})}{\partial \pi_{\alpha}} \right)$$

(44)

(45)
where (44) follows from \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_2 [u_M(\bar{y}) - u_M(\bar{z}_\ell)]}{\delta \tau_a} \); (45) because (i) \( \rho_{\ell_2} > \rho_{\ell_1} \) and (ii) \( \frac{\partial U^E_g(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq 0 \) implies the bracketed expression in (44) is positive; and (46) because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2}, \hat{x}_{g_1} = \hat{x}_{g_2} \), and
\[
\frac{\partial \sigma}{\partial \alpha_2} = \frac{\delta \rho_2 [u_M(\bar{y}) - u_M(\bar{z}_\ell)]}{\delta \tau_a \delta \rho_2_{(1-\delta)(\rho_{\ell_2} + \rho_{\ell_1} + \rho_{\ell_2})}}.
\]

- **Case 2**: Suppose \( z = \bar{\alpha} \) and \( y = \hat{y} \). In this case, \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(\bar{\alpha}) - u_M(\bar{\alpha})]}{\delta \tau_a \delta \rho_2_{(1-\delta)(\rho_{\ell_2} + \rho_{\ell_1} + \rho_{\ell_2})}} \), Arguments analogous to Case 1 show
\[
\frac{\partial U^E_g(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_g(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}. \text{ The argument for } z = -\bar{\alpha} \text{ and } y = \hat{y} \text{ is symmetric.}
\]

- **Case 3**: Suppose \( z = \hat{x}_{\ell} \) and \( y = \bar{\alpha} \). In this case, \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_1 \tau_a}{\delta \tau_a \delta \rho_2_{(1-\delta)(\rho_{\ell_2} + \rho_{\ell_1} + \rho_{\ell_2})}} \), Arguments analogous to Case 1 show
\[
\frac{\partial U^E_g(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_g(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}. \text{ The argument for } z = \hat{x}_{\ell} \text{ and } y = -\bar{\alpha} \text{ is symmetric.}
\]

- **Case 4**: Suppose \( z = \bar{\alpha} \) and \( y = -\bar{\alpha} \). In this case, \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_1 \tau_a}{\delta \tau_a \delta \rho_2_{(1-\delta)(\rho_{\ell_2} + \rho_{\ell_1} + \rho_{\ell_2})}} \), Arguments analogous to Case 1 show
\[
\frac{\partial U^E_g(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_g(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}. \text{ The argument for } z = -\bar{\alpha} \text{ and } y = \bar{\alpha} \text{ is symmetric.}
\]

- **Case 5**: Suppose \( z = -\bar{\alpha} \) and \( y = \bar{\alpha} \). Then, \( \frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha = 0} = \frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha = \alpha} = 0 \). The argument for \( z = -\bar{\alpha} \) and \( y = -\bar{\alpha} \) is symmetric. 

\[ \square \]

**Proposition 5.** Assume minority-party agenda exclusion and \( \ell \) is majority-leaning.

If either (i) \( g \) is more centrist than \( \ell \), or (ii) \( g \) is majority-leaning and more extreme than \( \ell \), then \( \frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha = 0} \) weakly increases in \( |\hat{x}_g - \hat{x}_\ell| \).

**Proof.** Without loss of generality, assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \).

First, \( g \)'s ex ante expected utility for \( \alpha \in [0,1] \) is
\[
U^E_g(\theta) = \rho_\ell \left( \alpha [u_g(y) + u_\ell(y)] + (1 - \alpha) u_\ell(z_\ell) \right) + \rho_M u_g(0) + \rho_R u_g(\bar{\alpha}). \tag{47}
\]
Thus, \( g \)'s willingness to acquire access to \( \ell \) is
\[
\frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha=0} = \rho_\ell \left( u_g(y) - u_g(z_\ell) + u_\ell(y) - u_\ell(z_\ell) \right) + \rho_R \frac{\partial u_g(x_0)}{\partial \pi_0} \frac{\partial \pi_0}{\partial \alpha}.
\] (48)

The cross-partial with respect to \( \hat{x}_g \) satisfies
\[
\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial \hat{x}_g} \bigg|_{\alpha=0} = \rho_\ell \left( \left( \frac{\partial u_g(y)}{\partial y} + \frac{\partial u_\ell(y)}{\partial y} \right) \frac{\partial y}{\partial \hat{x}_g} + \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(z_\ell)}{\partial \hat{x}_g} \right)
+ \rho_R \left( \frac{\partial^2 u_g(x_0)}{\partial \pi_0^2} \frac{\partial \pi_0}{\partial \alpha} + \frac{\partial u_g(x_0)}{\partial \pi_0} \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} + \frac{\partial u_\ell(x_0)}{\partial \pi_0} \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} \right)
\] (49)
\[
= \rho_\ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(z_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial^2 u_g(x_0)}{\partial \pi_0^2} \frac{\partial \pi_0}{\partial \alpha} + \frac{\partial u_g(x_0)}{\partial \pi_0} + \frac{\partial u_\ell(x_0)}{\partial \pi_0} \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} \right),
\] (50)

where (50) follows because either (i) \( y = \pi_0 \), which implies \( \frac{\partial y}{\partial \hat{x}_g} = 0 \), or (ii) \( y = \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \), which implies \( \frac{\partial u_g(y)}{\partial y} = -\frac{\partial u_\ell(y)}{\partial y} \).

**Part (i)** Consider \( \hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell] \). There are two cases.

**Case 1:** Suppose \( \hat{x}_\ell \geq \pi_0 \). Then \( z_\pi = \pi_0 \). Since \( \hat{x}_g \geq -\hat{x}_\ell \), we have \( \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \geq 0 \). There are two subcases.

- First, consider \( \hat{x}_g \geq 2\pi_0 - \hat{x}_\ell \). Then \( y = z_\ell = \pi_0 \). For \( \alpha \in [0, 1] \), if \( y = z_\ell = \pi_\alpha \), then \( \pi_\alpha \) solves
\[
0 = (1 - \delta) u_M(q) + \delta \rho_M u_M(0) - [1 - \delta(\rho_R + \rho_\ell)] u_M(\pi_\alpha).
\] (51)

Applying the implicit function theorem to (51) yields \( \frac{\partial \pi_\alpha}{\partial \alpha} = 0 \) and thus \( \frac{\partial \pi_\alpha}{\partial \alpha} = 0 \). Therefore \( \frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha=0} = 0 \) over \( \hat{x}_g \in [2\pi_0 - \hat{x}_\ell, \hat{x}_\ell] \).

- Second, consider \( \hat{x}_g < 2\pi_0 - \hat{x}_\ell \). Then \( y = \hat{y} \). For \( \alpha \in [0, 1] \), if \( y = \hat{y} \) and \( z_\ell = \pi_\alpha \), then \( \pi_\alpha \) solves
\[
0 = (1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \alpha \rho_\ell u_M(\hat{y}) \right) - \left( 1 - \delta(\rho_R + (1 - \alpha)\rho_\ell) \right) u_M(\pi_\alpha).
\] (52)
Applying the implicit function theorem to (52) yields

\[
\frac{\partial T_x}{\partial \alpha} = \delta \rho_x \frac{\partial u_M(\bar{y}) - u_M(\bar{T_x})}{\partial \bar{T_x}} \quad (53)
\]

\[
\frac{\partial T_x}{\partial \hat{x}_g} = \alpha \delta \rho_x \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{T_x}}{\partial \hat{x}_g} \quad (54)
\]

and

\[
\frac{\partial^2 T_x}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho_x}{(1 - \delta [\rho_R + (1 - \alpha)\rho_x])} \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{T_x}}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\bar{T_x})}{\partial \bar{T_x}^2} \frac{\partial \bar{T_x}}{\partial \hat{x}_g} \right) \left( \frac{\partial u_M(\bar{T_x})}{\partial \bar{T_x}} \right)^{-1} \quad (55)
\]

Inspecting (54) reveals \( \frac{\partial T_x}{\partial \hat{x}_g} = 0 \). Thus,

\[
\frac{\partial^2 T_x}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho_x \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{T_x}}{\partial \hat{x}_g}}{(1 - \delta [\rho_R + (1 - \alpha)\rho_x])} > 0, \quad (56)
\]

which follows because (i) \( \frac{\partial \bar{y}}{\partial \hat{x}_g} > 0 \) and (ii) \( 0 < \bar{y} < T_x \) implies \( 0 > \frac{\partial u_M(\bar{y})}{\partial \bar{y}} > \frac{\partial u_M(\bar{T_x})}{\partial \bar{T_x}} \). Thus,

\[
\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial \hat{x}_g} \bigg|_{\alpha = 0} = \rho_x \left( \frac{\partial u_g(\bar{y})}{\partial \bar{y}} - \frac{\partial u_g(T_x)}{\partial \bar{y}} \right) + \rho_R \frac{\partial u_g(T_x)}{\partial \bar{T_x}} \frac{\partial^2 T_x}{\partial \alpha \partial \hat{x}_g} \quad (57)
\]

\[
< \rho_x \left( \frac{\partial u_g(\bar{y})}{\partial \bar{y}} - \frac{\partial u_g(T_x)}{\partial \bar{y}} \right) \quad (58)
\]

\[
< 0, \quad (59)
\]

where (58) follows from (56) and \( \frac{\partial u_g(T_x)}{\partial \bar{T_x}} < 0 \); and (59) because \( \hat{x}_g < \bar{y} < T_x \) implies \( \frac{\partial u_g(\bar{y})}{\partial \bar{y}} < \frac{\partial u_g(T_x)}{\partial \bar{T_x}} \).

**Case 2:** Suppose \( \hat{x}_t < T_x \). Then \( z_t = \hat{x}_t \). Furthermore, \( \hat{x}_g \in [-\hat{x}_t, \hat{x}_t] \) implies \( y = \bar{y} \geq 0 \). For \( \alpha \in [0, 1] \), if \( y = \bar{y} \) and \( z_t = \hat{x}_t \), then \( T_x \) solves

\[
u_M(T_x) = \frac{(1 - \delta)u_M(q) + \delta \left( \rho_M u_M(0) + \rho_t \left( au_M(\bar{y}) + (1 - \alpha)u_M(\hat{x}_t) \right) \right)}{1 - \delta \rho_R}. \quad (60)
\]
Applying the implicit function theorem yields

\[
\frac{\partial x_\alpha}{\partial \alpha} = \frac{\delta \rho_x [u_M(y) - u_M(x_\ell)]}{(1 - \delta \rho_R) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}},
\]

(61)

\[
\frac{\partial x_\alpha}{\partial x_g} = \frac{\alpha \delta \rho_x \frac{\partial u_M(y)}{\partial y} \frac{\partial \hat{y}}{\partial x_g}}{(1 - \delta \rho_R) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}},
\]

(62)

and

\[
\frac{\partial^2 x_\alpha}{\partial \alpha \partial x_g} = \left( \frac{\delta \rho_x \frac{\partial u_M(y)}{\partial y} \frac{\partial \hat{y}}{\partial x_g} - \frac{\partial^2 u_M(x_\alpha)}{\partial x_g^2} \frac{\partial \hat{y}}{\partial x_g} \frac{\partial \hat{y}}{\partial \alpha} \right) \left( \frac{\partial u_M(x_\alpha)}{\partial x_\alpha} \right)^{-1}.
\]

Inspecting (62) reveals \(\frac{\partial x_\alpha}{\partial x_g} = 0\), which implies

\[
\frac{\partial^2 x_\alpha}{\partial \alpha \partial x_g} = \frac{\delta \rho_x \frac{\partial u_M(y)}{\partial y} \frac{\partial \hat{y}}{\partial x_g}}{(1 - \delta \rho_R) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}} > 0.
\]

(63)

Because \(0 \leq \hat{y} < x_\ell\), a inequalities analogous to (57)-(59) imply \(\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} < 0\).

In both cases, g’s willingness to acquire access weakly decreases in \(|\hat{x}_g - x_\ell|\).

**Part (ii)** Assume \(\hat{x}_g \geq x_\ell\). There are two cases.

**Case 1:** Consider \(\hat{x}_g \geq x_\ell\). Then \(y = z_\ell = x_0\) at \(\alpha = 0\), implying \(\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} = 0\).

**Case 2:** Consider \(\hat{x}_g < x_\ell\). Then \(z_\ell = \hat{x}_g\). There are three subcases.

- First, assume \(\hat{x}_g \in [\hat{x}_g, x_0]\). Then \(y = \hat{y}\). I show \(\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} \geq 0\) implies \(\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} > 0\). Since \(y = \hat{y}\) and \(z_\ell = \hat{x}_g\), case 2 of Part (ii) implies \(\frac{\partial x_\alpha}{\partial x_g} = 0\), and \(\frac{\partial^2 x_\alpha}{\partial \alpha \partial x_g} = 0\) is given by (61), \(\frac{\partial x_\alpha}{\partial x_g} = 0\), and \(\frac{\partial^2 x_\alpha}{\partial \alpha \partial x_g} = 0\) is (63). Therefore

\[
\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} = \rho_{\ell} \left( \frac{\partial u_g(y)}{\partial x_g} \frac{\partial \hat{y}}{\partial x_g} \right) + \rho_{R} \left( \frac{\partial u_g(x_0)}{\partial x_0} \frac{\partial^2 x_\alpha}{\partial \alpha \partial x_g} \right)
\]

\[
= \rho_{\ell} \left( \frac{\partial u_g(y)}{\partial x_g} \frac{\partial \hat{y}}{\partial x_g} \right) + \rho_{R} \left( \frac{\partial u_g(x_0)}{\partial x_0} \delta \rho_x \frac{\partial u_M(y)}{\partial y} \frac{\partial \hat{y}}{\partial x_g} \frac{\partial \hat{y}}{\partial \alpha} \right) \left( \frac{\partial u_M(x_\alpha)}{\partial x_\alpha} \right)^{-1}
\]

(64)

\[
\geq \rho_{\ell} \left( \frac{\partial u_g(y)}{\partial x_g} \frac{\partial \hat{y}}{\partial x_g} \right)
\]

(65)

\[
- \rho_{R} \left[ \rho_{\ell} \left( u_g(y) - u_g(x_\ell) + u_e(y) \right) \right] \left( \frac{\delta \rho_x \frac{\partial u_M(y)}{\partial y} \frac{\partial \hat{y}}{\partial x_g}}{(1 - \delta \rho_R) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}} \right)
\]

(64)
\[ = \rho \ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) - \rho \ell \frac{\partial u_M(y)}{\partial y} \frac{\partial y}{\partial \hat{x}_g} \left( \frac{u_g(y) - u_g(\hat{x}_\ell)}{u_M(y) - u_M(\hat{x}_\ell)} \right) \]  
\[ = \frac{5\rho \ell}{4} (\hat{x}_g - \hat{x}_\ell) \]  
\[ > 0, \]  
(66)

where (64) follows from using (63) to substitute for \( \partial^2 \tau_0 / \partial \alpha \partial \hat{x}_g \); (65) because (i) \( \partial^2 \tau_0 / \partial \alpha \partial \hat{x}_g > 0 \) and (ii) \( \partial U^E(\theta) / \partial \alpha \big|_{\alpha = 0} \geq 0 \) and \( \partial \tau_0 / \partial \alpha > 0 \) together imply \( \partial u_\theta(\tau_0) / \partial \tau_0 \geq -\rho \ell |u_\theta(\hat{y}) - u_\theta(\hat{x}_\ell) + u(\hat{y})| / \rho R \partial \tau_0 / \partial \alpha \); and (66) from using (61) to substitute for \( \partial \tau_0 / \partial \alpha \) and simplifying.

- Second, assume \( \hat{x}_g \in [\bar{x}_0, 2\bar{x}_0 - \hat{x}_\ell] \). Then \( y = \hat{y} \). Thus, \( \partial \tau_0 / \partial \alpha \), \( \partial \tau_0 / \partial \hat{x}_g \), and \( \partial^2 \tau_0 / \partial \alpha \partial \hat{x}_g \) are defined as in subcase 1. Therefore

\[ \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} \big|_{\alpha = 0} = \rho \ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g} \right) \]
\[ \geq \rho \ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) \]
\[ > 0, \]  
(69)

where (69) follows because (i) \( \partial^2 \tau_0 / \partial \alpha \partial \hat{x}_g > 0 \) and (ii) \( \hat{x}_g \geq \bar{x}_0 \) implies \( \partial u_\theta(\tau_0) / \partial \tau_0 \geq 0 \); and (70) because \( \hat{x}_\ell < \hat{y} < \hat{x}_g \) implies \( \partial u_\theta(\hat{x}_\ell) / \partial \hat{x}_g < \partial u_\theta(\hat{y}) / \partial \hat{x}_g \).

- Third, assume \( \hat{x}_g \geq 2\bar{x}_0 - \hat{x}_\ell \). Then \( y = \bar{x}_0 \). For \( \alpha \in [0, 1] \), \( y = \bar{x}_\alpha \) and \( z_\ell = \hat{x}_\ell \) imply \( \tau_\alpha \) solves

\[ = (1 - \delta) u_M(q) + \delta \left( \rho M u_M(0) + \rho \ell (1 - \alpha) u_M(\hat{x}_\ell) \right) - \left( 1 - \delta [\rho R + \alpha \rho \ell] \right) u_M(\bar{x}_\alpha). \]

(71)

Applying the implicit function theorem to (71) yields

\[ \frac{\partial \tau_\alpha}{\partial \alpha} = \frac{\delta \rho \ell |u_M(\bar{x}_\alpha) - u_M(\hat{x}_\ell)|}{(1 - \delta [\rho R + \alpha \rho \ell]) \partial u_M(\bar{x}_\alpha) / \partial \tau_\alpha}, \]
\[ \frac{\partial \tau_\ell}{\partial \hat{x}_g} = 0, \]  
and \( \partial^2 \tau_\alpha / \partial \alpha \partial \hat{x}_g = 0 \). Thus,

\[ \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} \big|_{\alpha = 0} = \rho \ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g} \right) \]
\[ = \rho \ell \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) > 0. \]  
(72)
Lemma 7. Assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \). There exists \( x' > \hat{x}_\ell \) such that \( \hat{x}_g \in (\hat{x}_\ell, x') \) implies \( \alpha = 0 \) is optimal. An symmetric result holds if \( \rho_R = 0 \) and \( \hat{x}_\ell \leq 0 \).

Proof. Without loss of generality, assume \( \rho_L = 0 \) and \( 0 \leq \hat{x}_\ell \).

If \( \hat{x}_\ell \geq \overline{x}_0 \), then \( \hat{x}_g > \hat{x}_\ell \) implies \( g \) is indifferent, so \( \alpha = 0 \) is optimal. As in the proof of Proposition 5, \( \frac{\partial U^E_g( \theta )}{\partial \alpha} \big|_{\alpha = 0} = 0 \) for all \( \alpha \in [0, 1] \). Setting \( x' = \infty \) delivers the result.

Next, suppose \( \hat{x}_\ell < \overline{x}_0 \). Lemma 5 implies that it suffices to show \( \frac{\partial U^E_g( \theta )}{\partial \alpha} \big|_{\alpha = 0} \leq 0 \) and \( \frac{\partial^2 U^E_g( \theta )}{\partial \alpha^2} < 0 \) for \( \hat{x}_g \) sufficiently close to \( \hat{x}_\ell \).

An argument analogous to Part 1 of Proposition 2 shows existence of \( x' > \hat{x}_\ell \) such that \( \frac{\partial U^E_g( \theta )}{\partial \alpha} \big|_{\alpha = 0} \leq 0 \) if \( \hat{x}_g \in [\hat{x}_\ell, x'] \). Also, \( x' \in (\hat{x}_\ell, \overline{x}_0) \) because \( \rho_L = 0 \) implies \( \frac{\partial U^E_g( \theta )}{\partial \alpha} \big|_{\alpha = 0} > 0 \) for \( \hat{x}_g \geq \overline{x}_0 \).

Consider \( \hat{x}_g \in [\hat{x}_\ell, x'] \). I show \( \frac{\partial^2 U^E_g( \theta )}{\partial \alpha^2} < 0 \). Applying the implicit function theorem to (60) yields

\[
\frac{\partial^2 \overline{x}_\alpha}{\partial \alpha^2} = -\frac{\frac{\partial^2 u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} \left( \frac{\partial \overline{x}_\alpha}{\partial \alpha} \right)^2}{\frac{\partial u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha}} < 0
\]  

(73)

because \( \frac{\partial^2 u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} < 0 \) and \( \hat{x}_g \in [\hat{x}_\ell, x'] \) implies \( \frac{\partial u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha} < 0 \).

We have \( y = \hat{y} \) and \( z_\ell = \hat{x}_\ell \), so

\[
\frac{\partial^2 U^E_g( \theta )}{\partial \alpha^2} = \rho_R \left( \frac{\partial^2 u_g( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} \left( \frac{\partial \overline{x}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_g( \overline{x}_\alpha )}{\partial \overline{x}_\alpha} \frac{\partial^2 \overline{x}_\alpha}{\partial \alpha^2} \right)
\]

(74)

\[
\leq \rho_R \left( \frac{\partial^2 u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} \left( \frac{\partial \overline{x}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha} \frac{\partial^2 \overline{x}_\alpha}{\partial \alpha^2} \right)
\]

(75)

\[
= 0,
\]

(76)

where (74) follows from \( \frac{\partial^2 u_g( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} = \frac{\partial^2 u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha^2} \); (75) because \( \frac{\partial^2 \overline{x}_\alpha}{\partial \alpha^2} < 0 \) and \( 0 < \hat{x}_g < \overline{x}_\alpha \) implies \( \frac{\partial u_M( \overline{x}_\alpha )}{\partial \overline{x}_\alpha} < \frac{\partial u_g( \overline{x}_\alpha )}{\partial \overline{x}_\alpha} < 0 \); and (76) from simplifying using (73). Thus, \( \frac{\partial U^E_g( \theta )}{\partial \alpha} \big|_{\alpha = 0} \leq 0 \) for all \( \alpha \in [0, 1] \). Proposition 5 delivers the result.

\[\square\]
Appendix B

A stationary strategy profile $\sigma = (\lambda, \pi, \varphi, \nu)$ is a stationary legislative lobbying equilibrium if it satisfies four conditions. First, for all $g \in NG$ and $\ell \in NL_g$, $\lambda_g$ places probability one on

$$\arg \max_{(y,m)} \nu \sigma(y) u_{\ell}(y) + [1 - \nu \sigma(y)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] - m$$

s.t. $\nu \sigma(y) u_{\ell}(y) + [1 - \nu \sigma(y)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] + m$

$$\geq \int_{X} \left[ \nu \sigma(x) u_{\ell}(x) + [1 - \nu \sigma(x)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_{\ell}(dx). \quad (77)$$

Second, for all $\ell \in NL$ and $(y, m) \in W$,

$$\nu \sigma(y) u_{\ell}(y) + [1 - \nu \sigma(y)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] + m$$

$$> \int_{X} \left[ \nu \sigma(x) u_{\ell}(x) + [1 - \nu \sigma(x)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_{\ell}(dx). \quad (78)$$

implies $\varphi_{\ell}(y, m) = 1$ and the opposite strict inequality implies $\varphi_{\ell}(y, m) = 0$. Third, for all $\ell \in NL$,

$$\pi_{\ell} \left( \arg \max_{x \in X} \nu \sigma(x) u_{\ell}(x) + [1 - \nu \sigma(x)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] \right) = 1. \quad (79)$$

Finally, for all $i \in NV$ and $x \in X$, $u_i(x) > (1 - \delta) u_i(q) + \delta V_i(\sigma)$ implies $\nu_i(x) = 1$ and the opposite strict inequality implies implies $\nu_i(x) = 0$.\footnote{Thus, voting strategies are stage-undominated (Baron and Kalai, 1993; Banks and Duggan, 2006a).}

Lemma B.1 shows surplus lobby payments never happen in equilibrium.

**Lemma B.1.** In every stationary legislative lobbying equilibrium, for all $\ell \in NL$ every $(y, m) \in \text{supp}(\lambda_g')$ satisfies

$$\nu \sigma(y) u_{\ell}(y) + [1 - \nu \sigma(y)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] + m$$

$$= \int_{X} \left[ \nu \sigma(x) u_{\ell}(x) + [1 - \nu \sigma(x)] [(1 - \delta) u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_{\ell}(dx). \quad (80)$$
The proof of Lemma 80 is straightforward and omitted.

From (11), recall $\xi_\ell(\alpha; \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_w [1 - \varphi_\ell(y, m)] \lambda_\ell^\ell(dw)$. Define

$$\hat{\chi}(X') = \sum_{\ell \in N^L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_{X'} \pi_\ell(dx) + \alpha_\ell \int_{X' \times R_+} \varphi_\ell(y, m) \nu_\ell^\ell(y, m) \lambda_\ell^\ell(dw) \right),$$

the probability some $x \in X' \subseteq X$ is passed in a given period under $\sigma$. Next, define

$$\tilde{\chi} = \sum_{\ell \in N^L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_{X'} [1 - \pi_\ell(dx)] \lambda_\ell^\ell(dw) + \alpha_\ell \int_{X' \times R_+} \varphi_\ell(y, m) \nu_\ell^\ell(y, m) \lambda_\ell^\ell(dw) \right),$$

the probability of a failed proposal in a given period under $\sigma$.

Following Banks and Duggan (2006a), each player’s continuation value can be expressed as a function of a common lottery over policy, denoted $\chi^\sigma$. Using (81) and (82), define $\chi^\sigma$ so that for all measurable $X' \subseteq X$: (i) if $q \not\in X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X')}{1 - \delta \tilde{\chi}}$, and (ii) if $q \in X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X') + (1 - \delta) \tilde{\chi}}{1 - \delta \tilde{\chi}}$.

Set $V_{\text{den}}(\sigma) = 1 - \delta \tilde{\chi}$ and define

$$V_{\text{num}}^i(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_{X} \left[ \pi_\ell(x) u_i(x) + [1 - \pi_\ell(x)] \right] (1 - \delta) u_i(q) \right) \pi_\ell(dx)$$

$$+ \alpha_\ell \int_{W} \varphi(y, m) \left[ \nu_\ell(y, m) u_i(x) + [1 - \nu_\ell(y, m)] \right] (1 - \delta) u_i(q) \lambda_\ell^\ell(dw).$$

For each $i \in N^V$, $i$’s continuation value defined in (12) satisfies $V_i(\sigma) = \frac{V_{\text{num}}^i(\sigma)}{V_{\text{den}}(\sigma)}$. Then we can express $V_i(\sigma)$ as a lottery over policy, $V_i(\sigma) = \int_X u_i(x) \chi^\sigma(dx)$.

The policy lottery $\chi^\sigma$ is common to all players, but committee members may receive payment and interest groups may make payments. Define

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \int_{W} m \varphi_\ell(y, m) \lambda_\ell^\ell(dw),$$

which is $\ell$’s expected lobby payment in each period until passage. For $\ell \in N^L$, re-arranging (13) yields

$$\tilde{V}_\ell(\sigma) = \frac{V_{\ell_{\text{num}}}(\sigma) + \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}$$

53
\[ = \int_X u_\ell(x) \chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \quad (84) \]

Similarly, for \( g \in N^G \) rearranging (14) yields

\[ \hat{V}_g(\sigma) = \frac{V_{\text{num}}^g(\sigma) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \]
\[ = \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N^L_g} \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \quad (85) \]

Finally, define

\[ \tilde{U}_\ell(\sigma) = \int_X \left[ \nu_{\sigma}(x) u_\ell(x) + \left(1 - \varphi_{\sigma}(x)\right) \left(1 - \delta\right) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \pi_\ell(dx), \quad (86) \]

which is \( \ell \)'s expected dynamic payoff under \( \sigma \) conditional on being recognized as the proposer and rejecting \( g_\ell \)'s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium \( \sigma \) such that \( \chi^\sigma \) is degenerate on \( q \).

**Proof.** Let \( \sigma \) denote an equilibrium. To show a contradiction, assume \( \chi^\sigma(q) = 1 \). Thus, \( V_M(\sigma) = u_M(q) \), which implies \( u_M(q) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma) \) and therefore \( q \in A(\sigma) \). Without loss of generality, assume \( q > 0 \).

By assumption, there exists \( \ell \in N^L \) such that \( \hat{x}_\ell < q \) and at least one of \( \hat{x}_{g_\ell} \leq q \) or \( \alpha_\ell < 1 \) holds. If \( \alpha_\ell < 0 \), then it is straightforward to show that \( \ell \) must have a profitable deviation, a contradiction.

For the other case, suppose \( \hat{x}_\ell < q, \hat{x}_{g_\ell} \leq q \), and \( \alpha_\ell = 1 \). Note that \( u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) is \( g_\ell \)'s expected dynamic payoff from any offer \((y,m)\) such that \( \nu_{\sigma}(y) = 1 \), \( \varphi_{\ell}(y,m) = 1 \), and \( \ell \) is indifferent between accepting and rejecting. We have \( \hat{y}_\ell = \arg \max_{y \in X} u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) and \( \hat{y}_\ell < q \). Strict concavity and continuity imply existence of \( \varepsilon > 0 \) and \( y^\varepsilon < q \) such that \( \nu_{\sigma}(y^\varepsilon) = 1 \), \( \varphi_{\ell}(y^\varepsilon,\tilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1 \), and

\[
\begin{align*}
&u_{g_\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \tilde{U}_\ell(\sigma) - \varepsilon > u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) \\
&\geq u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) - \delta \left( \sum_{j \in N^L_g} \frac{\hat{m}_j(\sigma)}{V_{\text{den}}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \right),
\end{align*}
\]

(87)
where (88) follows from \( \sum_{j \in N_L} \frac{m_j(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{m_x(\sigma)}{V_{\text{den}}(\sigma)} \). The RHS of (87) is weakly greater than \( g_\ell \)'s expected payoff from lobbying \( \ell \) to \( q \) if \( \nu_\sigma(q) = 1 \); and (88) is weakly greater than \( g_\ell \)'s expected payoff from lobbying \( \ell \) to any \( y' \) such that \( \nu_\sigma(y') = 0 \). Thus, \( g_\ell \) must have a profitable deviation, a contradiction. \( \square \)

**Lemma B.3.** Let \( \sigma \) denote a stationary legislative lobbying equilibrium. For all \( \ell \in N_L \) there exists \( (y, m) \in X \times \mathbb{R}_+ \) such that \( \nu_\sigma(y) = 1 \) and \( g_\ell \) strictly prefers \( (y, m) \) to any \( (y', m') \) such that \( \nu_\sigma(y') = 0 \).

**Proof.** Fix an equilibrium \( \sigma \). Let \( \chi^q \) denote a probability distribution degenerate on \( q \). Define the continuation distribution following rejection under \( \sigma \) as \( \chi = (1 - \delta)\chi^q + \delta\chi^\sigma \), which is non-degenerate because \( \delta \in (0, 1) \) and \( \chi^\sigma(q) < 1 \) by Lemma B.2.

For every player \( k \in N \), the expected dynamic policy payoff from a rejected policy proposal satisfies

\[
(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).
\]

Let \( x^\sigma \) denote the mean of \( \chi \). Since \( u \) is strictly concave and \( \chi \) is non-degenerate, Jensen’s Inequality implies

\[
u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma).
\]

(89)

Consider \( \ell \in N_L \). First, assume \( \phi_\ell(y, m) = 1 \) whenever \( \ell \) is indifferent. The condition for \( g_\ell \) to strictly prefer \( (y, m) \) such that \( \nu_\sigma(y) = 1 \), rather than \( (y', m') \) such that \( \nu_\sigma(y') = 0 \), is

\[
u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\sigma).
\]

Equivalently,

\[
u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma).
\]

(90)

Notice that

\[
\tilde{V}_{g_\ell}(\sigma) + \tilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{e' \in N_L} \hat{m}_{e'}(\sigma) V_{\text{den}}(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}
\]

(91)
\[
V_{g\ell}(\sigma) - \frac{\hat{m}_{\ell}(\sigma)}{V^{den}(\sigma)} = V_{g\ell}(\sigma) + \frac{\hat{m}_{\ell}(\sigma)}{V^{den}(\sigma)} \tag{92}
\]
\[
= V_{g\ell}(\sigma) + V_{\ell}(\sigma), \tag{93}
\]

where (91) follows from substituting for \(\hat{V}_{\ell}(\sigma)\) and \(\hat{V}_{g}(\sigma)\) using (84) and (85); and (92) from \(\sum_{\ell \in N^L_h} \frac{\hat{m}_{\ell}(\sigma)}{V^{den}(\sigma)} \geq \frac{\hat{m}_{\ell}(\sigma)}{V^{den}(\sigma)}\).

By (89), \(\mathbf{v}_{\sigma}(x^\sigma) = 1\) follows because \(u_M(x^\sigma) > (1-\delta)u_M(q) + \delta V_M(\sigma)\). Furthermore, (89) implies \(u_{g\ell}(x^\sigma) > (1-\delta)u_{g\ell}(q) + \delta V_{g\ell}(\sigma)\) and \(u_{\ell}(x^\sigma) > (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma)\). Thus, (93) implies that (90) holds because

\[
u_{g\ell}(x^\sigma) + u_{\ell}(x^\sigma) > (1-\delta)u_{g\ell}(q) + \delta V_{g\ell}(\sigma) + (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) \\
\geq (1-\delta)u_{g\ell}(q) + \delta \hat{V}_{g\ell}(\sigma) + (1-\delta)u_{\ell}(q) + \delta \hat{V}_{\ell}(\sigma),
\]

Next, assume \(\varphi_{\ell}(x^\sigma, m) < 1\) for \(m\) such that \(\ell\) is indifferent between accepting \((x^\sigma, m)\) and rejecting. For sufficiently small \(\varepsilon > 0\), \(\varphi_{\ell}(x^\sigma, m+\varepsilon) = 1\) and the preceding argument implies \(g_{\ell}\) strictly prefers \((x^\sigma, m+\varepsilon)\) over any \((y', m')\) such that \(\mathbf{v}_{\sigma}(y') = 0\).

**Lemma B.4.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

**Proof.** Let \(\sigma\) be an equilibrium. By Duggan (2014), \(M\) is decisive. Quadratic utility and \(\hat{x}_M = 0 \neq q\) together imply \(A(\sigma) = \{x \in X \mid u_M(x) \geq (1-\delta)u_M(q) + \delta V_M(\sigma)\}\) is a closed, non-empty interval symmetric about 0. Let \(A(\sigma) = [-\overline{x}(\sigma), \overline{x}(\sigma)]\). Then \(x \in (-\overline{x}(\sigma), \overline{x}(\sigma))\) implies \(\mathbf{v}_{\sigma}(x) = 1\).

Fix \(\ell \in N^L\). By Lemma B.2, \(\chi^\sigma(q) < 1\). Lemma B.3 implies existence of \((y, m) \in W\) such that \(\mathbf{v}_{\sigma}(y) = 1\) and \(g_{\ell}\) strictly prefers \((y, m)\) over all \((y', m')\) with \(\mathbf{v}_{\sigma}(y') = 0\). Thus, \(y \in A(\sigma)\) for all \((y, m) \in \text{supp}(\lambda_{g_{\ell}})\). Without loss of generality, assume \(\mathbf{v}_{\sigma}(-\overline{x}(\sigma)) < 1\). It suffices to check two cases.

- **Case 1:** If \(\hat{x}_\ell \leq -\overline{x}(\sigma)\) and \(u_{\ell}(-\overline{x}(\sigma)) > (1-\delta)u_{\ell}(q) + \delta \hat{V}_{\ell}(\sigma)\), then \(x \in A(\sigma)\) for all \(x \in \text{supp}(\pi_{\ell})\). Because \(u_{\ell}\) is strictly concave and continuous, and \(\mathbf{v}_{\sigma}(-\overline{x}(\sigma)) < 1\), there exists \(\varepsilon > 0\) such that \(\ell\) has a profitable deviation to \(-\overline{x}(\sigma) + \varepsilon\), a contradiction.

- **Case 2:** Assume \(\hat{y}_{\ell} \leq -\overline{x}(\sigma)\). Continuity, Lemma B.3, and \(\mathbf{v}_{\sigma}(-\overline{x}(\sigma)) < 1\) imply existence of \(\varepsilon, \varepsilon' > 0\) such that \(g_{\ell}\) has a profitable deviation to \((y', m') = (-\overline{x}(\sigma) + \varepsilon, \hat{U}_{\ell}(\sigma) - u_{\ell}(-\overline{x}(\sigma) + \varepsilon) + \varepsilon')\), a contradiction.

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It follows that either $\sigma$ must involve deferential voting, or $\sigma$ is equivalent in outcome distribution to an equilibrium with deferential voting. \hfill \Box

**Lemma B.5.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

**Proof.** Let $\sigma$ denote an equilibrium. By Lemma B.4, we can assume $\pi_{\sigma}(x) = 1$ iff $x \in A(\sigma)$. Fix $\ell \in N^L$ and define $y_{\ell}^* = \arg\max_{y \in A(\sigma)} u_{g_{\ell}}(y) + u_{\ell}(y) - \tilde{U}_{\ell}(\sigma)$, which is uniquely defined, and $m_{\ell}^* = \tilde{U}_{\ell}(\sigma) - u_{\ell}(y_{\ell}^*)$.

By Lemma B.2, $\chi^\sigma(q) < 1$. For sufficiently small $\varepsilon > 0$, Lemma B.3 implies $g$ strictly prefers $(y_{\ell}^*, m_{\ell}^* + \varepsilon)$ over every $(y', m')$ such that $y' \notin A(\sigma)$. Thus, if $\pi_{\ell}$ is not degenerate on $y_{\ell}^*$ and $\varphi_{\ell}(y_{\ell}^*, m_{\ell}^*) < 1$, then there exists $\varepsilon > 0$ such that $g_{\ell}$ has a profitable deviation to $(y_{\ell}^*, m_{\ell}^* + \varepsilon)$, a contradiction. Thus, $\sigma$ must satisfy either (i) $\pi_{\ell}(y_{\ell}^*) = 1$, or (ii) $\lambda_{g_{\ell}}(y_{\ell}^*, m_{\ell}^*) = 1$ and $\varphi_{\ell}(y_{\ell}^*, m_{\ell}^*) = 1$, as desired. \hfill \Box

A strategy profile $\sigma$ is no-delay if $\pi_{\sigma}(x) = 1$ for all $x \in \text{supp}(\pi_{\ell})$ and $\pi_{\sigma}(y) = 1$ for all $(y, m) \in \text{supp}(\lambda_{g_{\ell}}^\ell)$.

**Lemma B.6.** Every stationary legislative lobbying equilibrium is no-delay.

**Proof.** Fix an equilibrium $\sigma$. By Lemma B.2, $\chi^\sigma(q) < 1$. Thus, Lemma B.3 implies $g$ strictly prefers some $(y, m) \in W$ such that $\pi_{\sigma}(y) = 1$. Lemma B.4 implies we can assume $\pi_{\sigma}(x) = 1$ iff $x \in A(\sigma)$. Lemma B.5 implies we can assume all $\ell \in N^L$ use deferential acceptance strategies.

For each $\ell \in N^L$, the preceding observations and Lemma B.1 imply $\lambda_{g_{\ell}}^\ell$ puts probability one on $(y^*, m^*)$ such that $y^* = \arg\max_{y \in A(\sigma)} u_{g_{\ell}}(y) + u_{\ell}(y) - u_{\ell}(z_{\ell}; \sigma)$, which is unique. Lemmas B.4 and B.5 imply we can assume $\pi_{\sigma}(y^*) = 1$ and $\varphi_{\ell}(y^*, m^*) = 1$.

It remains to verify that $z_{\ell} \notin A(\sigma)$ cannot be optimal for any $\ell \in N^L$. To show a contradiction, assume proposing $z_{\ell} \notin A(\sigma)$ is optimal for some $\ell \in N^L$. Let $z^* = \arg\max_{x \in A(\sigma)} u_{\ell}(x)$. There are two steps. Step 1 establishes useful properties of $\ell$’s preferences over lotteries. Step 2 shows a contradiction.

**Step 1:** Recall the continuation lottery induced by $\sigma$, denoted $\chi = (1 - \delta)\chi^q + \delta\chi^\sigma$ with mean $x^\sigma$. Jensen’s inequality implies $u_i(x) > \int_x u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma)$ for all $i \in N$, so $x^\sigma \in \text{int} A(\sigma)$. 

Next, let $\chi^z$ denote the policy lottery nearly equivalent to $\chi$, but shifting probability $\frac{\delta \rho \alpha}{\text{den}(\sigma)}$ from $y^*$ to $z^*$. Let $x^z$ denote the mean of $\chi^z$. For all $i \in N$, Jensen’s inequality implies

$$u_i(x^z) > \int_X u_i(x) \chi^z(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho \alpha u_i(y^*)}{\text{den}(\sigma)} + \frac{\delta \rho \alpha u_i(z^*)}{\text{den}(\sigma)}.$$ 

Moreover, $x^z$ is located weakly between $x^\sigma$ and $z^*$, implying $x^z \in A(\sigma)$.

**Step 2:** Since $z_\ell \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_\ell(q) + \delta \tilde{V}_{\ell}(\sigma) - u_\ell(y^*)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{\text{den}(\sigma)} - u_\ell(y^*). \quad (94)$$

Using (83), $\hat{m}_\ell(\sigma)$ is expressed recursively as

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{\text{den}(\sigma)} - u_\ell(y^*) \right)$$

$$= \frac{\rho_\ell \alpha_\ell V_{\text{den}}(\sigma)}{V_{\text{den}}(\sigma)} \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*) \right). \quad (95)$$

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \tilde{V}(\sigma)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell}, \quad (96)$$

where (97) follows from the definition of $\tilde{V}_\ell(\sigma)$ and using (95) to substitute for $\hat{m}_\ell(\sigma)$. Next, we have $V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell > 0$, where the first inequality follows because Lemma B.3 implies all lobby offers are accepted and passed under $\sigma$, so $V_{\text{den}}(\sigma) \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j)$; and the second inequality follows from $\delta[\rho_\ell \alpha_\ell + \sum_{j \in N^L} \rho_j (1 - \alpha_j)] < 1$. Rearranging and simplifying (97),

$$0 \leq V_{\text{den}}(\sigma) \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \right) - \delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*) \left( V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \right)$$

$$\propto (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta \rho_\ell \alpha_\ell [u_\ell(y^*) - u_\ell(z^*)]}{V_{\text{den}}(\sigma)} - u_\ell(z^*)$$

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\[ = \int_X u_\ell(x) \chi^*_\ell(dx) - u_\ell(z^*), \]
a contradiction because \( u_\ell(z^*) \geq u_\ell(x^*) > \int_X u_\ell(x) \chi^*_\ell(dx). \]

**Lemma B.7.** Every stationary legislative lobbying equilibrium is such that \( \lambda_g \) is degenerate for all \( g \in N^G \) and \( \pi_\ell \) is degenerate for all \( \ell \in N^L \).

**Proof.** Let \( \sigma \) denote an equilibrium. By Duggan (2014), \( A_M(\sigma) = A(\sigma) \), which is nonempty, compact and convex.

First, consider \( g \in N^g \) and \( \ell \in N^L_g \). Recall \( \tilde{U}_\ell(\sigma) \) from (86). Lemmas B.1 and B.6 imply \( \lambda^\ell_g \) puts probability one on the unique \((y^*, m^*)\) satisfying \( y^* = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \), and \( m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*) \).

Second, consider \( \ell \in N^L \). Lemma B.6 implies \( \pi_\ell \) puts probability one on \( x^* = \arg\max_{x \in A(\sigma)} u_\ell(x) \), which is unique.

Proposition 1.2 corresponds to Part 2 of Proposition 1 in the text.

**Proposition 1.2** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

**Proof.** Follows from Lemmas B.4 - B.7.
Appendix C

Consider \( \ell \in N^L \). First, I define a function \( \zeta^\ell \) that relates to \( M \)'s equilibrium voting decision. Then, Lemmas C.3 - C.6 characterize \( \zeta^\ell \). Finally, Lemma 1 delivers a partitional characterization on \( \hat{x}_g \) that facilitates Proposition 2.

**Preliminaries to define \( \zeta^\ell \).** Recall \( \pi(0) = \pi(\hat{x}_g) \) for \( \hat{x}_g = 0 \). Let \( \hat{D}^{\ell,y} = \{ \hat{y}_j : |\hat{y}_j| > \pi(0), j \neq \ell \} \) and \( \hat{D}^{\ell,x} = \{ \hat{x}_j : |\hat{x}_j| > \pi(0), j \neq \ell \} \). Next, set \( D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y}\} \) and \( D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x}\} \). Define \( D^\ell \) as the unique elements of \( D^{\ell,y} \cup D^{\ell,x} \cup \{\pi(0)\} \). Let \( K \ell + 1 = |D^\ell| \). Denote the \( k \)-th element of \( D^\ell \) as \( d_k^\ell \). Index elements \( k = 0, \ldots, K_\ell \) of \( D^\ell \) in ascending order so that \( d_0^\ell = \pi(0) \) and \( k' > k \) implies \( d_k' > d_k^\ell \).

For each \( k \) and \( j \neq \ell \), let \( C_j^k = \mathbb{I}\{ \hat{x}_j \in [-d_k^\ell, d_k^\ell]\} \) and \( \tilde{C}_j^k = \mathbb{I}\{ \hat{y}_j \in [-d_k^\ell, d_k^\ell]\} \). Define

\[
I_j^k = (1 - \alpha_j)C_j^k u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^k u_M(\hat{y}_j)
\]

and

\[
O_j^k = (1 - \alpha_j)(1 - C_j^k) + \alpha_j(1 - \tilde{C}_j^k),
\]
suppressing dependence on \( \ell \). Let

\[
\hat{x}_k^\ell = \left( \frac{1}{\delta \rho_\ell} \left[ (1 - \delta)u_M(g) + \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell) \left( 1 - \delta \sum_{j \neq \ell} O_j^k \right) \right] \right)^{\frac{1}{2}}. \quad (98)
\]

Because \( d_0^\ell = \pi(0) \), rearranging (98) yields \( \hat{x}_0^\ell = 0 \).

**Lemma C.1.** For all \( \ell \in N^L \) and each \( k = 0, \ldots, K_\ell \), we have

\[
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) = \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^k \right).
\]

**Proof.** Consider \( \ell \in N^L \) and fix \( k < K_\ell \). Then,

\[
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right)
= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) + \delta u_M(d_{k+1}^\ell) \left( \sum_{j \neq \ell} \rho_j O_j^k - \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j O_j^k \right).
\]
\[ \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) \]

\[ = \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \tag{99} \]

\[ = \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k), \tag{100} \]

where (99) follows because \( u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \) by construction. \( \square \)

**Lemma C.2.** For all \( \ell \in N^\ell \), \( \hat{x}_k^\ell \) strictly increases in \( k \).

**Proof.** Consider \( \ell \in N^\ell \) and fix \( k < K^\ell \). Lemma C.1 and \( 0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell) \) together imply

\[ \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) \tag{101} \]

Thus, \( \hat{x}_k^\ell < \hat{x}_{k+1}^\ell \) follows from (98). \( \square \)

**Definition of \( \zeta^\ell \).** For \( k = 0, \ldots, K^\ell \), define \( \overline{x}_k^\ell : \mathbb{R}_+ \to \mathbb{R}_+ \) as

\[ \overline{x}_k^\ell(x) = \left( \frac{(1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \right)^{\frac{1}{2}} \tag{102} \]

and \( \zeta_k^\ell : \mathbb{R}_+ \to \mathbb{R} \) as

\[ \zeta_k^\ell(x) = u_M(x) - \left( (1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right). \]

By construction, \( \overline{x}_k^\ell(\hat{x}_k^\ell) = d_k^\ell \) for all \( k \). Adopt the convention \( d_{K^\ell+1}^\ell = \infty \). Define the piecewise function \( \zeta^\ell : \mathbb{R}_+ \to \mathbb{R} \) as

\[ \zeta^\ell(x) = \zeta_k^\ell(x) \text{ if } x \in [d_k^\ell, d_{k+1}^\ell). \]

**Lemma C.3.** For all \( \ell \in N^L \), \( \zeta^\ell(0) > 0 \) and \( \zeta^\ell(q) \leq 0 \).
Proof. Consider \( \ell \in N^L \). First, we have

\[
\zeta^\ell(0) = \zeta_0^\ell(0)
\]

\[
= u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho \bar{u}_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\bar{x}_0(0)) \sum_{j \neq \ell} \rho_j O_j^0 \right)
\]

\[
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right)
\]

(103)

\[
> 0,
\]

where (103) follows from \( u_M(0) = 0 \) and \( \bar{x}_0^\ell(0) = \hat{x}_0 \).

Next, I show \( \zeta^\ell(q) \leq 0 \). Let \( k' \) denote the largest \( k \) such that \( \hat{x}_k^\ell \leq q \).

- **Step 1**: Because \( \bar{x}_k^\ell(\hat{x}_k^\ell) = d_k^\ell \), we have

\[
u_M(d_k^\ell) = (1 - \delta)u_M(q) + \delta \rho \bar{u}_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^k
\]

(104)

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho \bar{u}_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}
\]

(105)

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho \bar{u}_M(q) + \delta u_M(d_k^\ell) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right]}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}
\]

(106)

\[
= \frac{(1 - \delta)u_M(q) + \delta \rho \bar{u}_M(q) + \delta u_M(d_k^\ell)(1 - \rho_k - \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k},
\]

(107)

where (104) follows from rearranging (102); (105) from \( \hat{x}_k^\ell \leq q \); (106) because for all \( j \) the construction of \( I_j^k \) implies \( I_j^k \geq u_M(d_k^\ell) \left[ (1 - \alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right] \); and (107) because \( \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right] = 1 - \rho_k - \sum_{j \neq \ell} \rho_j O_j^k \) by construction.

Rearranging and simplifying (107) yields \( u_M(d_k^\ell) \geq \frac{(1 - \delta + \delta \rho \bar{u}_M(q))}{1 - \delta + \delta \rho_k} = u_M(q) \). Thus,

\[
\sum_{j \neq \ell} \rho_j I_j^k = \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right] u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^k u_M(\hat{y}_j)
\]

(108)

\[
\geq u_M(d_k^\ell) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right]
\]

(109)

\[
= u_M(d_k^\ell)(1 - \rho_k - \sum_{j \neq \ell} \rho_j O_j^k)
\]

(110)
\[ \geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) \] (111)

where (108) follows from the definition of \( I_j^{k'} \); (109) from \( u_M(\hat{x}_j) \geq u_M(d_k') \) if \( C_j^{k'} = 1 \) and \( u_M(\hat{y}_j) \geq u_M(d_k') \) if \( \tilde{\bar{C}}_j^{k'} = 1 \); (110) because \( \sum_{j \neq \ell} \rho_j[(1 - \alpha_j)C_j^{k'} + \alpha_j\tilde{\bar{C}}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'} \) by construction; and (111) from \( u_M(d_k') \geq u_M(q) \).

- **Step 2:** We have

\[
u_M(\bar{x}_k'(q)) = \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta\sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta\sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta\sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{112}
\]

\[ = u_M(q). \tag{113} \]

where (112) follows from Step 1 and (113) from simplifying.

- **Step 3:** To see \( \zeta^{\ell}(q) \leq 0 \), note

\[
\zeta^{\ell}(q) = u_M(q) - \left( (1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta\sum_{j \neq \ell} \rho_j I_j^{k'} + \delta u_M(\bar{x}_k'(q))\sum_{j \neq \ell} \rho_j O_j^{k'} \right) \\
\leq u_M(q) - \left( (1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) + \delta u_M(q)\sum_{j \neq \ell} \rho_j O_j^{k'} \right) \tag{114}
\]

\[ = 0, \tag{115} \]

where (114) follows from Steps 1 and 2.

\[ \square \]

**Lemma C.4.** For all \( \ell \in N^L \), \( \zeta^{\ell} \) is continuous.

*Proof.* Consider \( \ell \in N^L \) and fix \( k \). Because \( \bar{x}_k'(x) \) is continuous, \( \zeta^{\ell} \) is continuous over \((\hat{x}_{k}, \bar{x}_{k+1})\). It suffices to show \( \zeta^{\ell}_k(\bar{x}_{k+1}) = \zeta^{\ell}_{k+1}(\bar{x}_{k+1}) \).

First, I establish \( d_{k+1} = \bar{x}_{k+1} \). Rearranging (98) for \( k + 1 \) yields

\[ 0 = u_M(d_{k+1}')(1 - \delta\sum_{j \neq \ell} \rho_j O_j^{k+1}) - (1 - \delta)u_M(q) - \delta\rho_\ell u_M(\bar{x}_{k+1}) - \delta\sum_{j \neq \ell} \rho_j I_j^{k+1} \]

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where (116) follows from Lemma C.1. Thus, \( u_M(d_{k+1}^\ell) = \frac{(1-\delta)u_M(q) + \delta \rho u_M(\bar{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1-\delta \sum_{j \neq \ell} \rho_j O_j^k} \), so \( d_{k+1}^\ell = \bar{x}_{k+1}^\ell(\bar{x}_{k+1}^\ell) \). Then,

\[
\zeta_k^\ell(\bar{x}_{k+1}^\ell) = u_M(\bar{x}_{k+1}^\ell) - \left( (1-\delta)u_M(q) + \delta \rho u_M(\bar{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta \sum_{j \neq \ell} \rho_j O_j^k \right) \\
= u_M(\bar{x}_{k+1}^\ell) - \left( (1-\delta)u_M(q) + \delta \rho u_M(\bar{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} + \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) \\
= \zeta_{k+1}^\ell(\bar{x}_{k+1}^\ell),
\]

where (117) follows from Lemma C.1 because \( d_{k+1}^\ell = \bar{x}_{k+1}^\ell(\bar{x}_{k+1}^\ell) \).

\( \square \)

**Lemma C.5.** For all \( \ell \in N^L \), \( \zeta^\ell \) is strictly decreasing.

**Proof.** Consider \( \ell \in N^L \) and fix \( k \). The proof shows that the derivative of \( \zeta^\ell \) is strictly negative at every \( x \in (\bar{x}_k^\ell, \bar{x}_{k+1}^\ell) \). Continuity then implies that \( \zeta^\ell \) is strictly decreasing.

Consider \( x \in (\bar{x}_k^\ell, \bar{x}_{k+1}^\ell) \). Then

\[
\zeta^\ell(x) = u_M(x) - \left( (1-\delta)u_M(q) + \delta \rho u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta \sum_{j \neq \ell} \rho_j O_j^k \right)
\]

and

\[
\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x\delta \rho_\ell + \frac{2x\delta \rho_\ell (\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1-\delta \sum_{j \neq \ell} \rho_j O_j^k} \\
\approx \delta \rho_\ell + \delta \sum_{j \neq \ell} \rho_j O_j^k - 1
\]

\( < 0, \)

where (119) follows from \( \frac{\partial u_M(x)}{\partial \bar{x}_k^\ell(x)} \frac{\partial \bar{x}_k^\ell(x)}{\partial x} = -\frac{2x\delta \rho_\ell}{1-\delta \sum_{j \neq \ell} \rho_j O_j^k} \); and (121) because \( \delta \in (0,1) \) and \( \rho_\ell + \sum_{j \neq \ell} \rho_j O_j^k \leq 1 \).

\( \square \)

**Lemma C.6.** For all \( \ell \in N^L \), there is a unique \( \bar{x}_\ell \in (0, q] \) such that \( \zeta^\ell(x) > 0 \) for all \( x \in [0, \bar{x}_\ell) \), \( \zeta^\ell(\bar{x}_\ell) = 0 \), and \( \zeta^\ell(x) < 0 \) for all \( x > \bar{x}_\ell \).
Proof. Consider \( \ell \in N^L \). Lemma C.3 implies \( \zeta^\ell(0) > 0 \) and \( \zeta^\ell(q) \leq 0 \). By Lemma C.5, \( \zeta^\ell \) is strictly decreasing. Thus, there is a unique \( \bar{x}_\ell \in (0, q] \) such that \( \zeta^\ell(x) > 0 \) for all \( x < \bar{x}_\ell \) and \( \zeta^\ell(x) < 0 \) for all \( x > \bar{x}_\ell \). Lemma C.4 implies \( \zeta^\ell(\bar{x}_\ell) = 0 \). 

**Lemma 1.** For all \( \ell \in N^L \), \( \hat{x}_g \in (\bar{x}_\ell, \bar{x}_\ell) \) implies \( \hat{x}_g \in \text{int}A(\hat{x}_g) \). Otherwise, \( A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell] \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_\ell = \hat{x}_g \).

**Part 1.** First, suppose \( \hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell) \) and assume \( \hat{x}_g \geq 0 \) without loss of generality. I show \( \hat{x}_g \in \text{int}A(\hat{x}_g) \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k \leq \hat{x}_g \). Define the strategy profile \( \sigma' \) such that it puts probability \( \rho_k \) on \( \hat{x}_g \) and for each \( j \neq \ell \) it (i) puts probability \( (1 - \alpha_j)\rho_j \) on: \( \hat{x}_j \) if \( \hat{x}_j \notin [d_k', d_k'] \), \( \bar{x}_\ell(\hat{x}_g) \) if \( \hat{x}_j > d_k' \), or \( -\bar{x}_\ell(\hat{x}_g) \) if \( \hat{x}_j < -d_k' \); and (ii) puts probability \( \alpha_j\rho_j \) on: \( \hat{y}_j \) if \( \hat{y}_j \notin [-d_k', d_k'] \), \( \bar{x}_\ell(\hat{x}_g) \) if \( \hat{y}_j > d_k' \), or \( -\bar{x}_\ell(\hat{x}_g) \) if \( \hat{y}_j < -d_k' \). By construction, \( \bar{x}(\sigma') = \bar{x}_{k'}(\hat{x}_g) \). Furthermore, proposal strategies are optimal given \( A(\sigma') = [-\bar{x}(\sigma'), \bar{x}(\sigma')] \).

I now check optimality for \( M \). Because \( \hat{x}_g \in [\hat{x}_{k'}, \hat{x}_{k'+1}] \), we have \( \bar{x}(\sigma') = \bar{x}_{k'}(\hat{x}_g) \in [d_{k'}, d_{k'+1}] \). Thus, \( M \) optimally accepts all offers by \( j \neq \ell \). Next, I verify \( \hat{x}_g \in \text{int}A(\sigma') \).

By Lemma C.6, \( \hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell) \) implies \( \zeta(\hat{x}_g) > 0 \), which is equivalent to \( u_M(\hat{x}_g) > (1 - \delta)u_M(q) + \delta \rho_k u_M(\hat{x}_g) + \delta \sum_\ell \rho_k I_k^\ell \). Under \( \sigma' \), this is equivalent to \( \hat{x}_g \in \text{int}A(\sigma') \).

Thus, \( \sigma' \) is equivalent to the equilibrium \( \sigma(\hat{x}_g) \) and \( \hat{x}_g \in \text{int}A(\hat{x}_g) \), as desired.

**Part 2.** Assume \( \hat{x}_g \notin (-\bar{x}_\ell, \bar{x}_\ell) \) and suppose \( \hat{x}_g \geq 0 \) without loss of generality. I verify \( A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell] \) in two steps. Step 1 shows \( \bar{x}(\hat{x}_g) \geq \bar{x}_\ell \). Step 2 shows \( \bar{x}(\hat{x}_g) \leq \bar{x}_\ell \).

**Step 1.** Suppose \( \bar{x}(\hat{x}_g) < \bar{x}_\ell \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k \leq \bar{x}(\hat{x}_g) \).

Because \( \hat{x}_g > \bar{x}_\ell > \bar{x}(\hat{x}_g) \), it follows that \( \sigma(\hat{x}_g) \) puts probability \( \rho_k \) on \( \bar{x}(\hat{x}_g) \). Thus, \( u_M(\bar{x}(\hat{x}_g)) = \left(1 - \delta\right)u_M(q) + \delta \sum_\ell \rho_k I_k^\ell \) and rearranging yields \( \zeta(\bar{x}(\hat{x}_g)) = 0 \). Lemma C.6 implies \( \bar{x}(\hat{x}_g) = \bar{x}_\ell \), a contradiction.

**Step 2.** Suppose \( \bar{x}(\hat{x}_g) > \bar{x}_\ell \). If \( \hat{x}_g \geq \bar{x}(\hat{x}_g) \), then the argument from Step 1 shows a contradiction. Assume \( \hat{x}_g < \bar{x}(\hat{x}_g) \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k(\hat{x}_g) \leq \bar{x}(\hat{x}_g) \).

Then \( \sigma(\hat{x}_g) \) puts probability \( \rho_k \) on \( \hat{x}_g \). Next, \( M \) optimally accepts \( \hat{x}_g \) under \( \sigma(\hat{x}_g) \) if \( u_M(\hat{x}_g) \geq \left(1 - \delta\right)u_M(q) + \delta \rho_k u_M(\hat{x}_g) + \delta \sum_\ell \rho_k I_k^\ell \). Rearranging, this condition is equivalent to \( \zeta(\hat{x}_g) \geq 0 \). By Lemma C.6, this requires \( \hat{x}_g \leq \bar{x}_\ell \), a contradiction. \( \square \)
References


Hall, Richard L. and Alan V. Deardorff, “Lobbying as Legislative Subsidy,” *American Political Science Review*, 2006, **100** (01), 69–84.


