Access and Lobbying in Legislatures∗

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Abstract

I study a model of legislative policymaking where interest groups can form connections with legislators to get access. Access provides opportunities to lobby those legislators if they control the agenda. In equilibrium, persistent access has spillover effects. It changes legislature-wide expectations, thereby changing which policies pass today and, in turn, can change proposals by other legislators. These endogenous spillovers encourage access to some legislators, but discourage access to others. Under broad conditions, groups forgo access to a range of more centrist legislators. But they are keen to access more extreme legislators. The results have implications for campaign finance and revolving door hiring. I also show that equilibrium lobbying expenditures increase with several measures of legislature polarization.

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Special interests and lobbying are widely maligned. A popular, cynical view is that wealthy interest groups frequently lobby key politicians to get favorable policy. Implicit in many discussions about this possibility are the relative preferences of groups and the politicians they influence. Specifically, a common concern is that ideologically extreme groups pull centrist politicians away from majority interests.

To lobby effectively, interest groups typically must get access by developing good working relationships with politicians. Understanding where such relationships form is thus crucial for anticipating who influences whom and resulting welfare implications. In isolation, the clear benefits of lobbying suggest groups always crave access. In large legislatures, however, groups cannot feasibly influence every legislator at all times. In such settings, groups may need to account for the possibility that access to particular legislator(s) indirectly affects behavior by other legislators. In principle, such spillovers can push in either direction, potentially discouraging or encouraging access.

To develop our understanding of these forces, I study the connections interest groups cultivate in legislatures. More precisely, which combinations of groups and legislators form relationships yielding access? In pursuing this question, I also address two related questions. First, given connections that form, how do various political conditions influence observed levels of lobbying? Second, what are the policy and welfare effects of access and lobbying?

I study a game-theoretic model where access provides interest groups with opportunities to influence legislative policy proposals. Although expanding the scope of application for the canonical legislative bargaining framework is of independent theoretical interest, I model a rich legislative environment to disentangle access-seeking incentives from lobbying. There are four primary contributions in this direction. First, I provide a microfoundation for spillover effects from access, as they arise endogenously in equilibrium. Beyond highlighting which features of legislative policymaking can produce spillovers, this microfoundation also permits characterization of how their magnitude and direction depend on various political considerations. Second, I highlight that the nature of access-driven spillovers depends on group and legislator ideology relative to the ideological distribution of all legislators. Specifically, for different group-legislator pairs, I shed light on whether these spillovers encourage or discourage access, relative to a setting without spillovers. Third, I show that some groups optimally forgo access to particular legislators and characterize when this behavior occurs. It does not

\[1\text{See, e.g., Wright (1989, 1990); Hall and Wayman (1990); Hansen (1991); Ainsworth (1993); de Figueiredo and Silverman (2006) and Powell (2014).}\]
require costly access and occurs when negative spillovers dominate gains from access. 

Fourth, I discuss how forgoing access, although counterintuitive, is consistent with 
several puzzling empirical regularities in US campaign finance.

The model has three key features. First, lobbying allows groups to influence policy 
proposals before they reach the floor. Second, to study which connections form, I 
distinguish between access and lobbying. Interest groups choose whether to access 
particular legislators before policymaking and, if they do so, have chances to lobby 
those legislators if they control the agenda. Finally, I unpack the legislative black box 
by modeling a well-studied legislative bargaining environment where failed proposals 
can be revisited, forward-looking legislators anticipate outside influence, agenda power 
can change hands unpredictably, and passage requires majority approval.\(^2\)

To illustrate how lobbying affects legislative policymaking, I first study a baseline 
model with exogenous access. I establish equilibrium existence and provide a sharp 
characterization with clear connections to equilibria policymaking without lobbying. 
As expected, groups pull policy in their favored direction whenever they have access 
to the proposer. Groups may be constrained, however, because successful policy pro-
posals must satisfy a legislative majority. Depending on the respective preferences of 
groups and their associated legislators, lobbying can increase or decrease policy extremism. The equilibrium characterization also yields clear comparative statics about 
relationships between lobbying expenditures and various legislative conditions.

The baseline analysis produces a key insight: access can have indirect effects on 
equilibrium policymaking under broad conditions. These spillovers arise because legis-
lators account for where connections form and anticipate potential policy implications 
of lobbying facilitated by those connections.

To study the consequences of these spillovers, I extend the baseline model so that 
groups choose access before policymaking. The sharp characterization of legislative 
behavior pins down how access affects a group’s welfare. Substantively, this extension 
reflects that access must be acquired well ahead of time to facilitate lobbying 
opportunities. The analysis reveals that access-driven spillovers can produce qualita-
tive differences in how groups value connections to different legislators. The relative 
extremism of an interest group and target legislator plays a key role.

Access-driven spillovers can create an important tradeoff for interest groups. On 
the one hand, access provides groups with more opportunities to lobby during policy-

\(^2\)The legislative setting follows Banks and Duggan (2006a) and Cho and Duggan (2003), which 
integrate key features of Baron and Ferejohn (1989) and Romer and Rosenthal (1978).
making. On the other hand, access can have a negative indirect effect on proposals by legislators the group does not access. The logic is as follows. Forward-looking legislators anticipate lobbying behavior following rejected proposals. Thus, a group’s access to some legislator affects every legislator’s expectation about policymaking. More precisely, access affects each legislator’s reservation value, which is generated endogenously by equilibrium expectations about future policymaking. This effect can change which policies pass. In turn, access can indirectly affect proposals of legislators constrained by majority approval. From a group’s perspective, this indirect effect can be either good or bad. Furthermore, its magnitude depends on various legislative conditions, including ideological polarization or the distribution of agenda setting power.

I show that some interest groups optimally forgo access to a range of legislators, even if that access is free. Specifically, non-extremist groups forgo access to neighboring, more centrist legislators. In such cases, access-driven spillovers increase policy extremism. This indirect effect outweighs the group’s gain from more lobbying opportunities. Thus, these connections are foregone even if they are free. In these cases, groups face a time inconsistency problem. They always want to lobby when given the opportunity. Ex ante, however, they forgo access because it polarizes the policymaking environment too much relative to their expected gain from lobbying.

To illustrate the logic, consider the following stylized example. A regional energy interest group anticipates national legislation regulating emissions. It prefers moderately tighter regulations to capitalize on recent investments in clean technology. The group’s local congressman wants to tighten existing regulations more than the group. If the group gets access, its chances of lobbying the congressman increase. Thus, moderate and pro-environment legislators are less optimistic about the eventual regulatory outcome. These legislators know that if the congressman drafts policy, then the group may be able to lobby. If so, the resulting policy will be more extreme than if the congressman had acted alone. Consequently, rejecting proposals is less attractive and these other legislators are willing to approve more extreme policies. The group’s access thus indirectly allows extreme pro-energy legislators to pass weaker emissions regulations if they draft policy. Such policies would reduce the group’s benefits from its recent technological investments. I show that this threat of greater extremism can worsen the group’s expectations about policymaking so that it prefers to forgo access altogether.

In contrast, groups always want access to nearby, more extreme legislators. In this case, access increases lobby opportunities and also favorably constrains extreme legis-
lators. Substantively, the analysis suggests that centrist and moderate interest groups have especially strong incentives to acquire access to a broad spectrum of legislators.

The analysis provides implications for welfare and empirical work. First, the consequences of access depend on the relative preferences of groups and targeted politicians. Many fear that groups pull otherwise public-minded politicians away from majority interests. Yet, some groups may moderate otherwise extreme politicians. Studying which pairs of groups and legislators are likely to form connections highlights when society may want to restrict access, encourage it, or do nothing. Second, empirical evidence suggests that campaign contributions and hiring lobbyists with revolving door connections are two ways that groups get access (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kalla and Broockman, 2015). Thus, identifying who groups want to access provides direct implications for both (i) how groups allocate contributions and (ii) who they hire as lobbyists.

I contribute to a large literature studying how strategic considerations influence interest groups. Lobbying has been modeled in many ways. I focus on lobbying to influence policy content.\(^3\) Specifically, I study *lobbying as exchange* in the spirit of Grossman and Helpman (1994), where groups provide resources to shape policy proposals.\(^4\) The lobbying technology here follows Bils, Duggan and Judd (2017), which studies lobbying in a model of repeated elections.\(^5\) There, officeholder ideology exogenously determines access and groups can always lobby officeholders to whom they have access. In contrast, I study whether groups want access and allow them to choose.\(^6\)

Existing work also explores implications of less cynical perspectives on lobbying, such as groups providing useful information (Austen-Smith, 1995; Prat, 2002) or services to politicians and voters (Hall and Deardorff, 2006). Studying these perspectives is worthwhile, but I focus on a cynical form of influence for several reasons. First, it aligns with a widespread outlook and underlies public concern about special interests. Second, I aim to strengthen positive theory under this perspective to parse its empirical

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\(^3\)Other work studies lobbying to influence voting on a fixed agenda. Many have analyzed vote buying in legislatures, mostly studying distributive policies or public goods (Snyder Jr., 1991; Groseclose and Snyder, 1996; Banks, 2000; Dal Bó, 2007; Dekel et al., 2009). Others allow groups to influence votes by strategically providing information (Bennedsen and Feldmann, 2002; Jackson and Tan, 2013; Schnakenberg, 2015, 2017; Alonso and Câmara, 2016; Awad, 2018).

\(^4\)See Grossman and Helpman (2002) for an extensive overview of this setting, which they apply to campaign contributions.

\(^5\)See Martimort and Semenov (2008) and an extension in Acemoglu et al. (2013) for recent studies using a similar approach to model lobbying.

\(^6\)Another notable difference is that I allow partial access, which does not guarantee lobbying opportunities. Furthermore, I consider legislative, rather than executive, policymaking.
implications and welfare consequences.

Scholars have studied access acquisition in static, informational lobbying environments (Austen-Smith, 1995; Cotton, 2012, 2016). Closest to this paper is Schnakenberg (2017), where groups can buy access in a legislature. There, groups try to influence a legislative vote over exogenous policy proposals in a static setting. Access allows groups to provide information. As in this paper, influencing a legislature is quite different from influencing a solitary policymaker. In contrast, I study a complete information setting where lobbying affects endogenous policy proposals and policymaking continues after failed proposals. Moreover, groups sometimes optimally forgo free access in this paper, which never happens in Schnakenberg (2017).

I also contribute to a literature taking access as given and analyzing lobbying to influence the agenda within various legislative institutions. Helpman and Persson (2001) introduce interest groups into a static version of Baron and Ferejohn (1989). They compare the consequences of lobbying in different legislative institutions. As in this paper, groups can lobby particular legislators when they control the agenda and lobbying influences proposals. Unlike this paper, they study distributive policies, groups can also lobby to influence votes, and bargaining does not continue after rejected proposals. Moreover, they do not study access acquisition. In this paper, the prospect of future bargaining creates endogenous spillovers from access.


An exception is Levy and Razin (2013), who study a dynamic setting with an endogenous status quo in a one-dimensional policy space. In each period, a continuum of groups compete in an all-pay auction for temporary agenda control. They provide conditions for policies to moderate over time. They do not address which connections form, as they do not model politicians and instead implicitly treat them as homogeneous. Furthermore, they do not study persistent access, as groups instead vie for

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temporary access throughout policymaking. In contrast, I study targeted and persistent access acquired before policymaking. There are several other differences. Here, bargaining ends when a proposal passes and I abstract from head-to-head competition for access.

Finally, a key result of this paper is that groups may forgo access to more centrist legislators. The logic connects to moderation results in spatial models of dynamic bargaining with endogenous status quo (Baron, 1996; Zápal, 2014; Forand, 2014; Buisseret and Bernhardt, 2017). There, legislators prefer to propose more centrist policies to constrain future proposers in equilibrium. They forgo the full power of their current agenda control to constrain the scale of policy changes by future proposers who may have substantially different preferences. I study a different setting, as here policymaking ends once a proposal passes. Yet, the incentive to forgo access arises from the same desire to constrain potential future proposers who are ideologically distant.

Model of Legislative Bargaining with Lobbying

To study access, it is important to firmly understand its downstream effects through lobbying. Thus, I first present and analyze the legislative environment with access fixed exogenously, having implicitly arisen from previous efforts to create connections. This handle on legislative behavior sets the stage to subsequently study access acquisition and then, given access, how lobbying expenditures vary with legislative features.

In the model, legislators bargain to set a common policy. Throughout policymaking, ideological interest groups may receive opportunities to influence policy by providing favors. The logic for the main results can be illustrated in a streamlined setting with four legislators and one interest group. I provide several comments after describing the baseline model.

There are four legislators: a left-partisan $L$, a moderate $M$, a right-partisan $R$, and a generic legislator $\ell$. The interest group is denoted $g$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Each legislator $i$ has associated ideal point $\hat{x}_i \in X$ and $g$’s ideal point is $\hat{x}_g \in X$. Throughout, I normalize $\hat{x}_M = 0$. Furthermore, I assume $\hat{x}_L < 0 < \hat{x}_R$. To reflect that partisans are staunchly ideological, I maintain $\min\{|\hat{x}_L|, \hat{x}_R| > |q|$. Although not crucial, this assumption clarifies key tradeoffs.

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8Although Forand (2014) is cast as a model of elections, it can be interpreted as a spatial bargaining model with an endogenous status quo and endogenous proposers.

9Appendix A presents a more general setting.
Legislative bargaining occurs over an infinite horizon, with periods discrete and indexed by \( t \in \{1, 2, \ldots \} \). Let \( \rho_i > 0 \) denote the probability that legislator \( i \) is chosen to propose policy in any period \( t \). Then \( \rho = (\rho_L, \rho_M, \rho_R) \) denotes the distribution of legislator recognition probabilities, which sum to one. The interest group, \( g \), has opportunities to influence \( \ell \)'s policy proposals. Specifically, \( g \) has \( \alpha \in [0, 1] \) access to \( \ell \).

Access determines the probability that \( g \) can lobby \( \ell \), conditional on \( \ell \) being recognized to propose. This technology reflects the standard view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714).\(^{10}\) To reflect targeted access, \( g \) does not have access to legislators other than \( \ell \) in the baseline model. Additionally, \( g \)'s access is exogenously endowed for now. Later, to study when groups seek access, I allow \( g \) to choose \( \alpha \).

In each period \( t \), bargaining proceeds as follows. If no policy has passed before \( t \), then each legislator \( i \) is recognized as the period-\( t \) proposer with probability \( \rho_i \). The identity of the period-\( t \) proposer, \( i_t \), is publicly observed. If \( i_t \neq \ell \), then \( g \) is not active and \( i_t \) proposes any policy \( x_t \in X \). If \( i_t = \ell \), then \( g \) can lobby \( \ell \) with probability \( \alpha \). If \( g \) lobbies, then \( g \) offers \( \ell \) a binding contract \((y_t, m_t)\) consisting of a policy \( y_t \in X \) and a transfer \( m_t \geq 0 \). After observing \( g \)'s offer, \( \ell \) decides to accept or reject. If \( \ell \) accepts, then \( g \) is committed to propose \( x_t = y_t \) and \( m_t \) transfers from \( g \) to \( \ell \). If \( \ell \) rejects, then \( g \) can propose any \( x_t \in X \) and \( g \) keeps \( m_t \). With probability \( 1 - \alpha \), \( g \) cannot lobby in \( t \). In this case, \( \ell \) simply proposes any \( x_t \in X \) and \( g \) does not make an offer.

In each case, all legislators observe the period-\( t \) proposal, \( x_t \). Next, the moderate legislator, \( M \), chooses to accept or reject the proposal. If \( M \) accepts, then the proposal passes and bargaining ends with \( x_t \) enacted in \( t \) and all subsequent periods. If \( M \) rejects, then the status quo \( q \in \mathbb{R} \) is enacted in \( t \) and bargaining proceeds to \( t + 1 \). This setup captures the spirit of a more general setting where all legislators vote and \( M \) is a decisive median legislator.\(^{11}\)

If \( i_t = \ell \), \( \ell \) accepts \( g \)'s offer \((y, m)\), and \( x_t \) is the enacted policy in \( t \),\(^{12}\) then \( g \)'s stage payoff is \( u_g(x_t) - m \) and \( \ell \)'s stage payoff is \( u_\ell(x_t) + m \). All players have quadratic policy utility and discount streams of stage utility by the common discount factor \( \delta \in (0, 1) \). See Appendix A for explicit expressions of dynamic payoffs. Figure 1 illustrates the within-period interaction and accumulation of payoffs for a period in which \( \ell \) proposes

\(^{10}\) Also see, e.g., Milbrath (1976); Hall and Wayman (1990); Hansen (1991); Grossman and Helpman (2002); Hall and Deardorff (2006); Gordon et al. (2007) and Powell (2014).

\(^{11}\) Under the assumptions maintained here, \( M \)'s decision corresponds to the outcome of majority voting over policy lotteries (Banks and Duggan, 2006b; Duggan, 2014).

\(^{12}\) Notice \( x_t = q \) if \( y \) does not pass in period \( t \).
and $g$ can lobby. For a period in which $\ell$ does not propose, or $g$ cannot lobby, the within-period interaction is analogous to Figure 1 following $\ell$ rejecting $g$’s offer.

Figure 1: A period in which the interest group can lobby

Model Discussion

I make several comments before proceeding to the analysis.

First, there are multiple interpretations for access, $\alpha$. One is personal connections possessed by $g$’s lobbyists affecting their chances of meeting with $\ell$ (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Cain and Drutman, 2014; Kang and You, 2015; McCrain, 2018). Another is access gained from campaign contributions to $\ell$ during a preceding, yet unmodeled, election.\textsuperscript{13} A third is $\ell$’s value from using policy proposals to appeal to constituents, which may affect her propensity to meet with lobbyists.

Second, $g$ has access to only one legislator. I relax this assumption in the appendices, but it reflects the idea that groups are unable to access some legislators due to exogenous factors. For example, regional groups may not be able to access legislators absent a geographic connection (Wright, 1989). Alternatively, voters in some districts

\textsuperscript{13}See, e.g., Langbein (1986); Romer and Snyder Jr. (1994); Kalla and Broockman (2015); Barber (2016); Grimmer and Powell (2016) and Fournaies and Hall (2017) for evidence suggesting that many interest groups use campaign contributions to buy access.
may be strongly opposed to the group’s mission or tactics (Stratmann, 1992). Finally, the group simply may not be able to afford access to many different legislators.

Third, I model access as $g$’s probability of being able to lobby when $\ell$ controls the agenda. Qualitatively similar results hold if access is binary, or if access is modeled as $\ell$’s marginal value of money.

Fourth, I abstract from lobbying to influence legislative votes. Ignoring this channel isolates considerations related to lobbying over policy details “in committee.” Furthermore, the median ideology is a robust statistic in large legislatures, and meaningful vote buying likely requires coordinating deals with several legislators.

Finally, I interpret the lobbying technology. In the model, groups offer binding contracts exchanging resources for policy. In practice, groups spend substantial effort drafting legislation (Schlozman and Tierney, 1986) and frequently present legislators with model bills (Levy and Razin, 2013; Kroeger, 2016). Formally, this corresponds to the policy offer, $y$. In exchange, legislators may gain an inside track on future employment opportunities (Diermeier et al., 2005). Moreover, legislators are freed to pursue other tasks such as constituent service and fundraising, in the spirit of Hall and Deardorff (2006). Finally, groups may also provide valuable political intelligence, or write speeches to help sell policies to the legislator’s constituents and co-partisans (Schlozman and Tierney, 1983, 1986; Hall and Wayman, 1990; Wright, 1996). The group’s transfer, $m$, captures these benefits.

**Equilibrium Policies and Lobbying Activity**

I study a selection of the model’s subgame perfect equilibria (SPE), applying standard refinements from the legislative bargaining literature. In particular, I focus on no-delay stationary legislative lobbying equilibria. A strategy profile $\sigma$ is no-delay if it specifies that all legislators propose socially acceptable policy and the interest group’s policy offer is socially acceptable. Informally, a no-delay stationary legislative lobbying equilibrium requires four conditions: (i) $g$’s policy offer is socially acceptable and $g$ cannot profitably deviate to another offer; (ii) legislator $\ell$ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal; (iii) conditional on not receiving a payment from $g$, each legislator proposes socially acceptable policy and cannot profitably deviate to a different proposal; and (iv) $M$
supports a policy if and only if she weakly prefers it relative to rejecting and extending bargaining. Stationarity implies that $g$’s offers to $\ell$ are independent of previous play; $\ell$ accepts or rejects $g$’s offer based only on the terms of the current offer, and $\ell$’s policy proposals in lieu of acceptance are independent of the preceding history; legislators other than $\ell$ propose policy independent of preceding play; and $M$’s voting decision depends only on current proposal.

Several features of this equilibrium concept are noteworthy. First, although players use straightforward behavioral rules, no player can profitably deviate to any other strategy. Second, $g$ must make an offer in each period that $\ell$ proposes and $g$ can lobby. This requirement is innocuous, however, as $g$ can effectively forgo lobbying by offering a contract composed of $\ell$’s default proposal and no payment. Third, $\ell$ always accepts $g$’s offer when indifferent, but this restriction is without loss of generality. Finally, I focus on no-delay strategy profiles for convenience, as this restriction is inconsequential.

Proposition 1 provides three results. First, I show existence of a no-delay stationary legislative lobbying equilibrium. Along the way, I obtain a sharp characterization of equilibrium behavior. Next, I show that a larger class of equilibria are equivalent in a strong sense to these equilibria. Finally, I prove there is a unique equilibrium outcome distribution, capitalizing on Cho and Duggan (2003). This property ensures that endogenizing access does not require consequential equilibrium selection.

**Proposition 1.**

1. There exists a no-delay stationary legislative lobbying equilibrium.

2. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay strategy legislative lobbying equilibrium.

3. Every stationary legislative lobbying equilibrium has the same outcome distribution.

In light of Proposition 1, I simply refer to equilibria throughout the rest of the analysis. As is standard in the legislative bargaining literature, equilibria can be char-

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15Specifically, $M$ uses stage-undominated voting strategies (Baron and Kalai, 1993).

16See Appendix B for more details.

17In Appendix B, I define mixed strategy stationary legislative lobbying equilibrium and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.
acterized by their social acceptance set, which is denoted $A(\sigma)$ and corresponds to the set of policies that $M$ accepts under $\sigma$.\footnote{See, e.g., Banks and Duggan (2000) and Banks and Duggan (2006a).}

In an equilibrium $\sigma$, the boundaries of $A(\sigma)$ are the two policies that $M$ is indifferent between approving and rejecting. Formally, the upper bound of $A(\sigma)$, denoted $\pi(\sigma)$, is the positive solution to

$$u_M(x) = (1 - \delta)u_M(q) + \delta V_M(\sigma),$$

where $V_M(\sigma)$ denotes $M$’s continuation value under $\sigma$. The acceptance set is $A(\sigma) = [-\pi(\sigma), \pi(\sigma)]$, facilitating a sharp characterization of proposal strategies. In the hypothetical legislature illustrated in Figure 2, $M$ proposes $\hat{x}_M = 0$ if recognized, legislator $L$ proposes $-\pi(\sigma)$, and $R$ proposes $\pi(\sigma)$. The partisans, $L$ and $R$, are thus constrained by $M$’s voting power because their respective ideal policies will not pass. In equilibrium, each compromises by proposing its favorite passable policy.

If legislator $\ell$ is recognized and does not accept a lobby offer, either because $g$ cannot lobby or because $\ell$ rejects $g$’s offer, then $\ell$ proposes $z_\ell$, her favorite policy in $A(\sigma)$. In equilibrium, however, $\ell$ never rejects $g$’s offers because $g$ always makes an offer $\ell$ accepts. Specifically, $g$’s equilibrium lobby payment exactly satisfies $\ell$’s acceptance condition given $g$’s policy offer, as $g$ is strictly worse off giving $\ell$ a surplus transfer. Additionally, $g$ always offers policy that passes and is skewed away from $\hat{x}_\ell$ towards $\hat{x}_g$.

Formally, $g$’s equilibrium offer $(y_g, m_g)$ consists of the policy

$$y_g = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - u_\ell(z_\ell)$$

and transfer $m_g = u_\ell(z_\ell) - u_\ell(y)$. Because $u_\ell(z_\ell)$ does not depend on $g$’s offer,

$$y_g = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y),$$

which uniquely maximizes the joint surplus of $g$ and $\ell$, subject to the constraint that $y_g$ passes.\footnote{Uniqueness follows because $u_g$ and $u_\ell$ are strictly concave, and $A(\sigma)$ is compact, convex, and nonempty.} It is always feasible for $g$ to offer $\ell$’s independent proposal, $z_\ell$, with zero payment. By $z_\ell \in A(\sigma)$, $g$ weakly prefers to make successful offers that $\ell$ accepts. For
convenience, define $g$’s *unconstrained policy offer* as

$$\hat{y} = \arg \max_{y \in X} u_g(y) + u_\ell(y).$$

(4)

Because $u_g$ and $u_\ell$ are quadratic, $\hat{y} = \frac{x_g + x_\ell}{2}$. If $\hat{y} \in A(\sigma)$, then $y_g = \hat{y}$. Otherwise, $y_g$ equals the boundary of $A(\sigma)$ closest to $\hat{y}$ by strict concavity.

The model, although complicated by lobbying, can be reinterpreted as a one-dimensional, spatial bargaining environment with an additional legislator endowed with $\alpha\rho_\ell$ recognition probability and an ideal point, $\hat{y}$, located between $\hat{x}_g$ and $\hat{x}_\ell$. After expanding the legislature to add this additional proposer representing the effect of $g$’s lobbying, legislators propose bills that are closest to their ideal point among those that are acceptable. Uniqueness follows from applying Cho and Duggan (2003) to this fictitious enlarged legislature.

Figure 2 illustrates the equilibrium social acceptance set, $A(\sigma)$, for a hypothetical legislature, along with corresponding equilibrium proposals.

Figure 2: Equilibrium characterization

![Equilibrium Acceptance Set](image)

Figure 2 depicts equilibrium policy proposals. Arrows point from legislator ideal points to proposals. The bold interval is the acceptance set, $A(\sigma)$. If legislator $\ell$ is recognized, then she proposes the acceptable policy closest to $\hat{y} = \frac{x_g + x_\ell}{2}$ with probability $\alpha$ and otherwise proposes the acceptable policy closest to $\hat{x}_\ell$.

In general, the characterization implies that $M$’s continuation value from rejecting a proposal in equilibrium is

$$V_M(\sigma) = \rho_M u_M(\hat{x}_M) + \alpha \rho_\ell u_M(y_g) + (1 - \alpha) \rho_\ell u_M(z_\ell) + \rho_L u_M(-\pi(\sigma)) + \rho_R u_M(\pi(\sigma)).$$

(5)
Using (5) and \( u_M(\hat{x}_M) = 0 \), we can recursively express

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\bar{x}(\sigma) = \left( - \frac{(1 - \delta)u_M(q) + \delta\left(\alpha \rho_L u_M(y_g) + (1 - \alpha)\rho_R u_M(z_\ell)\right)}{1 - \delta(\rho_L + \rho_R)} \right)^{1/2},
\]

which is the unique \( x \geq 0 \) making \( M \) indifferent between accepting and rejecting. Inspection of (6) shows that \( \alpha \), and several other legislative parameters, can affect the boundaries of \( A(\sigma) \) and, consequently, policies proposals by the partisans \( L \) and \( R \). This indirect effect requires dynamic concerns, i.e. \( \delta > 0 \), and plays a key role in the analysis.

Before proceeding, I define terminology used to characterize the ideologies of \( g \) and \( \ell \) within the context of an equilibrium.

**Definition 1.** Legislator \( \ell \) is *extremist* if \( \hat{x}_\ell \notin \text{int}A(\sigma) \) and *centrist* otherwise. Analogously, the interest group, \( g \), is extremist if \( \hat{x}_g \notin \text{int}A(\sigma) \) and centrist otherwise.

**Definition 2.** Legislator \( \ell \) and the interest group, \( g \), are *aligned* if their ideal points are on the same side of \( \hat{x}_M = 0 \), e.g. \( \max\{\hat{x}_\ell, \hat{x}_g\} \leq 0 \). Otherwise, \( \ell \) and \( g \) are *opposed*.

Two conditions are necessary for non-trivial lobbying in equilibrium. Of course, \( g \) must have positive access, i.e. \( \alpha_\ell > 0 \). Furthermore, \( g \) and \( \ell \) cannot be aligned extremists because then \( g \) cannot profitably lobby to improve upon \( \ell \)'s independent policy proposal.

**Who do Interest Groups Want to Access?**

To study where connections form, I now allow the group, \( g \), to choose \( \alpha_\ell \), its access to legislator \( \ell \). Proposition 1 ensures that \( g \)'s choice of \( \alpha_\ell \) pins down equilibrium expected payoffs in the legislature. To isolate key tradeoffs of durable access, I focus on a one-time choice of perfectly persistent access. I discuss other possibilities later. Substantively, this setup can be interpreted as \( g \) using campaign contributions or hiring connected lobbyists to form solid working relationships.
I abstract from the particular mapping that determines access by allowing \( g \) to choose \( \alpha \) freely. In practice, the cost of acquiring access almost certainly depends on idiosyncratic factors such as the connections of the interest group’s lobbyists (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kang and You, 2015), constituent interests within the legislator’s district (Stratmann, 1992), or the number of voters affiliated with the interest group (Bombardini and Trebbi, 2011).\(^{20}\) The following results are driven purely by policy considerations and hold for standard cost functions.

Propositions 2 and 3 fix \( \hat{x}_g \) and study whether \( g \) wants access, as a function of \( \hat{x}_\ell \). As noted above, \( \hat{x}_\ell \) and \( \alpha \) can affect equilibrium legislative behavior by changing policy proposals and the social acceptance set. Propositions 2 and 3 are distinguished by whether or not \( g \) is centrist for all \( \alpha \in [0,1] \) if \( \hat{x}_g \) and \( \hat{x}_\ell \) are sufficiently close.\(^{21}\) Lemma 1 shows that this distinction has a simple partitional characterization. Define

\[
\bar{\pi} = \left( \frac{(1 - \delta)u_M(q)}{1 - \delta(\rho_L + \rho_R + \rho_\ell)} \right)^{\frac{1}{2}},
\]

which satisfies \( \bar{\pi} \geq \pi(\sigma) > 0 \).

**Lemma 1.** If \( \hat{x}_g \in (-\bar{\pi}, \bar{\pi}) \), then \( \hat{x}_g \in \text{int}A(\sigma(\hat{x}_g)) \). Otherwise, \( A(\sigma(\hat{x}_g)) = [-\bar{\pi}, \bar{\pi}] \).

Leveraging Lemma 1, I define a notion of ideological extremism pinned down by primitives.

**Definition 3.** The interest group, \( g \), is a non-ideologue if \( \hat{x}_g \in (-\bar{\pi}, \bar{\pi}) \). Otherwise, \( g \) is an ideologue.

Notably, \( g \)'s status as an ideologue or non-ideologue does not vary with \( \alpha \) or \( \hat{x}_\ell \), even though these features can change whether \( g \) is extremist or centrist. For example, suppose \( g \) is a non-ideologue. Then, \( g \) is extremist if \( \alpha \) is low and \( \ell \) is sufficiently centrist, but \( g \) is centrist if \( \ell \) is sufficiently extreme or \( \alpha \) is sufficiently large. If \( g \) is an ideologue, however, then it is always extremist.

### Non-ideologue Interest Groups

I first consider non-ideologue interest groups, focusing on \( \hat{x}_g \in (0, \bar{\pi}) \), as illustrated in Figure 3. The symmetric case is analogous. Recall \( g \) and \( \ell \) are aligned if \( \hat{x}_g \) and \( \hat{x}_\ell \) are

\(^{20}\)For example, La Raja and Schaffner (2015) emphasize that contributions do not translate into influence the same way for different pairs of interest groups and legislators.

\(^{21}\)Recall that \( g \) is extremist if \( \hat{x}_g \notin \text{int}A(\sigma) \) and centrist otherwise, and similarly for \( \ell \).
on the same side of $\hat{x}_M$.

Let $\overline{\pi}(\alpha; \hat{x}_\ell)$ denote the upper bound of the equilibrium acceptance set if $g$ has $\alpha$ access, given $\hat{x}_\ell$. For non-ideologue $g$, there exists $\bar{x} < \hat{x}_g$ such that $\hat{x}_\ell \geq \bar{x}$ implies $g$’s ex ante expected utility from $\alpha$ access is

$$U^E_g(\alpha; \hat{x}_\ell) = \rho_\ell \left( \alpha [u_g(\hat{y}) + u_\ell(\hat{y})] + (1 - \alpha) u_g(\hat{x}_\ell) \right) + \rho_L u_g(-\overline{\pi}(\alpha; \hat{x}_\ell)) + \rho_R u_g(\overline{\pi}(\alpha; \hat{x}_\ell)) + \rho_M u_g(0), \quad (8)$$

where $u_g(\hat{y}) + u_\ell(\hat{y})$ is $g$’s lobbying return from transferring $m = u_\ell(\hat{y})$ to $\ell$ in exchange for proposing $\hat{y}$. The second line in (8) is $g$’s expected policy utility if a legislator other than $\ell$ proposes.

Qualitatively, $\alpha$ can affect $U^E_g(\alpha; \hat{x}_\ell)$ in two ways. Of course, it directly changes the probability that $g$ lobbies $\ell$. Second, it can change the proposals of $L$ and $R$ by shifting $\pi(\alpha; \hat{x}_\ell)$. More explicitly, the marginal effect of increasing $\alpha$ is

$$\frac{\partial U^E_g(\alpha; \hat{x}_\ell)}{\partial \alpha} = \rho_\ell \left( u_g(\hat{y}) + u_\ell(\hat{y}) - u_g(\hat{x}_\ell) \right) + \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} \left( \rho_L \frac{\partial u_g(-\overline{\pi}(\alpha; \hat{x}_\ell))}{\partial \overline{\pi}(\alpha; \hat{x}_\ell)} + \rho_R \frac{\partial u_g(\overline{\pi}(\alpha; \hat{x}_\ell))}{\partial \overline{\pi}(\alpha; \hat{x}_\ell)} \right). \quad (9)$$

The first bracketed term in (9) is $g$’s direct benefit from lobbying more frequently, which is always positive.

The second bracketed term is the indirect effect of expanding the acceptance set, i.e., increasing $\pi(\alpha; \hat{x}_\ell)$, which allows $L$ and $R$ to pass more extreme policies. It is negative because $\hat{x}_\ell \geq \bar{x}$ implies $\hat{x}_g \in (0, \overline{\pi}(\alpha; \hat{x}_\ell))$ for all $\alpha \in [0, 1]$. Crucially, however, the direction of $\alpha$’s overall indirect effect depends on whether the acceptance set shrinks or expands with $\alpha$, i.e., the sign of $\frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha}$. If $\frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} > 0$, which holds if and only if $g$ is more extreme than $\ell$, then $\alpha$’s indirect effect is negative. Otherwise, it is positive.

Proposition 2 uses these observations to characterize whether non-ideologue groups want access to a range of aligned legislators. One key takeaway is that $g$ optimally forgoes access to moderately more centrist legislators. Another takeaway is that $g$ is especially keen on access to moderately more extreme legislators.

**Proposition 2.** Suppose the interest group, $g$, satisfies $\hat{x}_g \in (0, \overline{\pi})$. There exist $x', x''$ satisfying $0 < x' < \hat{x}_g < \overline{\pi} < x''$ such that:

(i) if legislator $\ell$ satisfies $\hat{x}_\ell \in (x', \hat{x}_g)$, then $g$ forgoes access;
(ii) if $\hat{x}_\ell \in (\hat{x}_g, x'')$, then $g$ acquires access;

(iii) if $\hat{x}_\ell \geq x''$, then $g$ is indifferent over access.

An analogous result holds if $\hat{x}_g \in (-\bar{x}, 0)$.

Figure 3 depicts Proposition 2.

Figure 3: Do non-ideologue interest groups want access?

Figure 3 illustrates Proposition 2 if interest group $g$ is right leaning. If $\hat{x}_\ell \in (x', \hat{x}_g)$, then $g$ forgoes access, i.e. $\alpha = 0$. If $\hat{x}_\ell \in (\hat{x}_g, x'')$, then $\alpha > 0$. If $\hat{x}_\ell \geq x''$, then $g$ is indifferent. If $\hat{x}_\ell \in [0, x']$, then $g$’s preference is ambiguous and depends on parameters.

If $\hat{x}_\ell \in (\hat{x}, \hat{x}_g)$, then $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$ for all $\alpha \in [0, 1]$. Increasing $\alpha$ raises the probability that $\ell$ proposes $\hat{y}$, at the expense of $\hat{x}_\ell$. Because $\hat{x}_\ell \in [0, \hat{x}_g)$, $M$ prefers $\hat{x}_\ell$ to $\hat{y}$. Thus, $M$’s continuation value from rejection decreases with $\alpha$. Therefore $\bar{\pi}(\alpha; \hat{x}_\ell)$ increases and $M$ passes more extreme policies. Because $L$ and $R$ are always partisan, their legislative proposals are thus more extreme. Figure 4 illustrates these effects on the ex ante distribution of equilibrium policy.

As $\hat{x}_\ell$ increases to $\hat{x}_g$, $g$’s lobbying surplus shrinks faster than the indirect loss from enabling more extreme policies. Proposition 2 simply shows existence of $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies the indirect cost of increasing $\alpha$ outweighs $g$’s direct benefit. To see this more concretely, $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$ yield a more explicit expression of (9) as

$$
\frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} = \frac{\rho_\ell (\hat{x}_g - \hat{x}_\ell)^2}{2} + \frac{\delta \rho_\ell (\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g)}{4\bar{\pi}(\alpha; \hat{x}_\ell)[1 - \delta(\rho_L + \rho_R)]} \left[ \hat{x}_g(\rho_R - \rho_L) - \bar{\pi}(\alpha; \hat{x}_\ell)(\rho_L + \rho_R) \right],
$$

which is proportional to

$$
(\hat{x}_g - \hat{x}_\ell) + \frac{\delta (3\hat{x}_\ell + \hat{x}_g)}{2\bar{\pi}(\alpha; \hat{x}_\ell)[1 - \delta(\rho_L + \rho_R)]} \left[ \hat{x}_g(\rho_R - \rho_L) - \bar{\pi}(\alpha; \hat{x}_\ell)(\rho_L + \rho_R) \right].
$$
Figure 4: Forgoing access to a more centrist legislator

Figure 4 illustrates why the group, \( g \), forgoes access (\( \alpha = 0 \)) to legislator \( \ell \) if \( \hat{x}_g \in (-x, x) \) and \( \hat{x}_\ell \in (x', \hat{x}_g) \). Part (a) displays equilibrium behavior for \( \alpha = 0 \). Part (b) illustrates \( \alpha > 0 \). Increasing \( \alpha \) has two immediate effects: (i) lobbying is more likely, and (ii) \( M \)'s expectations worsen. Effect (ii) expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If \( \hat{x}_g \) and \( \hat{x}_\ell \) are close, then effect (ii) dominates and \( g \) prefers \( \alpha = 0 \).

As \( \hat{x}_\ell \) increases to \( \hat{x}_g \), the first term is positive and goes to zero. The second term is bounded away from zero and negative because \( \hat{x}_\ell \in (\hat{x}, \hat{x}_g) \) implies \( \hat{x}_g < \overline{x}(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Thus, the indirect cost dominates as \( \hat{x}_\ell \) approaches \( \hat{x}_g \) from below.

Proposition 2 does not preclude \( g \) forgoing access to sufficiently centrist \( \ell \), i.e. \( \hat{x}_\ell \in [0, x') \). In this case, \( g \) receives a larger benefit from lobbying and may want access.

Proposition 2 implies non-ideologue groups do not want access to some nearby \( \ell \), but does not imply \( g \) forgoes access to all nearby \( \ell \). Instead, if \( \ell \) is moderately more extreme, i.e. \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( g \) craves access. In this case, there is \( \overline{\alpha} \in (0, 1] \) such that \( y = \hat{y} < z_\ell \) for \( \alpha < \overline{\alpha} \) and \( y = z_\ell = \overline{x} \) otherwise.\(^{22}\) Thus, \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < 0 \) for \( \alpha < \overline{\alpha} \), so the second term in (9) is positive. For \( \alpha \geq \overline{\alpha} \), \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} = 0 \) because \( y = z_\ell \), lobbying is inconsequential. Intuitively, \( g \) strictly prefers \( \alpha > 0 \) because it (i) increases \( g \)'s lobbying opportunities and (ii) further constrains partisan legislators to propose policies more favorable to \( g \). The direct and indirect effects both work in \( g \)'s favor. Figure 5 depicts these forces.

Finally, if \( \hat{x}_\ell > x'' \), then \( y = z_\ell = \overline{x} \) for all \( \alpha \). Thus, \( g \) cannot profitably lobby to change \( \ell \)'s policy proposal. Consequently, \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} = 0 \) and \( g \) is indifferent over \( \alpha \).

\(^{22}\)If \( \hat{y} < \overline{x} \), then \( \overline{\alpha} = 1 \). Otherwise, \( \overline{\alpha} < 1 \).
Figure 5: Seeking access to a more extreme legislator

Figure 5 illustrates why the group, \( g \), prefers strictly positive access, \( \alpha > 0 \), if \( \hat{x}_g \in (-\bar{x}, \bar{x}) \) and \( \hat{x}_\ell \in (\hat{x}_g, x'') \). Part (a) displays equilibrium behavior if \( \alpha = 0 \). Part (b) illustrates \( \alpha > 0 \). Access has two immediate effects: (i) \( g \)'s probability of lobbying increases, and (ii) \( M \)'s expectations improve. Effect (ii) causes the acceptance set to shrink, as shown in (b). Thus, partisans propose more centrist policy. Both effects improve \( g \)'s expected payoff.

**Ideologue Interest Groups**

Next, I study ideologue interest groups, i.e., \( \hat{x}_g \notin (-\bar{x}, \bar{x}) \). If \( \ell \) and \( g \) are aligned ideologues, then lobbying is inconsequential and \( g \) is indifferent over \( \alpha \). Otherwise, if \( \ell \) is a non-ideologue, or \( \ell \) and \( g \) are opposed ideologues, then lobbying is consequential. Yet, \( g \)'s preferences over access are ambiguous in general. Specifically, if \( \alpha \) changes the acceptance set, then one of \( L \) or \( R \)'s equilibrium proposal grows worse for \( g \) but the other's proposal becomes more favorable. The specific balance of partisan proposal power determines which change dominates. Thus, it is difficult to draw strong conclusions about an ideologue group's preference for access without restricting the balance of partisan proposal power.

Accordingly, I study a substantively motivated restriction on relative partisan power. In U.S. legislatures, majority parties typically exercise substantial control over committee assignments and committee leadership positions (Cox and McCubbins, 2005, 2007). To reflect this observation, I restrict proposal power to one side of the moderate legislator, \( M \). Formally, either \( \rho_L = 0 < \rho_R \) or \( \rho_R = 0 < \rho_L \).

**Definition 4.** The legislature exhibits *minority-party agenda exclusion* if one of the partisans, \( L \) or \( R \), has no agenda setting power, while the other, *majority*, partisan has positive recognition probability.
Definition 4 reflects the widespread belief that majority parties carefully allocate agenda setting power in the US. Yet, the model also aligns with empirical work suggesting that individual legislators possess some freedom from their party and thus can be influenced by interest groups (Fouriinaies, 2017).

**Definition 5.** Legislator \( \ell \) is *majority-leaning* if aligned with the majority partisan and similarly for the interest group, \( g \).

Proposition 3 shows that a majority-leaning ideologue group is indifferent over \( \alpha \) if \( \ell \) is also a majority-leaning ideologue under minority-party exclusion. But if \( \ell \) is a majority-leaning non-ideologue, then \( g \) wants access. The result focuses on majority-leaning legislators because minority-leaning legislators do not have proposal power and thus \( g \) is indifferent.

**Proposition 3.** Assume there is minority-party agenda exclusion and the interest group, \( g \), is a majority-leaning ideologue.

(i) If legislator \( \ell \) is a majority-leaning ideologue, then \( g \) is indifferent over access.

(ii) If \( \ell \) is a majority-leaning non-ideologue, then \( g \) acquires access.

Part (i) follows because \( g \) cannot profitably lobby to change \( \ell \)'s proposal. Part (ii) follows because access provides two benefits for \( g \) under minority-party exclusion. First, lobbying is profitable and greater \( \alpha \) increases \( g \)'s chances of enjoying that profit. Second, greater \( \alpha \) diminishes \( M \)'s expectations about future policy, and thus expands the acceptance set. Partisans can thus pass more extreme policy. Because minority partisans are unable to propose policy under minority-party agenda exclusion, \( g \) benefits from emboldening aligned partisan legislators without risking more extreme proposals by opposing partisans.

**Access and the Welfare of Legislators and Society**

*Legislator Welfare:* Ex ante, access can be good or bad from legislator \( \ell \)'s perspective.\(^{23}\) These effects arise entirely from changes in expected extremism. Whenever group \( g \) lobbies \( \ell \), it compensates \( \ell \) for any policy loss she suffers. Thus, access affects \( \ell \) only through the indirect effect on partisan proposals. The relative extremism of \( \ell \) and \( g \)

\(^{23}\)This is also true in, e.g., Schnakenberg (2017).
determine whether $\ell$ is better off. For example, $\ell$ may improve her expected welfare by giving access if $g$ is slightly more centrist. Here, access is mutually desirable because it acts as a commitment device on $\ell$’s proposals. In contrast, $\ell$ is always weakly worse off giving access if $g$ is more extreme and they are aligned, because extremism increases. These observations suggest that legislators may price discriminate based on group ideology when providing access.

**Social Welfare:** To measure social welfare, I use $M$’s expected payoff. This approach is appropriate if the median citizen is close to $\hat{x}_M$, so that the legislature suitably represents the ideological distribution of the unmodeled citizenry. Then $M$’s expected payoff corresponds to majority welfare in this ordered setting (Banks and Duggan, 2006b).

Connections shifting policy towards $\hat{x}_M$ also reduce expected extremism, and vice versa. Social welfare thus improves if groups access more extreme legislators and decreases if they access relative centrists. Therefore Propositions 2 and 3 have immediate welfare implications for a wide range of aligned group-legislator pairs. In some cases, groups do not acquire access and thus do not effect welfare. Highlighting this possibility, and the conditions producing it, is a key benefit of the formal analysis.

**Willingness to Acquire Access**

Thus far, the analysis considers whether groups want access. But what about contribution amounts? I provide two results in this direction. First, and consistent with a large body of empirical work, I demonstrate that groups are willing to pay more for access to legislators with greater proposal power. Second, under broad conditions I show that ideologically distant groups are willing to pay more to access a given legislator.

The results study $g$’s willingness to pay (WTP) for access. Ideally, we would characterize $g$’s optimal amount of access and compare the cost of that access under different conditions. This task requires specifying a cost function for access. Instead of restricting this class of functions, I study $g$’s WTP for access.24

**Proposition 4.** All else equal, an interest group’s willingness to pay for $\alpha$ access weakly increases with the targeted legislator’s proposal power.

Proposition 4 does not depend on the respective ideologies of the legislator and interest group. Furthermore, it does not require majority party control. Proposal

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24See, e.g., Denzau and Munger (1986) and Hall and Deardorff (2006) for previous work on access-seeking campaign contributions studying willingness to pay.
power amplifies the marginal benefit of access by increasing the probability that $g$ can extract surplus via lobbying. This increases the value of additional access. On the other hand, greater proposal power also increases how sensitive the acceptance set is to $\alpha$, which may help or harm the interest group. The overall effect is proportional to the legislator’s recognition probability if $g$’s WTP is strictly positive. Thus, if the group is willing to pay for access, then greater proposal power amplifies this desire.

Proposition 4 suggests groups will pay a higher price to access more powerful legislators. This implication fits the empirical regularity that legislators on important and relevant committees, especially committee chairmen, attract more contributions (Ainsworth, 2002; Grimmer and Powell, 2016; Berry and Fowler, 2018; Fourrinaies, 2017).

Next, I analyze how $g$’s ideology affects its willingness to buy access to a majority-leaning legislator under minority-party exclusion. Proposition 5 fixes $\hat{x}_\ell$ and analyzes $g$’s willingness to pay for increasing $\alpha$ from zero, which I refer to as $g$’s willingness to acquire access (WTA). Under broad conditions, $g$’s WTA weakly increases as its ideology diverges from $\hat{x}_\ell$ in either direction.

**Proposition 5.** Suppose there is minority-party agenda exclusion and legislator $\ell$ is majority-leaning. If either (i) the interest group, $g$, is more centrist than $\ell$, or (ii) $g$ is majority-leaning and more extreme than $\ell$, then $g$’s willingness to acquire access weakly increases as $g$ becomes less ideologically similar to $\ell$.

I discuss the logic using the case with right-party control, as illustrated in Figure 6. Part (i) assumes $g$ is more centrist than $\ell$. Two forces increase $g$’s WTA as $|\hat{x}_g - \hat{x}_\ell|$ increases. First, $g$’s lobbying surplus grows, so it has more to gain from increasing access. Second, $g$’s access forces majority-party partisans to moderate their policy proposals further because $g$’s policy offer gets better for $M$. Thus, $g$ gains more from inducing partisan moderation. These effects increase $g$’s WTA as $\hat{x}_g$ decrease away from $\hat{x}_\ell$. 

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Figure 6: Willingness to acquire access

The logic for part (ii) of Proposition 5 is best described in two cases.

First, suppose $g$ is partisan when $\alpha = 0$. In Figure 6, this corresponds to $\hat{x}_g \geq \pi_0$. If $\ell$ is centrist, as pictured in Figure 6, then $g$’s WTA decreases as $\hat{x}_g$ shifts towards $\hat{x}_\ell$ for reasons analogous to those described above: (i) $g$’s lobbying surplus decreases and (ii) $g$’s benefit from inciting more extreme partisan proposals also decreases. If $\ell$ is partisan, which corresponds to $\hat{x}_\ell \geq \pi_0$ in Figure 6, then $g$’s lobbying is inconsequential. Therefore $g$’s WTA is zero and thus constant as $\hat{x}_g$ approaches $\hat{x}_\ell$.

Second, suppose $g$ is centrist when $\alpha = 0$, which corresponds to $\hat{x}_g \in (\hat{x}_\ell, \pi_0)$ in Figure 6. Access now has competing effects. By logic similar to Proposition 2, $g$ forgoes access if $\hat{x}_g$ is close to $\hat{x}_\ell$. But $g$’s WTA increases as $\hat{x}_g$ increases away from $\hat{x}_\ell$. Specifically, whenever $g$’s WTA is positive, lobbying surplus grows faster than the loss from inciting more extreme partisan proposals.

An additional observation is that if $g$’s WTA is zero, then $g$ is not willing to pay for any positive amount of access. Proposition 5 thus implies that a majority-leaning group forgoes access if it is slightly more extreme than $\ell$, mirroring Proposition 2.\(^{25}\)

**Persistent vs. Short-term Access**

Thus far, I have studied persistent access, which has direct and indirect effects on the interest group’s ex ante welfare. The direct effect, opportunities to lobby, always

\(^{25}\)See Lemma 7 in Appendix A for more details.
benefits the group. The spillover effect, changing partisan proposals, can be good or bad. Any amount of persistence can produce these spillovers.

Temporary access can avoid spillovers and produce universal access seeking. First, suppose groups choose access only once and it lasts one period. Then access today does not affect expectations about future policymaking and the acceptance set, along with partisan proposals, do not change. The group always wants immediate, one-shot access because it provides only a direct benefit. Alternatively, if the group can set access each period, then it chooses full access every period of every stationary equilibrium.26

In some cases, a group’s optimal access contract is one period of immediate access followed by no future access. Consider the stationary equilibria mentioned above. Groups choose full access every period. Expected payoffs are thus equal to those from persistent full access. The main analysis implies some groups would rather commit to forgo access. But these groups most prefer immediate, one-shot access with no chance for later access. This arrangement has a direct benefit without any indirect cost because the acceptance set is unchanged. Thus, one of two possibilities must hold for these groups to pursue access. Either (i) access is temporary and commitment is impossible, or (ii) groups can contract against future access.

The preceding discussion has focused on group-legislator pairs in which legislative considerations discourage access. But recall some groups enjoy the indirect effect of persistent access. These groups always want access, regardless of its durability or contractability. Moreover, legislative considerations increase their desire for persistent access. In sum, the main analysis distinguishes groups that covet access from those less inclined.

Multiple Groups and Conceding Access

The main analysis abstracts from competition over access to isolate how legislative considerations affect access-seeking. Interest group competition has been studied elsewhere (e.g. Chamon and Kaplan, 2013; Levy and Razin, 2013) and a full analysis is outside the scope of this paper. Yet, the analysis offers several insights about when competition is unlikely.

In many policy areas, interest groups frequently lobby unopposed (Baumgartner and Leech, 2001; Leaver and Makris, 2006; Dal Bó, 2007). Where are competing interests?

26In a stationary strategy profile, a one-shot access deviation does not change expectations about future policymaking. Thus, the group always has a profitable deviation if it is not choosing full access.
Existing explanations include collective action problems, free-riding incentives, and entry costs. This paper sheds further light on when these competitive voids can arise, as legislative considerations can produce strong anti-competitive incentives.

To fix ideas, say that group \( g \) concedes access to group \( g' \) if \( g \) strictly prefers letting \( g' \) optimally choose access. Some groups concede durable access purely for policy reasons. Specifically, some groups prefer conceding to more centrist groups even if both access and lobbying are free. Moreover, this preference can arise regardless of whether the group wants access in isolation.

For example, suppose \( g \) is a non-ideologue contemplating access to a slightly more centrist legislator, \( \ell \). By Proposition 2, \( g \) does not want access to \( \ell \). Furthermore, if there is another group \( g' \) slightly more centrist than \( \ell \), then \( g \) strictly prefers to concede access to \( g' \). Why? Group \( g \) forgoes access to avoid increasing expected extremism, but is even better off reducing expected extremism. Because \( g' \) is more centrist than \( \ell \), it seeks access by Proposition 2, thereby reducing expected extremism. Thus, if \( g' \) is not too far from \( \ell \), then \( g \) concedes access to \( g' \). In this case, conceding access reduces expected extremism beyond what \( g \) achieves by simply forgoing access.

This illustration reveals how legislative forces can discourage competition. Conceding to more centrist groups may be attractive because it allows otherwise unobtainable moderation. As discussed, groups may even concede access to groups on the opposite side of a given legislator. Substantively, this could be why some industries lobby through trade associations funded by diverse interests.

### Effects of Legislative Conditions on Expenditures

Having studied access acquisition, I now fix access and characterize how equilibrium lobbying expenditures vary with legislative features. In general, lobbying expenditures weakly increase as the acceptance set expands. Thus, the characterization of \( \pi(\sigma) \) in (6) has direct implications for expenditures. I begin by cataloging the relevant legislative features and describing their effects on the acceptance set.

The first legislative feature I study is the distribution of agenda power, \( \rho \). Given \( \rho \) and access \( \alpha \), let the moderate legislator’s unconstrained extremism lottery be the lottery putting probability \( \alpha \rho_\ell \) on \(|\hat{y}|\), probability \( \rho_\ell (1 - \alpha) \) on \(|\hat{x}_\ell|\), and probability \( \rho_j \) on \(|\hat{x}_j|\) for each legislator \( j \neq \ell \). Thus, the outcomes of an unconstrained extremism lottery are measured in terms of absolute distance between each player’s ideal proposal.
and \( \hat{x}_M = 0 \). Say that legislative extremism under \((\rho', \alpha')\) is higher than \((\rho, \alpha)\) if \( M \)'s unconstrained extremism lottery induced by \((\rho', \alpha')\) first order stochastically dominates the lottery induced by \((\rho, \alpha)\).  \(^{27}\) For example, legislative extremism increases if proposal power shifts away from \( M \) to other legislators. This change lowers \( M \)'s reservation value because extreme policy proposals become more likely, without an offsetting increase in the chance of moderate policy proposals. Thus, \( M \) is willing to accept more extreme policy proposals and the acceptance set expands.

Second, I vary the location of the status quo, \( q \). More extreme status quo lower \( M \)'s reservation value because she is more averse to enduring the status quo until a new policy passes. Thus, the acceptance set expands.

Third is legislator patience, \( \delta \). As \( \delta \) increases, \( M \) is less bothered by enduring the status quo and places more weight on policies she will pass. Thus, greater patience shrinks the acceptance set.

As noted above, expanding the acceptance set weakly increases ex post lobbying payments. The preceding observations thus characterize how expenditures vary.

**Proposition 6.** The interest group’s equilibrium lobbying expenditures weakly increase as either (i) legislative extremism increases, holding constant \( \hat{x}_g \) and \( \hat{x}_\ell \); (ii) the status quo policy becomes more extreme; or (iii) legislator patience decreases.

To see the logic for Proposition 6, recall \( g \)'s equilibrium transfer to \( \ell \) is \( m = u_\ell(z_\ell) - u_\ell(y) \). Therefore equilibrium lobbying expenditures increase if either (i) \( g \)'s policy offer becomes worse for \( \ell \) or (ii) \( \ell \) can pass more favorable policy after rejecting \( g \)'s overtures. Thus, there are two ways that a larger acceptance set can increase expenditures: (i) more slack for \( g \) to shift \( \ell \)'s proposal, or (ii) a better outside option for \( \ell \).

First, if \( g \) is extremist and \( M \) constrains \( g \)'s equilibrium policy offer, then greater legislative extremism gives \( g \) more slack to lobby \( \ell \) to more extreme policy. Consequently, lobbying expenditures increase because \( g \)'s policy offer is worse for \( \ell \). Figure 7 displays this case. If \( g \) and \( \ell \) are aligned extremists, however, then \( z_\ell = y \) and lobbying expenditures are constant for small enough changes in legislative extremism.

Second, if \( \ell \) is extremist, then increasing legislative extremism improves \( \ell \)'s outside option because \( z_\ell \) equals the boundary of \( A(\sigma) \) closest to \( \hat{x}_\ell \). This boundary shifts

\(^{27}\)In this context, the unconstrained extremism lottery \((\rho', \alpha')\) first order stochastically dominates another unconstrained extremism lottery \((\rho, \alpha)\) if: (i) for all \( x \in X \), \((\rho', \alpha')\) puts weakly greater probability on \( x' \) such that \(|x'| \geq |x|\) and (ii) for some \( x \in X \), \((\rho', \alpha')\) puts strictly greater probability on \( x' \) such that \(|x'| \geq |x|\).
Figure 7: Increasing lobbying expenditures – extreme group

(a) Figure 7(a) displays legislator ℓ’s proposals for a baseline acceptance set where g’s offer is constrained and ℓ’s proposal is unconstrained. Figure 7(b) displays ℓ’s proposals if the acceptance set expands. Expenditures increase because g pays more for a better offer.

(b)

Figure 8: Increasing lobbying expenditures – centrist group

(a) Figure 7(a) displays legislator ℓ’s proposals for a baseline acceptance set where g’s offer is unconstrained and ℓ’s proposal is constrained. Figure 7(b) displays ℓ’s proposals if the acceptance set expands. Expenditures increase because ℓ’s reservation value increases.

(b)

An important special case of changing legislative extremism is varying g’s access,
α. Greater access causes $M$ to anticipate more frequent lobbying by $g$. Thus, α’s effect on the acceptance set depends on $g$ and $\ell$’s relative ideology. If $g$ is more extreme, then increasing α raises legislative extremism and the acceptance set expands. This relationship flips if $g$ is more centrist. Given a group-legislator pair, Proposition 6 yields an immediate corollary on the relationship between access and lobbying expenditures.

**Corollary 1.** Suppose the interest group, $g$, is aligned with legislator $\ell$. If $g$ is more extreme than $\ell$, then equilibrium lobbying expenditures weakly increase with access. Otherwise, they weakly decrease with access.

Next, I state two corollaries of Proposition 6 showing how substantively meaningful features of the model affect extremism and, in turn, lobbying expenditures. First, Corollary 2 establishes that lobbying expenditures grow if $M$ loses proposal power, which weakly increases legislative extremism. Substantively, this result suggests that weakening centrist agenda setting power encourages more vigorous lobbying.

**Corollary 2.** If proposal power transfers away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.

Corollary 3 states that lobbying expenditures grow weakly as $\ell$ shifts away from $M$, which weakly increases legislative extremism. This result suggests that groups spend more on lobbying in legislatures that are more polarized, in the colloquial sense of having greater ideological spread among legislators.

**Corollary 3.** If legislator $\ell$ shifts farther away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.

**Conclusion**

I study which legislators and interest groups form connections to facilitate lobbying. To do so, I analyze a model where groups choose access before policymaking. Access provides opportunities for groups to influence policy proposals by lobbying during policymaking. The model provides a tractable framework to explore how access-seeking depends on the larger legislative context.

Interest groups weigh various institutional and political factors when deciding whether to pursue access. Does greater access increase or decrease policy extremism in the legislature? Is the targeted legislator likely to have much control over policymaking? Are
partisan legislators likely to draft policy? I refine our understanding of how groups weigh these questions when evaluating who they want to access.

I highlight conditions under which lobbying increases or decreases policy extremism. Then, I show that groups avoid access to particular legislators under broad conditions. Specifically, if groups are not too extreme, they forego access to a range of more centrist legislators. Policy considerations drive this behavior, as access to these legislators generates increased policy polarization that counteracts better lobbying prospects. More generally, the analysis unpacks a neglected consequence of access: the prospect of lobbying can spill over and affect policies proposed by other legislators.

The analysis has implications for campaign finance, revolving-door hiring, and lobbying expenditures. First, which legislators do access-seeking interest groups direct campaign contributions towards? And whose associates do they hire through the revolving door? Second, which groups lobby which legislators? Third, what can lobbying expenditures tell us about access. Finally, why do many groups contribute so little (Tullock, 1972; Ansolabehere et al., 2003)? Distinguishing between access and lobbying is key for these implications. I now elaborate on each.

First, analyzing endogenous access suggests which connections are likely to form. In practice, campaign contributions and revolving-door hiring are prominent channels for access. Propositions 2 and 3 thus have implications for both (i) whose staffers interest groups hire and (ii) to whom they contribute. Data exist for both. Yet, using revolving-door hiring data may be a better starting point because most measures of group and legislator ideology use contributions to locate the actors on a common scale. Given group ideology, the model suggests a curvilinear, and possibly multimodal, relationship between hiring/contributions and legislator ideology. An especially strong prediction is that groups never access very extreme legislators.

Second, and reflecting with the widespread view, access is necessary for lobbying in the model. The analysis thus suggests immediate qualitative predictions about who lobbies whom. In some instances, detailed lobbying data specify targeted legislators. Another possibility is data on “points of contact,” which may provide information about which legislators groups target.

Third, the relationship between observed expenditures and access is conditional on relative ideology. Thus, empirical work should control for relative ideology when evaluating the connection between access and lobbying expenditures. Otherwise, offsetting observations may obscure a meaningful effect. Related, recovering a negative relation-
ship between access and lobbying expenditures for centrist groups targeting extreme legislators need not imply that groups get less for their money. Instead, expenditures may decline because the targeted legislator’s outside option is worse. Another implication is that ceteris paribus changes in lobbying expenditures can indicate changes in access amounts. With information about relative ideology, we can infer the direction of the change.

Finally, the analysis speaks to Tullock’s puzzle, that many groups do not contribute at all and those that do rarely reach legal limits (Tullock, 1972). Some view this empirical regularity as evidence that either contributions are not valuable, or donors are unsophisticated (Ansolabehere et al., 2003). Previous work has shown that strategic forces can lead sophisticated groups to contribute small amounts (Chamon and Kaplan, 2013). I provide a new strategic mechanism for such behavior, legislative considerations, which can reduce contributions by sophisticated groups precisely because they are valuable for gaining access and generate adverse spillover effects.
Appendix A

Model

I prove the main results in a more general version of the model, relaxing restrictions on the number of legislators and interest groups. There are three disjoint sets of players: $n^V$ (finite and odd) voting legislators in $N^V$; $n^L \geq 3$ committee members in $N^L$; and $n^G \leq n^L$ interest groups in $N^G$. Let $N = N^V \cup N^L \cup N^G$. Throughout, voting legislators are called voters and denoted by $i$. To align with the main text, $M$ denotes the median voter. I denote committee members by $\ell$ and interest groups by $g$.

Each $\ell \in N^L$ is associated with only one group, $g_\ell$. Each group $g$ can have access to multiple $\ell \in N^L$ and this set is $N^L_g \subseteq N^L$. Let $\alpha_\ell \in [0,1]$ denote $g_\ell$’s access to $\ell$.28

Legislative bargaining occurs over an infinite number of periods $t \in \{1,2,\ldots\}$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Let $\rho = (\rho_1,\ldots,\rho_{n^L}) \in \Delta([0,1])^{n^L}$ be the distribution of recognition probability among $\ell \in N^L$.29 In each period $t$, bargaining proceeds as follows. If no policy has passed before $t$, then $\ell$ proposes with probability $\rho_\ell > 0$. All players observe the period-$t$ proposer, $\ell_t$. With probability $1 - \alpha_\ell$, $g_\ell$ cannot lobby and $\ell_t$ freely proposes any $x_t \in X$. Conversely, with probability $\alpha_\ell$, $g_\ell$ can lobby and offers $\ell_t$ a binding contract $(y_t,m_t) \in X \times \mathbb{R}_+$. Next, $\ell_t$ accepts or rejects. Let $a_t \in \{0,1\}$ denote $\ell_t$’s period-$t$ acceptance decision, where $a_t = 1$ indicates acceptance and $a_t = 0$ if either $\ell_t$ rejects or $g_\ell$ is unable to lobby in $t$. If $\ell_t$ accepts, then $\ell_t$ is committed to propose $x_t = y_t$ in $t$ and $g_\ell$ transfers $m_t$ to $\ell_t$. If $\ell_t$ rejects, then she can propose any $x_t \in X$ and $g_\ell$ keeps $m_t$. All players observe $x_t$. There is a simultaneous vote by $i \in N^V$ using simple majority rule. If $x_t$ passes, then bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $x_t$ fails, then $q$ is enacted in $t$ and bargaining proceeds to $t+1$.

Each player $j \in N$ has quadratic policy utility with ideal point $\hat{x}_j \in X$. As in the main text, I normalize $\hat{x}_M = 0$ and assume $q \neq 0$. Additionally, I assume there exists $\ell \in N^L$ on the same side of $q$ as $M$ such that either $\alpha_\ell = 0$ or $g_\ell$ is on the same side of $q$. For example, assume $q > 0$. Then some $\ell \in N^L$ satisfies $\hat{x}_\ell < q$ and either (i) $\hat{x}_{g_\ell} < q$, or (ii) $\alpha_\ell = 0$. This assumption ensures some $\ell$ who wants to move policy in the same direction as $M$ and, moreover, $g_\ell$’s lobbying does not counteract this preference.

Players discount streams of stage utility by common discount factor $\delta \in (0,1)$.28 An independent legislator is accommodated by setting $\alpha_\ell = 0$.29 Where $\Delta([0,1])^{n^L}$ denotes the $n^L$-dimensional unit simplex.
For convenience, I normalize per-period payoffs by \((1 - \delta)\). Let \(I_t^\ell \in \{0, 1\}\) equal one iff \(\ell\) is the period-\(t\) proposer and \(g_\ell\) can lobby in \(t\). Given a sequence of offers \((y_1, m_1), (y_2, m_2), \ldots\), a sequence of proposers \(\ell_1, \ell_2, \ldots\) a sequence of acceptance decisions \(a_1, a_2, \ldots\), and a sequence of independent policy proposals \(x_1, x_2, \ldots\) such that bargaining continues until \(t\), the discounted sum of per-period payoffs for \(i \in N^V\) is

\[
(1 - \delta^{t-1})u_i(q) + \delta^{t-1}\left[(1 - a_t)u_i(x_t) + a_t u_i(y_t)\right];
\]

for \(\ell \in N^\ell\),

\[
(1 - \delta)\sum_{t'=1}^{t-1} \delta^{t'-1}[u_\ell(q) + I_{t'}^\ell a_{t'} m_{t'}] + \delta^{t-1}\left[(1 - a_t)u_\ell(x_t) + a_t \left(u_\ell(y_t) + I_t^\ell m_t\right)\right];
\]

and for \(g \in N^g\),

\[
(1 - \delta)\sum_{t'=1}^{t-1} \delta^{t'-1}\left[u_g(q) - a_{t'} m_{t'} \sum_{\ell \in N^g} I_{t'}^\ell\right] + \delta^{t-1}\left[(1 - a_t)u_g(x_t) + a_t \left(u_g(y_t) - m_t \sum_{\ell \in N^g} I_t^\ell\right)\right].
\]

Unless noted otherwise, results are proved for this more general setting. The model in the main text is a special case featuring: one voter with ideal point \(\hat{x}_M\); four committee members with ideal points \(\hat{x}_L, \hat{x}_M, \hat{x}_R\); and one group at \(\hat{x}_g\) with access \(\alpha_\ell \geq 0\) and \(\alpha_j = 0\) for all \(j \neq \ell\).

### Strategies

I study a class of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.\(^{30}\)

Let \(\Delta(X)\) be the set of probability measures on \(X\). Let \(W = X \times \mathbb{R}_+\) denote the offer space and \(\Delta(W)\) denote the set of probability measures on \(W\). A stationary mixed strategy for \(g \in N^G\) is a probability measure \(\lambda_g \in \Delta(W)^{|N^G|}\) over \(g\)'s offers \((y, m) \in W\) to each \(\ell \in N^L_g\). A stationary mixed legislative strategy for \(\ell \in N^L_g\) is

\[^{30}\text{In Appendix B, I define mixed strategy stationary legislative lobbying equilibrium and show that every mixed strategy stationary legislative lobbying equilibrium is equivalent in outcome distribution to a pure strategy no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.}\]
a pair \((\pi_\ell, \varphi_\ell)\); where \(\pi_\ell \in \Delta(X)\) specifies a probability measure over \(\ell\)'s independent proposals and \(\varphi_\ell : W \to [0, 1]\) is the probability \(\ell\) accepts each \((y, m) \in W\). Finally, voter \(i\)'s stationary mixed strategy \(\nu_i : X \to [0, 1]\) specifies the probability \(i\) votes for \(x \in X\).

Let \(\lambda\) denote a profile of interest group strategies, \((\pi, \varphi)\) a profile of committee member strategies, and \(\nu\) a profile of voter strategies. A stationary strategy profile is \(\sigma = (\lambda, \pi, \varphi, \nu)\). Finally, let \(\nu_\sigma(x)\) represent be the probability that \(x\) passes in a given period under \(\sigma\).

**Continuation Values**

Let \(w = (y, m) \in W\) denote an arbitrary interest group offer. For convenience, define

\[
\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda_\ell^y(dw),
\]

(12)

which is the probability under \(\sigma\) that \(\ell\) makes an independent policy proposal in any period she is recognized. Given \(\sigma\), \(i \in NV\) has continuation value

\[
V_i(\sigma) = \sum_{\ell \in NL} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta V_i(\sigma)] \right] \lambda_\ell^y(dw)
\]

\[
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)][(1 - \delta)u_\ell(q) + \delta V_i(\sigma)] \right] \pi_\ell(dx) \right),
\]

(13)

the continuation value of \(\ell \in NL\) is

\[
\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \int_W \varphi_j(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda_\ell^y(dw)
\]

\[
+ \xi_j(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right),
\]

\[
+ \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m \right] \lambda_\ell^y(dw) \right)
\]

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\[ + \xi(\alpha, \sigma) \int_X \left[ \varphi(y) u_g(y) + [1 - \varphi(y)] [(1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \pi(x) dx, \]

and the continuation value of \( g \in N^G \) is

\[ \hat{V}_g(\sigma) = \sum_{\ell \in N^L_g} \rho_{\ell} \left( \alpha \int_W \varphi(y, m) \left[ \varphi(y) u_g(y) + [1 - \varphi(y)] [(1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \lambda_{g\ell}(dw) \right. \\
+ \xi(\alpha, \sigma) \int_X \left[ \varphi(y) u_g(y) + [1 - \varphi(y)] [(1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \pi(x) dx, \]

\[ + \sum_{\ell \in N^L_g} \rho_{\ell} \left[ \alpha \int_W \varphi(y, m) \left[ \varphi(y) u_g(y) + [1 - \varphi(y)] [(1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma)] - m \right] \lambda_{g\ell}(dw) \right. \\
+ \xi(\alpha, \sigma) \int_X \left[ \varphi(y) u_g(y) + [1 - \varphi(y)] [(1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \pi(x) dx. \]

**Stationary Legislative Lobbying Equilibrium**

A stationary pure strategy for \( g \in N^G \) is \((y, m, z, a) \in X^{|N^L_g|} \times \mathbb{R}^{|N^L_g|}, \) where \( y \) is a profile of policy offers and \( m \) is a profile of monetary offers. A pure stationary strategy for \( \ell \in N^L_g \) is \((z, a) \); where \( z \in X \) specifies \( \ell \)'s independent proposals, and \( a : X \times \mathbb{R} \rightarrow \{0, 1\} \) equals one iff \( \ell \) accepts \( g \)'s offer. Finally, for each \( i \in N^V, v_i : X \rightarrow \{0, 1\} \) equals one iff \( i \) supports the proposal.

Given \( \sigma \), the set of policies that pass is constant across periods by stationarity and denoted \( A(\sigma) \subset X \). For \( \ell \in N^L_g \), define

\[ \tilde{U}_\ell(x; \sigma) = \begin{cases} 
  u_\ell(x) & \text{if } x \in A(\sigma) \\
  (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) & \text{else.} 
\end{cases} \]

Formally, \( \sigma = (y, m, z, a, v) \) is a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance if it satisfies five conditions. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), \((y^\ell_g, m^\ell_g) \) satisfies

\[ y^\ell_g = \arg \max_{y \in A(\sigma)} u_{g\ell}(y) + u_\ell(y) - u_\ell(z_\ell) \] (17)
and
\[ m_g^\ell = u_\ell(z_\ell) - u_\ell(y_g^\ell). \] (18)

Second, for all \( \ell \in N^L \) and \( (y, m) \in W \), \( a_\ell(y, m) = 1 \) iff
\[ \tilde{U}_\ell(y; \sigma) + m \geq \tilde{U}_\ell(z_\ell; \sigma). \] (19)

Third, for each \( \ell \in N^L \), \( z_\ell \) solves
\[ \max_{x \in A(\sigma)} u_\ell(x). \] (20)

Finally, for each \( i \in N^V \), \( v_i(x) = 1 \) iff
\[ u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma). \] (21)

Appendix B shows that all stationary mixed strategy legislative lobbying equilibria are equivalent in outcome distribution to strategy profiles satisfying (17)-(21).

**Existence**

The following shows part 1 of Proposition 1 from the main text.

**Proposition 1.1.** There exists a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.

**Proof.** There are three parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for \( g \in N^G \) and \( \ell \in N^L \). Part 2 uses the fixed point to construct a strategy profile \( \sigma \). Part 3 checks that \( \sigma \) satisfies (17) - (21).

**Part 1:** Let \( (y, z) = (y_1, \ldots, y_{n^L}, z_1, \ldots, z_{n^L}) \in X^{2n^L} \) and for each \( j \in N \) define
\[ r_j(y, z) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell u_j(y_\ell) + (1 - \alpha_\ell)u_j(z_\ell) \right). \] (22)

Set \( A(r(y, z)) = \{ x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta r_M(y, z) \} \), which is non-empty, convex, and compact because \( u_M \) is strictly concave, \( X \) is compact and convex, \( q = 0 \),
and \( \delta \in (0, 1) \). Moreover, \( A(r(y, z)) \) is continuous in \((y, z)\).

For each \( \ell \in N^L \), define

\[
\tilde{\phi}_\ell(y, z) = \arg \max_{x \in A(r(y, z))} u_{\ell}(x) + u_\ell(x),
\]

which is unique for all \((y, z)\) because the objective function is strictly concave and continuous and \( A(r(y, z)) \) is non-empty, compact and convex. Because \( A(r(y, z)) \) is continuous, the Theorem of the Maximum implies continuity of \( \tilde{\phi}_\ell(y, z) \). Next, define

\[
\phi_\ell(y, z) = \arg \max_{x \in A(r(y, z))} u_\ell(x),
\]

which is unique for all \((y, z)\) and continuous by the Theorem of the Maximum.

Define the mapping \( \Phi : X^{2n_L} \to X^{2n_L} \) as \( \Phi(y, z) = \prod_{\ell \in N_L} \tilde{\phi}_\ell(y, z) \times \prod_{\ell \in N_L} \phi_\ell(y, z) \), which is a product of continuous functions and thus continuous in \((y, z)\). By Brouwer’s theorem, a fixed point \((y^*, z^*) = \Phi(y^*, z^*) \) exists because \( \Phi \) is a continuous function mapping a non-empty, convex, and compact set into itself.

**Part 2:** Define a stationary pure strategy profile \( \sigma \) as follows. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), set \( y^\ell_g = y^*_\ell \) and \( m^\ell_g = u_\ell(z^*_\ell) - u_\ell(y^*_\ell) \). Next, for \( \ell \in N^L \), set \( z_\ell = z^*_\ell \) and define

\[
a_\ell(y, m) = \begin{cases} 
  1 & \text{if } u_\ell(y) + m \geq u_\ell(z^*_\ell), \text{ for } y \in A(r(y^*, z^*)) \\
  1 & \text{if } (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell), \text{ for } y \notin A(r(y^*, z^*)) \\
  0 & \text{else.}
\end{cases}
\]

Finally, for each \( i \in N^V \) define \( v_i \) so that \( v_i(x) = 1 \) if \( u_i(x) \geq (1 - \delta)u_i(q) + \delta r_\ell(y^*, z^*) \) and \( v_i(x) = 0 \) otherwise.

**Part 3:** I check that \( \sigma \) satisfies (17)-(21).

First, I verify (21) to show \( A(\sigma) = A(r(y^*, z^*)) \). Note that for each \( g \in N^G \) and all \( \ell \in N^L_g \), \( a_\ell(y^\ell_g, m^\ell_g) = 1 \) and \( y^\ell_g = y^*_\ell \in A(r(y^*, z^*)) \). Moreover, \( z_\ell = z^*_\ell \in A(r(y^*, z^*)) \) for all \( \ell \in N^L \). Thus, voter \( i \)'s continuation value under \( \sigma \) is \( V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell [\alpha_\ell u_i(y^*_\ell) + (1 - \alpha_\ell)u_i(z^*_\ell)] = r_i(y^*, z^*) \). Thus, each voter \( i \)'s strategy satisfies (21). Banks and Duggan (2006b) and Duggan (2014) apply, so \( M \) is decisive over lotteries and \( A(\sigma) = A(r(y^*, z^*)) \).
To check (17), consider \( g \in N^G \) and \( \ell \in N^L_g \). Focusing on acceptable offers is without loss of generality because \( a_\ell(z_\ell, 0) = 1 \). Because \( A(\sigma) = A(r(y^*, z^*)) \), (23) implies \( \tilde{\phi}_\ell(y^*, z^*) = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell^*) \). Thus, (17) holds because \( \tilde{\phi}_\ell(y^*, z^*) = y_\ell^* = y_\ell^2 \). Lemma B.6 in Appendix B implies \( y \notin A(\sigma) \) is not a profitable deviation for any \( g \in N^G \).

It is immediate that \( m_\ell^\ell \) satisfies (18).

To check (19), note that \( \ell \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( \tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z_\ell^*) \). Thus, \( \ell \) weakly prefers to accept any \((y, m)\) satisfying \( y \in A(r(y^*, z^*)) \) iff \( u_\ell(y) + m \geq u_\ell(z_\ell^*) \). If \( y \notin A(r(y^*, z^*)) \), then \( \ell \) weakly prefers to accept \((y, m)\) iff \( (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell\alpha m_\ell^\ell) + m \geq u_\ell(z_\ell^*) \). Thus, \( a_\ell \) satisfies (19).

To check (20), note that (24) implies \( \phi_\ell(y^*, z^*) = \arg\max_{x \in A(\sigma)} u_\ell(x) \) because \( A(\sigma) = A(r(y^*, z^*)) \). Thus, (20) holds because \( \phi_\ell(y^*, z^*) = z^* = z_\ell \) for each \( \ell \in N^L \). By Lemma B.6 in Appendix B, \( x \notin A(\sigma) \) is not a profitable deviation for any \( \ell \in N^L \).

\[ \hat{y}_\ell = \arg\max_{y \in X} u_{g_\ell}(y) + u_\ell(y) = \frac{\hat{x}_{g_\ell} + \hat{x}_\ell}{2}. \] (26)

Recall \( u_\ell(z_\ell) \) is \( \ell \)'s expected dynamic payoff in equilibrium, conditional on rejecting \( g_\ell \)'s offer. By (17), in equilibrium

\[ y_\ell^\ell = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y). \] (27)

If \( \hat{y}_\ell \in A(\sigma) \), then \( y_\ell^\ell = \hat{y}_\ell \). Otherwise, strict concavity implies \( y_\ell^\ell \) equals the boundary of \( A(\sigma) \) closest to \( \hat{y}_\ell \). As this characterization applies to every equilibrium, there is a clear connection to the characterization in Cho and Duggan (2003), where lobbying is absent.

Proposition 1.3 establishes Part 3 of Proposition 1 from the main text.
Proposition 1.3. Every stationary legislative lobbying equilibrium has the same outcome distribution.

Proof. Let $\sigma$ and $\sigma'$ be stationary legislative lobbying equilibria. It suffices to show $(y_g, m_g) = (y'_g, m'_g)$ for all $g \in N^C$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Assume $y_{g\ell} \neq y'_{g\ell}$ or $z_\ell \neq z'_\ell$ for some $\ell \in N^L$. Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, $A(\sigma) = A(\sigma')$. Because $\sigma$ and $\sigma'$ are no-delay by Lemma B.6, $\ell$'s expected dynamic payoff from rejecting $g_\ell$'s offer is $u_\ell(z_\ell)$ under both $\sigma$ and $\sigma'$. Lemma B.1 implies $m^\ell_g = u_\ell(y^\ell_g) - u_\ell(z_\ell)$. Therefore $(y_g, m_g) = (y'_g, m'_g)$ and $z_\ell = z'_\ell$.

Comparative Statics on Lobbying Expenditures

To facilitate the analysis of endogenous access, it is useful to first prove Proposition 6. Set $\theta = (\hat{x}, \rho, \alpha)$. Let $\mu_\theta$ denote the unconstrained extremism lottery, which puts probability $\rho_b \alpha_\ell$ on $|\hat{y}_\ell|$ and probability $\rho_b(1 - \alpha_\ell)$ on $|\hat{x}_\ell|$ for each $\ell \in N^L$. Given $\theta$ and $\theta'$, legislative extremism is greater under $\theta'$ if $\mu_{\theta'}$ first order stochastically dominates $\mu_\theta$.

Lemma 2. The equilibrium acceptance set weakly expands with legislative extremism.

Proof. Consider $\theta$ and $\theta'$, with legislative extremism greater under $\theta'$. By Proposition 1.3, $\theta$ and $\theta'$ each induce a unique equilibrium acceptance set. Let $\overline{x}_\theta$ and $\overline{x}_{\theta'}$ denote the respective upper bounds of these sets. I show $\overline{x}_{\theta'} \geq \overline{x}_\theta$.

For $b \geq 0$, let $C^b_j = \mathbb{I}\{\hat{x}_j \in (-b, b)\}$ and $\tilde{C}^b_j = \mathbb{I}\{\hat{y}_j \in (-b, b)\}$. Define $C^b$ and $\tilde{C}^b$ analogously for $\hat{x}_j$ and $\hat{y}_j$. For all $b \geq 0$,

$$
(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)C^b_j u_M(\hat{x}_j) + \alpha_j \tilde{C}^b_j u_M(\hat{y}_j) \right)
+ \delta u_M(b) \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)(1 - C^b_j) + \alpha_j(1 - \tilde{C}^b_j) \right)
\geq (1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho'_j \left( (1 - \alpha'_j)C^b_j u_M(\hat{x}'_j) + \alpha'_j \tilde{C}^b_j u_M(\hat{y}'_j) \right)
+ \delta u_M(b) \sum_{j \in N^L} \rho'_j \left( (1 - \alpha'_j)(1 - C^b_j) + \alpha'_j(1 - \tilde{C}^b_j) \right),
$$

where (29) follows because $\mu_{\theta'}$ FOSD $\mu_\theta$ and $u_M$ is negative quadratic. The equilibrium characterization, and construction of $C_j$ and $\tilde{C}_j$, implies $\overline{x}_\theta$ is the unique $b \geq 0$ such
that \( u_M(b) \) equals (28). Analogously, \( \pi_{\theta'} \) is the unique \( b \geq 0 \) such that \( u_M(b) \) equals (29). Thus, \( \pi_{\theta'} \geq \pi_{\theta} \).

**Proposition 6.** For all \( \ell \in N^L \), \( g_{\ell} \)'s equilibrium lobbying expenditures increase as either (i) legislative extremism increases, fixing \( \hat{x}_{\ell} \) and \( \hat{x}_{g_{\ell}} \); (ii) \( |q| \) increases; or (iii) \( \delta \) decreases.

**Proof.** (i) Increase legislative extremism. Let \( \sigma \) denote an equilibrium, suppressing dependence on legislative extremism. By Lemma 2, \( \pi(\sigma) \) weakly increases with legislative extremism. There are two cases.

- **Case 1.** Suppose \( \hat{x}_{\ell} \in A(\sigma) \). Then \( z_{\ell} = \hat{x}_{\ell} \). There are two subcases.

  First, assume \( \hat{y}_{g_{\ell}} \in A(\sigma) \). Thus, \( y_{g_{\ell}} = \hat{y}_{g_{\ell}} \). Lemma B.1 and (17) imply \( m_{g_{\ell}} = u_{t}(\hat{x}_{\ell}) - u_{t}(\hat{y}_{g_{\ell}}) \). From Lemma 2, \( z_{\ell} = \hat{x}_{\ell} \) and \( y_{g_{\ell}} = \hat{y}_{g_{\ell}} \) as legislative extremism increases, so \( m_{g_{\ell}} \) is constant.

  Second, assume \( \hat{y}_{g_{\ell}} \notin A(\sigma) \). Since \( \hat{x}_{\ell} \in A(\sigma) \), this requires \( \hat{x}_{g_{\ell}} \notin [-\pi(\sigma), \pi(\sigma)] \).

  Without loss of generality, assume \( \hat{x}_{g_{\ell}} > \pi(\sigma) \). Then \( \hat{x}_{\ell} \in A(\sigma) \) and \( \hat{y}_{g_{\ell}} \notin A(\sigma) \) imply \( \hat{y}_{g_{\ell}} > \pi(\sigma) \). Thus, \( z_{\ell} = \hat{x}_{\ell} \) and \( y_{g_{\ell}} = \hat{y}_{g_{\ell}} \). Lemma B.1 and (17) imply \( m_{g_{\ell}} = u_{t}(\hat{x}_{\ell}) - u_{t}(\pi(\sigma)) \). From Lemma 2, \( \pi(\sigma) \) increases in legislative extremism. Thus, \( m_{g_{\ell}} \) increases.

- **Case 2.** Suppose \( \hat{x}_{\ell} \notin A(\sigma) \). Without loss of generality, assume \( \hat{x}_{\ell} > z_{\ell} = \pi(\sigma) \). There are three subcases.

  First, assume \( \hat{y}_{g_{\ell}} < -\pi(\sigma) \). Then \( y_{g_{\ell}} = -\pi(\sigma) \). By Lemma B.1 and (17), \( m_{g_{\ell}} = u_{t}(\pi(\sigma)) - u_{t}(-\pi(\sigma)) \). By Lemma 2, increasing legislative extremism increases \( \pi(\sigma) \) and decreases \( -\pi(\sigma) \). Thus, \( m_{g_{\ell}} \) increases because \( -\pi(\sigma) < \pi(\sigma) < \hat{x}_{\ell} \).

  Second, assume \( \hat{y}_{g_{\ell}} \in A(\sigma) \). Thus, \( y_{g_{\ell}} = \hat{y}_{g_{\ell}} \) and \( y_{g_{\ell}} \) is constant as legislative extremism increases. Arguments similar to subcase 2 of Case 1 imply \( m_{g_{\ell}} \) increases.

  Third, assume \( \hat{y}_{g_{\ell}} \geq \pi(\sigma) \), which implies \( y_{g_{\ell}} = \pi(\sigma) \). By Lemma B.1 and (17), \( m_{g_{\ell}} = u_{t}(\pi(\sigma)) - u_{t}(\pi(\sigma)) \), which is constant in legislative extremism.

  Altogether, \( m_{g_{\ell}} \) weakly increases in legislative extremism.

(ii) Increase \( |q| \). First, let \( C_{j}(\hat{x}_{\ell}) = \mathbb{I}\{\hat{x}_{j} \in \text{int}A(\sigma)\} \). Similarly, let \( \tilde{C}_{j}(\hat{y}_{j}) = \mathbb{I}\{\hat{y}_{j} \in \text{int}A(\sigma)\} \).
Then

\[
\pi(\sigma) = \left( -\frac{(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left[ C_j(\hat{x}_\ell)(1 - \alpha_j)u_M(\hat{x}_j) + \tilde{C}_j(\hat{y}_j)\alpha_j u_M(\hat{y}_j) \right]}{1 - \delta \sum_{j \in N^L} \rho_j \left[ (1 - C_j(\hat{x}_\ell))(1 - \alpha_j) + (1 - \tilde{C}_j(\hat{y}_j))\alpha_j \right]} \right)^{\frac{1}{\delta}},
\]

(30)

Inspection of (30) shows \(\pi(\sigma)\) strictly increases in \(|q|\) and thus \(A(\sigma)\) expands. Arguments analogous to Part (i) imply \(m_g^\ell\) weakly increases in \(|q|\).

(iii) Decrease \(\delta\). Inspection of (30) shows \(\pi(\sigma)\) strictly decreases as \(\delta\) increases and thus \(A(\sigma)\) shrinks. Arguments analogous to Part (i) imply \(m_g^\ell\) weakly increases as \(\delta\) decreases.

Endogenous Access

Fix \(\ell \in N^L\). Recall \(\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}\). For convenience, refer to \(g_\ell\) as \(g\). The results fix \(\hat{x}_g\). Let \(\sigma(\alpha_\ell; \hat{x}_\ell)\) denote an equilibrium, given \(\hat{x}_\ell\) and \(\alpha_\ell\). Denote the corresponding social acceptance set as \(A(\alpha_\ell; \hat{x}_\ell)\), with upper bound \(\pi(\alpha_\ell; \hat{x}_\ell)\). Also, let \(A(\hat{x}_g)\) denote the equilibrium acceptance set if \(\hat{x}_\ell = \hat{x}_g\), suppressing \(\alpha_\ell\) because it is inconsequential.

First, I establish properties used to state analogues of Propositions 2 and 3.

Building upon Lemmas C.1–C.6 in Appendix C, Lemma 1 partitions whether \(\hat{x}_g \in \text{int}A(\hat{x}_g)\). See Appendix C for the proof. I state the result here for reference when proving Lemma 3.

**Lemma 1.** For all \(\ell \in N^L\), there exists \(\pi_\ell \in (0, q]\) such that \(\hat{x}_g \in (-\pi_\ell, \pi_\ell)\) implies \(\hat{x}_g \in \text{int}A(\hat{x}_g)\). Otherwise, \(A(\hat{x}_g) = [-\pi_\ell, \pi_\ell]\).

**Lemma 3.** Suppose \(\hat{x}_g \in (0, \pi_g)\). There exists \(\bar{x} \in [0, \hat{x}_g)\) such that \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). A symmetric result holds if \(\hat{x}_g \in (\pi_\ell, 0)\).

**Proof.** Consider \(\hat{x}_g \in (0, \pi_g)\). By Lemma 1, \(\hat{x}_\ell = \hat{x}_g\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies \(A(0; \hat{x}_\ell)\) is continuous in \(\hat{x}_\ell\). Thus, there exists \(\bar{x} \in [0, \hat{x}_g)\) such that \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). By Lemma 2, \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) thus implies \(A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). 

\[\square\]
For each \( j \in N^L \setminus \{\ell\} \), define \( E_j^{LB}(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \leq -\bar{\pi}(\alpha; \hat{x}_\ell)\} \), \( E_j^{UB}(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \geq \bar{\pi}(\alpha; \hat{x}_\ell)\} \), and \( C_j(\alpha; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \in \text{int}A(\alpha; \hat{x}_\ell)\} \). Define \( \tilde{E}_j^{LB}(\alpha; \hat{x}_\ell), \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \), and \( \tilde{C}_j(\alpha; \hat{x}_\ell) \) analogously for \( \hat{y}_j \). Let \( I_g \in \{0,1\} \) indicate whether \( j \in N^L_g \).

**Assumption A.1.** There exists \( j \in N^L \setminus \{\ell\} \) such that \( \alpha_j < 1 \) and \( \hat{x}_j \notin A(\sigma(\hat{x}_g)) \).

**Assumption A.2.** There exists \( j \in N^L \setminus \{\ell\} \) such that \( \alpha_j > 0 \) and \( \hat{y}_j \notin A(\sigma(\hat{x}_g)) \).

Next, define

\[
v_1^g(\alpha; \hat{x}_\ell) = \rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right)
\]

and

\[
v_2^g(\alpha; \hat{x}_\ell) = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_j^{LB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \right] u_g(-\bar{\pi}(\alpha; \hat{x}_\ell))
\]

\[
+ \left[ \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \right] u_g(\bar{\pi}(\alpha; \hat{x}_\ell))
\]

\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_\ell^g(\alpha; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j).
\]

**Lemma 4.** If \( \hat{x}_\ell \neq \hat{x}_g \), then \( \frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} > 0 \).

**Proof.** Suppose \( \hat{x}_\ell \neq \hat{x}_g \). From (31) and \( \hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2} \), \( \frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{2} (\hat{x}_g - \hat{x}_\ell)^2 > 0 \). \( \square \)

**Lemma 5.** Suppose \( 0 \leq \hat{x}_\ell < \hat{x}_g < \bar{\pi}_\ell \) and at least one of Assumption A.1 or A.2 holds. Then \( v_2^g(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha_\ell \). A symmetric result holds for \( \hat{x}_g < 0 \).

**Proof.** Assume \( 0 \leq \hat{x}_\ell < \hat{x}_g < \bar{\pi}_\ell \) and at least one of Assumption A.1 or A.2 holds. It suffices to show that

\[
\left[ \alpha_j \tilde{E}_j^{LB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \right] u_g(-\bar{\pi}(\alpha; \hat{x}_\ell))
\]

\[
+ \left[ \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \right] u_g(\bar{\pi}(\alpha; \hat{x}_\ell))
\]

\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_\ell^g(\alpha; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j)
\]

decreases in \( \alpha_\ell \) for all \( j \in N^L \setminus \{\ell\} \) and strictly decreases for some \( j \).
Without loss of generality, consider \( \hat{x}_j \geq 0 \). By Lemma 2, \( 0 \leq \hat{x}_\ell < \hat{x}_g \) implies \( \overline{\alpha}(\alpha; \hat{x}_\ell) \) increases in \( \alpha_\ell \). There are two implications. First, \( \hat{x}_g \in (0, \overline{\alpha}(0; \hat{x}_\ell)) \) by Lemma 1, so \( u_g(\overline{\alpha}(\alpha; \hat{x}_\ell)) \) and \( u_g(-\overline{\alpha}(\alpha; \hat{x}_\ell)) \) both decrease in \( \alpha_\ell \). Second, either: \( E_j^{UB}(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha; C_j(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha \); or there is a unique \( \overline{\alpha}_\ell \in (0, 1] \) such that \( \alpha_\ell \in [0, \overline{\alpha}_\ell] \) implies \( E_j^{UB}(\alpha; \hat{x}_\ell) = 1 \), and \( \alpha_\ell \in (\overline{\alpha}_\ell, 1] \) implies \( C_j(\alpha; \hat{x}_\ell) = 1 \). An analogous observation holds for \( \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) \) and \( \tilde{C}_j(\alpha; \hat{x}_\ell) \).

Thus, both

\[
E_j^{LB}(\alpha; \hat{x}_\ell) u_g(-\overline{\alpha}(\alpha; \hat{x}_\ell)) + E_j^{UB}(\alpha; \hat{x}_\ell) u_g(\overline{\alpha}(\alpha; \hat{x}_\ell)) + C_j(\alpha; \hat{x}_\ell) u_g(\hat{x}_\ell)
\]

and

\[
\tilde{E}_j^{LB}(\alpha; \hat{x}_\ell) u_g(-\overline{\alpha}(\alpha; \hat{x}_\ell)) + \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) u_g(\overline{\alpha}(\alpha; \hat{x}_\ell)) + \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{x}_\ell)
\]

decrease in \( \alpha_\ell \). Furthermore, at least one of (34) and (35) strictly decreases for some \( j \in N^L \setminus \{\ell\} \) because at least one of Assumptions A.1 or A.2 holds. Inspection of (30) reveals \( m_j^g(\alpha; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N^L \). Altogether, (33) decreases in \( \alpha_\ell \) for all \( j \in N^L \setminus \{\ell\} \) and strictly decreases for some \( j \), as desired. \( \square \)

For \( g \in N^G \), define

\[
U^E_g(\alpha; \hat{x}_\ell) = v_1^g(\alpha; \hat{x}_\ell) + v_2^g(\alpha; \hat{x}_\ell).
\]

**Lemma 6.** Assume \( \hat{x}_g \in (0, \overline{\alpha}) \) and at least one of Assumption A.1 or A.2 holds. There exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U^E_g(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha_\ell \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_g \in (0, \overline{\alpha}) \) and at least one of Assumption A.1 or A.2 holds. I show \( \left| \frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} \right| > \frac{\partial v_2^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} \) for \( \hat{x}_\ell \) sufficiently close to \( \hat{x}_g \).

By Lemma 3, there exists \( \bar{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int} A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Fix \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) and \( \alpha \in [0, 1] \).

First, I characterize a lower bound on \( \left| \frac{\partial v_1^g(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} \right| \). Define

\[
\Gamma = \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \tilde{E}_j^{LB}(\hat{x}_g) + (1 - \alpha_j) E_j^{LB}(\hat{x}_g) \right] \frac{\partial u_g(-\overline{\alpha}(\hat{x}))}{\partial \overline{\alpha}(\hat{x})} \\
+ \left[ \alpha_j \tilde{E}_j^{UB}(\hat{x}_g) + (1 - \alpha_j) E_j^{UB}(\hat{x}_g) \right] \frac{\partial u_g(\overline{\alpha}(\hat{x}))}{\partial \overline{\alpha}(\hat{x})} \right),
\]

(37)
Note \( \Gamma < 0 \) because (i) \( \hat{x}_g \in (-\overline{x}(\hat{x}), \overline{x}(\hat{x})) \) implies \( \frac{\partial u_g(\overline{x}(\hat{x}))}{\partial \alpha(\hat{x})} < 0 \) and \( \frac{\partial u_g(-\overline{x}(\hat{x}))}{\partial \alpha(\hat{x})} < 0 \), and (ii) at least one of Assumptions A.1 and A.2 hold.

I claim \( \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha(\alpha; \hat{x}_\ell)} < \Gamma \), where

\[
\frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha(\alpha; \hat{x}_\ell)} = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}^\ell UB(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^\ell LB(\alpha; \hat{x}_\ell) \right] \frac{\partial u_g(-\overline{x}(\alpha; \hat{x}_\ell))}{\partial \overline{x}(\alpha; \hat{x}_\ell)} + \left( \alpha_j \tilde{E}^\ell UB(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^\ell UB(\alpha; \hat{x}_\ell) \right) \frac{\partial u_g(\overline{x}(\alpha; \hat{x}_\ell))}{\partial \overline{x}(\alpha; \hat{x}_\ell)}
\]

\[- I^j \frac{\partial m_j^\ell(\alpha; \hat{x}_\ell)}{\partial \alpha(\alpha; \hat{x}_\ell)} \right) \tag{38} \]

Three steps show the claim. First, note \( \hat{x}_\ell \in (\hat{x}, \hat{x}_g) \) implies \( \overline{x}(\hat{x}_g) \geq \overline{x}(\alpha; \hat{x}_\ell) \). Thus, we have \( \tilde{E}^\ell UB(\hat{x}_g) \leq \tilde{E}^\ell UB(\alpha; \hat{x}_\ell), E_j^\ell UB(\hat{x}_g) \leq E_j^\ell UB(\alpha; \hat{x}_\ell), \tilde{E}^\ell LB(\hat{x}_g) \leq \tilde{E}^\ell LB(\alpha; \hat{x}_\ell) \), and \( E_j^\ell LB(\hat{x}_g) \leq E_j^\ell LB(\alpha; \hat{x}_\ell) \) for all \( j \neq \ell \). Next, \( \hat{x}_g < \overline{x}(\hat{x}_g) < \overline{x}(\hat{x}_g, \alpha) \) implies \( \frac{\partial u_g(\overline{x}(\alpha; \hat{x}_\ell))}{\partial \overline{x}(\alpha; \hat{x}_\ell)} \) and \( \frac{\partial u_g(-\overline{x}(\alpha; \hat{x}_\ell))}{\partial \overline{x}(\alpha; \hat{x}_\ell)} \) are both negative. Finally, \( \frac{\partial m_j^\ell(\alpha; \hat{x}_\ell)}{\partial \alpha(\alpha; \hat{x}_\ell)} \geq 0 \) for all \( j \in N^G \) as shown in Proposition 6.

For almost all \( \alpha \in [0, 1] \), \( \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha x} = \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha x} \). Define \( C_j(\alpha; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_j(\alpha; \hat{x}_\ell)) + \alpha_j (1 - \tilde{C}_j(\alpha; \hat{x}_\ell))] \). Then,

\[
\frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha x} < \Gamma \frac{\partial \overline{x}(\alpha; \hat{x}_\ell)}{\partial \alpha x} \tag{39}
\]

\[
= \frac{\delta \rho \Gamma}{2 \overline{x}(\alpha; \hat{x}_\ell)} \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right] \tag{40}
\]

\[
\leq \frac{\delta \rho \Gamma}{2 \overline{x}(\alpha; \hat{x}_\ell)} \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right] \tag{41}
\]

\[
= \frac{\delta \rho \Gamma}{2 \overline{x}(\alpha; \hat{x}_\ell)} \left[ \frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g) \right], \tag{42}
\]

where (39) follows from \( \frac{\partial \overline{x}(\alpha; \hat{x}_\ell)}{\partial \alpha x} > 0 \) and \( 0 > \Gamma > \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha x} \); (40) from applying the implicit function theorem to \( \overline{x}(\alpha; \hat{x}_\ell) \), which is possible for almost all \( \alpha \in [0, 1] \); (41) because \( \Gamma[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)] < 0, \overline{x}(\alpha; \hat{x}_\ell) > 0 \), and \( \delta \sum_{j \in N^G} \rho_j C_j^\ell(\alpha; \hat{x}_\ell) \in (0, 1) \); and (42) from using \( \hat{y}_\ell = \frac{\overline{x}_g + \hat{x}_\ell}{2} \) and simplifying.

By Lemma 4, \( \frac{\partial v_1(\alpha; \hat{x}_\ell)}{\partial \alpha x} = \frac{\alpha}{2}(\hat{x}_g - \hat{x}_\ell)^2 \). By (42), \( \frac{\partial U^2(\alpha; \hat{x}_\ell)}{\partial \alpha x} < \frac{\partial v_1(\alpha; \hat{x}_\ell)}{\partial \alpha x} + \Gamma \frac{\partial \overline{x}(\alpha; \hat{x}_\ell)}{\partial \alpha x} \)
for almost all $\alpha_\ell \in [0, 1]$. Therefore $\frac{\partial U^E_\ell(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ if

$$
\frac{\rho_\ell}{2} (\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_\ell \Gamma}{2 \phi_\ell} \left[ \frac{1}{4} (\hat{x}_g - \hat{x}_\ell) (3 \hat{x}_\ell + \hat{x}_g) \right] < 0,
$$

which holds for $\hat{x}_\ell > \hat{x}_g \left( \frac{4\pi \ell + \delta \Gamma}{4\pi \ell - 3\delta \Gamma} \right)$. Define $x' = \max\{ \tilde{x}, \hat{x}_g \left( \frac{4\pi \ell + \delta \Gamma}{4\pi \ell - 3\delta \Gamma} \right) \}$. Then $x' < \hat{x}_g$ because (i) $\tilde{x} < \hat{x}_g$ and (ii) $\delta \Gamma < 0$ implies $\frac{4\pi \ell + \delta \Gamma}{4\pi \ell - 3\delta \Gamma} < 1$. Thus, $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\frac{\partial U^E_\ell(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ for almost all $\alpha_\ell \in [0, 1]$. Continuity implies $U^E_\ell(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$ for such $\hat{x}_\ell$.

Next, I prove the analogue of Proposition 2.

**Proposition A.2** Assume $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ and at least one of Assumptions A.1 and A.2 holds. If $\hat{x}_g > 0$, then there exist $x'$ and $x''$ satisfying $x' < \hat{x}_g < \bar{x}_\ell < x''$ such that

1. if $\hat{x}_\ell \in (x', \hat{x}_g)$, then $\alpha_\ell = 0$ is uniquely optimal;
2. if $\hat{x}_\ell \in (\hat{x}_g, x'')$, then $\alpha_\ell = 0$ is not optimal; and
3. if $\hat{x}_\ell \geq x''$, then $g$ is indifferent over $\alpha_\ell$.

A symmetric result holds for $\hat{x}_g < 0$.

**Proof.** Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_g \in (0, \bar{x}_\ell)$ and at least one of Assumptions A.1 and A.2 hold.

1. By Lemma 3, there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. By Lemma 6, there exists $\tilde{x}' < \hat{x}_g$ such that $\hat{x}_\ell \in (\tilde{x}', \hat{x}_g)$ implies $U^E_\ell(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$. Consider $\hat{x}_\ell \in (\max\{\tilde{x}, \tilde{x}'\}, \hat{x}_g)$. Then $z_\ell = \hat{x}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ and $y^\ell_g = \hat{y}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. Thus, $g$’s ex ante expected utility equals $U^E_\ell(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. It follows that $g$ strictly prefers $\alpha_\ell = 0$.

2. Assume $\hat{x}_\ell \in (\hat{x}_g, x'')$, where $x'' = 2\bar{x}_\ell - \hat{x}_g$. It suffices to show $g$’s ex ante expected utility strictly increases at $\alpha_\ell = 0$. There are two cases.

- **Case 1:** If $\hat{x}_\ell < \bar{x}_\ell$, then $g$’s ex ante expected payoff equals $U^E_\ell(\alpha_\ell; \hat{x}_\ell)$ for sufficiently small $\alpha_\ell$. By Lemma 4, $\frac{\partial U^E_\ell(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$. To complete this case, I argue that $\nu^E_2(\alpha_\ell; \hat{x}_\ell)$ increases for sufficiently small $\alpha_\ell$. Under the maintained assumptions, $\hat{x}_g \in (-\bar{x}(0; \hat{x}_\ell), \bar{x}(0; \hat{x}_\ell))$ and $\hat{y}_\ell \in (\hat{x}_g, \bar{x}(0; \hat{x}_\ell))$. Thus, $\bar{x}(\alpha_\ell; \hat{x}_\ell)$ strictly
decreases for sufficiently small $\alpha_\ell$. Therefore $u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell))$ are strictly increasing for such $\alpha_\ell$. Arguments from Proposition 6 imply $m^j_g(\alpha_\ell; \hat{x}_\ell)$ weakly decreases in $\alpha_\ell$ for all $j \in N^L_g \\setminus \{\ell\}$. Thus, $v^2_g(\alpha_\ell; \hat{x}_\ell)$ strictly increases over sufficiently small $\alpha_\ell$.

- **Case 2:** If $\hat{x}_\ell > \pi_\ell$, then $\pi(0; \hat{x}_\ell) = \pi_\ell$. Thus, $g$’s ex ante expected payoff from $\alpha_\ell = 0$ is

$$
\rho_\ell \left( \alpha_\ell \left[ u_g(\bar{y}_\ell) + u_e(\bar{y}_\ell) - u_e(\pi_\ell) \right] - u_g(\pi_\ell) \right) + \sum_{j \neq \ell} \rho_j \left( \alpha_j \bar{E}_j^{LB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(0; \hat{x}_\ell) \right) u_g(-\bar{x}_\ell)
$$

$$
+ \left( \alpha_j \bar{E}_j^{UB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(0; \hat{x}_\ell) \right) u_g(\bar{x}_\ell)
$$

$$
+ \alpha_j C_j(0; \hat{x}_\ell) u_g(\bar{y}_j) + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\bar{y}_j)
$$

$$
- I_j^g \alpha_j m^j_g(0; \hat{x}_\ell).
$$

Arguments analogous to Case 1 show (43) strictly increases in $\alpha_\ell$ at $\alpha_\ell = 0$.

3. Assume $\hat{x}_\ell \geq x''$, where $x''$ is defined as in Case 2 of Part 2. Then $z_\ell = y^\ell_g = \pi(\alpha_\ell; \hat{x}_\ell) = \pi_\ell$ for all $\alpha_\ell \in [0, 1]$ and $g$’s ex ante expected payoff is constant. □

I prove the analogue of Proposition 3 from the main text.

**Proposition A.3**  Assume $\hat{x}_g \geq \pi_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\bar{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_\ell \geq \pi_\ell$, then $g$ is indifferent over $\alpha_\ell$.

2. If $\hat{x}_\ell \in [0, \pi_\ell)$, then $\alpha_\ell = 0$ is not optimal.

A symmetric result holds if $\hat{x}_g \leq -\pi_\ell$ and $\max\{\hat{x}_j, \hat{y}_j\} < \bar{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

**Proof.** Suppose $\hat{x}_g \geq \pi_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\bar{x}(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_\ell \geq \pi_\ell$, then apply arguments analogous to Part 3 of Proposition A.2.

2. Assume $\hat{x}_\ell \in [0, \pi_\ell)$. I show $g$’s ex ante expected utility strictly increases at $\alpha_\ell = 0$. 

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We have \( \hat{x}_\ell \in [0, \pi(0; \hat{x}_\ell)) \) and \( \hat{y}_\ell > \hat{x}_\ell \). Therefore \( 0 \leq z_\ell(0; \hat{x}_\ell) = \hat{x}_\ell < y^k_g(0; \hat{x}_\ell) \leq \hat{y}_\ell \). Furthermore, no \( j \in N^L \) proposes \(-\pi(0; \hat{x}_\ell)\) because \( \min\{\hat{x}_j, \hat{y}_j\} > -\pi(0; \hat{x}_\ell) \). Thus, \( g \)'s ex ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(y^1_g(0; \hat{x}_\ell)) + u_\ell(y^2_g(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right) \\
+ \sum_{j \neq \ell} \rho_j \left( \alpha_j \tilde{E}_{j}^{UB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_{j}^{UB}(0; \hat{x}_\ell) \right) u_g(\pi(0; \hat{x}_\ell)) \\
+ \alpha_j \left[ \tilde{C}_{j}(0; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m^j_g(0; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{y}_j). \tag{44}
\]

Three steps show (44) strictly increases at \( \alpha_\ell = 0 \).

- First, \( 0 \leq \hat{x}_\ell < y^k_g(0; \hat{x}_\ell) \leq \hat{y}_\ell \) implies \( y^k_g(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Therefore \( u_g(y^k_g(\alpha; \hat{x}_\ell)) \) weakly increases and \( u_\ell(y^k_g(\alpha; \hat{x}_\ell)) \) weakly decreases. Because \( u \) is quadratic and \( \hat{x}_\ell < y^k_g(0; \hat{x}_\ell) \), \( \hat{y}_\ell = x_g + x_\ell < \hat{x}_g \), it follows that \( u_g(y^k_g(\alpha; \hat{x}_\ell)) \) increases weakly faster than \( u_\ell(y^k_g(\alpha; \hat{x}_\ell)) \) decreases. Therefore \( u_g(y^k_g(0; \hat{x}_\ell)) + u_\ell(y^k_g(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Furthermore, \( \hat{x}_\ell < y^k_g(0; \hat{x}_\ell) \leq \hat{y}_\ell \) implies \( y^k_g(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Since \( \hat{x}_\ell < y^k_g(0; \hat{x}_\ell) \) also implies \( u_g(y^k_g(0; \hat{x}_\ell)) + u_\ell(y^k_g(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0 \). This shows \( \alpha_\ell \left[ u_g(y^k_g(0; \hat{x}_\ell)) + u_\ell(y^k_g(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \) weakly increases at \( \alpha_\ell = 0 \).

- Second, \( 0 \leq z_\ell < y^k_g(0; \hat{x}_\ell) \leq \pi(0; \hat{x}_\ell) \) implies \( \pi(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \). Since \( \pi(0; \hat{x}_\ell) < \hat{x}_g \), it follows that \( u_g(\pi(0; \hat{x}_\ell)) \) increases at \( \alpha_\ell = 0 \).

- Third, Proposition 6 implies \( m^j_g(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N^L_g \). However, \( \hat{y}_j > \pi(0; \hat{x}_\ell) \) for all \( j \in N^L_g \) such that \( m^j_g(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \), which implies \( g \)'s lobbying surplus weakly increases in \( \alpha_\ell \) for any such \( j \in N^L_g \).

Thus, (44) strictly increases at \( \alpha_\ell = 0 \). \( \square \)

**Willingness to Pay for Access**

The following results apply to the model in the main text. Recall \( \theta = (\hat{x}, \rho, \alpha) \). Let \( U^{EB}_g(\theta) \) be \( g \)'s ex ante expected utility. Additionally, let \( \bar{\pi}_\alpha = \bar{\pi}(\alpha; \hat{x}_\ell) \) denote the upper bound of \( A(\alpha; \hat{x}_\ell) \). Define \( \frac{\partial \bar{\pi}_\alpha}{\partial \alpha} \bigg|_{\alpha=0} = \frac{\partial \bar{\pi}_\alpha}{\partial \alpha} \bigg|_{\alpha=0} = \frac{\partial \bar{\pi}_\alpha}{\partial x_g} \bigg|_{\alpha=0} = \frac{\partial^2 \bar{\pi}_\alpha}{\partial \alpha \partial x_g} \bigg|_{\alpha=0} \).
To state Proposition 4, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, consider the baseline model, but replace $\ell$ with two legislators, $\ell_1$ and $\ell_2$, and replace $g$ with two groups, $g_1$ and $g_2$. To isolate differences in proposal power, assume $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, but $\rho_{\ell_1} \neq \rho_{\ell_2}$. Also, assume $\hat{x}_{g_1} = \hat{x}_{g_2}$. I use two identical groups to avoid complications arising if one group has access to two legislators, because the group accounts for how access to one legislator affects its offer to the other. These modifications do not qualitatively change the equilibrium characterization.

**Proposition 4.** Consider the modified baseline model with: $\ell_1$ and $\ell_2$ such that $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, and $g_1$ and $g_2$ satisfying $\hat{x}_{g_1} = \hat{x}_{g_2}$. For all $\alpha \in [0, 1]$, $\rho_{\ell_2} > \rho_{\ell_1}$ implies $\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_2} |_{\alpha_2=0} \geq \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_1} |_{\alpha_1=0}.$

**Proof.** Consider the modified setting with (i) $\ell_1$ and $\ell_2$ such that $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, and (ii) $g_1$ and $g_2$ such that $\hat{x}_{g_1} = \hat{x}_{g_2}$. Assume $\rho_{\ell_2} > \rho_{\ell_1}$ and fix $\alpha \in [0, 1]$. It suffices to show $\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_2} |_{\alpha_2=0} \geq \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_1} |_{\alpha_1=0}$ for $\alpha \in [0, 1]$.

Because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$ and $\hat{x}_{g_1} = \hat{x}_{g_2}$, we have $y_{g_1} = y_{g_2}$ and $z_{\ell_1} = z_{\ell_2}$. Thus, $m_{g_1} = m_{g_2}$. For convenience, let $y = y_{g_1}$, $z = z_{\ell_1}$, and $m = m_{g_1}$. Assume $\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_2} |_{\alpha_2=0} \geq 0$. There are five cases.

- **Case 1:** Consider $\hat{x}_\ell$ and $\hat{x}_g$ such that $z = \hat{x}_\ell$ and $y = \hat{y}$. Then,

$$\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1} |_{\alpha_1=0} = \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)$$

$$- \frac{\partial}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)$$

$$= \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)$$

$$+ \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M(\bar{x}_\alpha)} \left[ \frac{1}{1 - \delta(\rho_L + \rho_R)} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \right]$$

$$\leq \rho_{\ell_2} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)$$

$$+ \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M(\bar{x}_\alpha)} \left[ \frac{1}{1 - \delta(\rho_L + \rho_R)} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \right]$$

(45)

(46)
\[
\frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha},
\]

where (45) follows from \( \frac{\partial \pi_a}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(y)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R)]} \); (46) because (i) \( \rho_{\ell_1} > \rho_{\ell_2} \) and (ii) \( \frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq 0 \) implies the bracketed expression in (45) is positive; and (47) because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2}, \hat{x}_{g_1} = \hat{x}_{g_2} \), and \( \frac{\partial \pi_a}{\partial \alpha_1} = \frac{\delta \rho_2 [u_M(y)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R)]} \).

- **Case 2:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \pi_a \) and \( y = \hat{y} \). In this case, \( \frac{\partial \pi_a}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(y)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \) and \( \frac{\partial \pi_a}{\partial \alpha_2} = \frac{\delta \rho_2 [u_M(y)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \). Arguments analogous to Case 1 show \( \frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_E^L(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \). The argument for \( z = \pi_a \) and \( y = -\pi_a \) is symmetric.

- **Case 3:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \hat{x}_\ell \) and \( y = \pi_a \). In this case, \( \frac{\partial \pi_a}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(\pi_a)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \) and \( \frac{\partial \pi_a}{\partial \alpha_2} = \frac{\delta \rho_2 [u_M(\pi_a)-u_M(\hat{x})]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \). Arguments analogous to Case 1 show \( \frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_E^L(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \). The argument for \( z = \pi_a \) and \( y = -\pi_a \) is symmetric.

- **Case 4:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \pi_a \) and \( y = -\pi_a \). In this case, \( \frac{\partial \pi_a}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(\pi_a)-u_M(\pi_a)]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \) and \( \frac{\partial \pi_a}{\partial \alpha_2} = \frac{\delta \rho_2 [u_M(\pi_a)-u_M(\pi_a)]}{\delta \pi_a [1-\delta(\rho_L+\rho_R+\rho_1+\rho_2)]} \). Arguments analogous to Case 1 show \( \frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_E^L(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \). The argument for \( z = -\pi_a \) and \( y = \pi_a \) is symmetric.

- **Case 5:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \pi_a \) and \( y = \pi_a \). Then, \( \frac{\partial U_E^L(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} = \frac{\partial U_E^L(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} = 0 \). The argument for \( z = -\pi_a \) and \( y = -\pi_a \) is symmetric.

\( \square \)

**Proposition 5.** Assume minority-party agenda exclusion and \( \ell \) is majority-leaning. If either: \( g \) is more centrist than \( \ell \), or \( g \) is majority-leaning and more extreme than \( \ell \), then \( \frac{\partial U_E^L(\theta)}{\partial \alpha}|_{\alpha=0} \) weakly increases in \( |\hat{x}_g - \hat{x}_\ell| \).

**Proof.** Without loss of generality, assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \).
First, g’s ex ante expected utility for \( \alpha \in [0, 1] \) is

\[
U^E_g(\theta) = \rho_\ell \left( \alpha[u_y(y) + u_\ell(y) - u_\ell(z_\ell)] + (1 - \alpha)u_\ell(z_\ell) \right) + \rho_M u_\ell(0) + \rho_R u_y(x_\alpha). \tag{48}
\]

Thus, g’s willingness to acquire access to \( \ell \) is

\[
\frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha=0} = \rho_\ell \left( u_y(y) - u_\ell(z_\ell) + u_\ell(y) - u_\ell(z_\ell) \right) + \rho_R \frac{\partial u_\ell(x_0)}{\partial x_0} \frac{\partial x_0}{\partial \alpha}. \tag{49}
\]

The cross-partial with respect to \( \hat{x}_g \) satisfies

\[
\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial \hat{x}_g} \bigg|_{\alpha=0} = \rho_\ell \left( \left( \frac{\partial u_y(y)}{\partial y} + \frac{\partial u_\ell(y)}{\partial y} \right) \frac{\partial y}{\partial \hat{x}_g} + \frac{\partial u_\ell(y)}{\partial \hat{x}_g} - \frac{\partial u_\ell(z_\ell)}{\partial \hat{x}_g} \right)
+ \rho_R \left( \frac{\partial^2 u_\ell(x_0)}{\partial x_0^2} \frac{\partial x_0}{\partial \alpha} + \frac{\partial u_\ell(x_0)}{\partial \hat{x}_g} \frac{\partial^2 x_0}{\partial \alpha} \right)
= \rho_\ell \left( \frac{\partial u_y(y)}{\partial \hat{x}_g} - \frac{\partial u_\ell(z_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial^2 u_\ell(x_0)}{\partial x_0^2} \frac{\partial x_0}{\partial \alpha} + \frac{\partial u_\ell(x_0)}{\partial \hat{x}_g} \frac{\partial^2 x_0}{\partial \alpha} \right), \tag{50}
\]

where (51) follows because either (i) \( y = x_0 \), which implies \( \frac{\partial y}{\partial \hat{x}_g} = 0 \), or (ii) \( y = \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \), which implies \( \frac{\partial u_y(y)}{\partial y} = -\frac{\partial u_\ell(y)}{\partial \hat{x}_g} \).

**Part (i)** Consider \( \hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell] \). There are two cases.

**Case 1:** Suppose \( \hat{x}_\ell \geq x_0 \). Then \( z_\ell = x_0 \). Since \( \hat{x}_g \geq -\hat{x}_\ell \), we have \( \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \geq 0 \).

There are two subcases.

- First, consider \( \hat{x}_g \geq 2x_0 - \hat{x}_\ell \). Then \( y = z_\ell = x_0 \). For \( \alpha \in [0, 1] \), if \( y = z_\ell = x_0 \), then \( x_\alpha \) solves

  \[
  0 = (1 - \delta) u_M(q) + \delta \rho_M u_M(0) - [1 - \delta(\rho_R + \rho_\ell)] u_M(x_\alpha). \tag{52}
  \]

  Applying the implicit function theorem to (52) yields \( \frac{\partial x_\alpha}{\partial \alpha} = 0 \) and thus \( \frac{\partial x_0}{\partial \alpha} = 0 \). Therefore \( \frac{\partial U^E_g(\theta)}{\partial \alpha} \bigg|_{\alpha=0} = 0 \) over \( \hat{x}_g \in [2x_0 - \hat{x}_\ell, \hat{x}_\ell] \).

- Second, consider \( \hat{x}_g < 2x_0 - \hat{x}_\ell \). Then \( y = \hat{y} \). For \( \alpha \in [0, 1] \), if \( y = \hat{y} \) and \( z_\ell = x_0 \),
then $\bar{x}_\alpha$ solves

$$0 = (1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \alpha \rho_\ell u_M(\hat{y}) \right) - \left( 1 - \delta [\rho_R + (1 - \alpha) \rho_\ell] \right) u_M(\bar{x}_\alpha).$$

(53)

Applying the implicit function theorem to (53) yields

$$\frac{\partial \bar{x}_\alpha}{\partial \alpha} = \frac{\delta \rho_\ell \left[ u_M(\hat{y}) - u_M(\bar{x}_\alpha) \right]}{(1 - \delta [\rho_R + (1 - \alpha) \rho_\ell]) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}},$$

(54)

$$\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = \frac{\alpha \delta \rho_\ell \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g}}{(1 - \delta [\rho_R + (1 - \alpha) \rho_\ell]) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}}.$$  

(55)

and

$$\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho_\ell}{(1 - \delta [\rho_R + (1 - \alpha) \rho_\ell])} \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} \frac{\partial x_g}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha \partial \bar{x}_\alpha} \frac{\partial x_g}{\partial \bar{x}_g} \frac{\partial \bar{x}_\alpha}{\partial \bar{x}_\alpha} \right) \left( \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)^{-1}.$$  

(56)

Inspecting (55) reveals $\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = 0$. Thus,

$$\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho_\ell \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g}}{(1 - \delta [\rho_R + \rho_\ell]) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}} > 0,$$

(57)

which follows because (i) $\frac{\partial \hat{y}}{\partial \hat{x}_g} > 0$ and (ii) $0 < \hat{y} < \bar{x}_\alpha$ implies $0 < \frac{\partial u_M(y)}{\partial \hat{x}_g} > \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}$.

Thus,

$$\frac{\partial^2 \mathcal{U}^E(\hat{\theta})}{\partial \alpha \partial \hat{x}_g} \bigg|_{\alpha = 0} = \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\bar{x}_\alpha)}{\partial \hat{x}_g} \right) + \rho_R \frac{\partial u_g(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g},$$  

(58)

$$< \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\bar{x}_\alpha)}{\partial \hat{x}_g} \right),$$  

(59)

$$< 0,$$

(60)

where (59) follows from (57) and $\frac{\partial u_g(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} < 0$; and (60) because $\hat{x}_g < \hat{y} < \bar{x}_\alpha$ implies $\frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} < \frac{\partial u_g(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}$.

Case 2: Consider $\hat{x}_\ell < \bar{x}_\alpha$. Then $z_\ell = \hat{x}_\ell$. Furthermore, $\hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell]$ implies
$y = \hat{y} \geq 0$. For $\alpha \in [0,1]$, if $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$, then $\bar{x}_\alpha$ solves

$$u_M(\bar{x}_\alpha) = \frac{(1-\delta)u_M(q) + \delta \left( \rho_M u_M(0) + \rho_\ell [\alpha u_M(\hat{y}) + (1-\alpha)u_M(\hat{x}_\ell)] \right)}{(1-\delta \rho_R)}. \quad (61)$$

Applying the implicit function theorem yields

$$\frac{\partial \bar{x}_\alpha}{\partial \alpha} = \frac{\delta \rho_\ell [u_M(\bar{y}) - u_M(\hat{x}_\ell)]}{(1-\delta \rho_R) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}}, \quad (62)$$

$$\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = \frac{\alpha \delta \rho_\ell \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g}}{(1-\delta \rho_R) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}}, \quad (63)$$

and

$$\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho_\ell}{(1-\delta \rho_R)} \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial x^2} \frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} \frac{\partial \bar{x}_\alpha}{\partial \alpha} \left( \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)^{-1} \right). \quad (64)$$

Inspecting (63) reveals $\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = 0$, which implies

$$\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho_\ell}{(1-\delta \rho_R) \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}} > 0. \quad (64)$$

Because $0 \leq \hat{y} < \hat{x}_\ell$, a inequalities analogous to (58)-(60) imply $\frac{\partial^2 U_g^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} < 0$.

In both cases, $g$’s willingness to acquire access weakly decreases in $|\hat{x}_g - \hat{x}_\ell|$.

**Part (ii)** Assume $\hat{x}_g \geq \hat{x}_\ell$. There are two cases.

**Case 1:** Consider $\hat{x}_\ell \geq \bar{x}_0$. Then $y = z_\ell = \bar{x}_0$ at $\alpha = 0$, implying $\frac{\partial^2 U_g^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} = 0$.

**Case 2:** Consider $\hat{x}_\ell < \bar{x}_0$. Then $z_\ell = \hat{x}_\ell$. There are three subcases.

- First, assume $\hat{x}_g \in [\hat{x}_\ell, \bar{x}_0)$. Then $y = \hat{y}$. I show $\frac{\partial^2 U_g^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} \geq 0$ implies $\frac{\partial^2 U_g^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} > 0$. Since $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$, case 2 of Part (ii) implies $\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g}$ is given by (62), $\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = 0$, and $\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g}$ is (64). Therefore

$$\frac{\partial^2 U_g^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} = \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} \right) \quad (65)$$

$$= \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \left( \frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} \frac{\partial \bar{x}_\alpha}{\partial \alpha} \right)^{-1} \right).$$

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Third, assume \( \hat{x}_\ell \) defined as in subcase 1. Therefore

\[
0 \quad \text{and (ii) } 0 = (1-\delta) u_M(q) + \delta \left( \rho_M u_M(0) + \rho_\ell (1-\alpha) u_M(\hat{x}_\ell) \right) - \left( 1 - \delta [\rho_R + \alpha \rho_\ell] \right) u_M(\bar{x}_\alpha). \tag{72}
\]

Applying the implicit function theorem to (72) yields
Proof. Without loss of generality, assume \( \alpha \) and Lemma 7.

Because \( \partial \) to (61) yields such that \( \alpha \). An argument analogous to Part 1 of Proposition 2 shows existence of \( \hat{x}_g \in (\hat{x}_\ell, x') \) implies \( \alpha = 0 \) is optimal. An symmetric result holds if \( \rho_R = 0 \) and \( \hat{x}_\ell \leq 0 \).

Lemma 7. Assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \). There exists \( x' > \hat{x}_\ell \) such that \( \hat{x}_g \in (\hat{x}_\ell, x') \) implies \( \alpha = 0 \) is optimal. An symmetric result holds if \( \rho_R = 0 \) and \( \hat{x}_\ell \leq 0 \).

Proof. Without loss of generality, assume \( \rho_L = 0 \) and \( 0 \leq \hat{x}_\ell \).

If \( \hat{x}_\ell \geq \bar{x}_0 \), then \( \hat{x}_g > \hat{x}_\ell \) implies \( \alpha = 0 \). As in the proof of Proposition 5, \( \frac{\partial U^E(\theta)}{\partial \alpha} |_{\alpha = 0} = 0 \) for all \( \alpha \in [0, 1] \). Setting \( x' = \infty \) delivers the result.

Next, suppose \( \hat{x}_\ell < \bar{x}_0 \). Lemma 5 implies that it suffices to show that \( \frac{\partial U^E(\theta)}{\partial \alpha} |_{\alpha = 0} \leq 0 \) and \( \frac{\partial^2 U^E(\theta)}{\partial \alpha^2} < 0 \) for \( \hat{x}_g \) sufficiently close to \( \hat{x}_\ell \).

An argument analogous to Part 1 of Proposition 2 shows existence of \( x' > \hat{x}_\ell \) such that \( \frac{\partial U^E(\theta)}{\partial \alpha} |_{\alpha = 0} \leq 0 \) if \( \hat{x}_g \in (\hat{x}_\ell, x') \). Also, \( x' \in (\hat{x}_\ell, \bar{x}_0) \) because \( \rho_L = 0 \) implies \( \frac{\partial U^E(\theta)}{\partial \alpha} |_{\alpha = 0} > 0 \) for \( \hat{x}_g \geq \bar{x}_0 \).

Consider \( \hat{x}_g \in [\hat{x}_\ell, x'] \). I show \( \frac{\partial^2 U^E(\theta)}{\partial \alpha^2} < 0 \). Applying the implicit function theorem to (61) yields

\[
\frac{\partial^2 \bar{\tau}_\alpha}{\partial \alpha^2} = -\frac{\partial^2 u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha^2} \left( \frac{\partial \bar{\tau}_\alpha}{\partial \alpha} \right)^2 < 0
\]

because \( \frac{\partial^2 u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha^2} < 0 \) and \( \hat{x}_g \geq \bar{x}_\alpha \) implies \( \frac{\partial u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha} < 0 \).

We have \( y = \hat{y} \) and \( z_\ell = \hat{x}_\ell \), so

\[
\frac{\partial^2 U^E(\theta)}{\partial \alpha^2} = \rho_R \left( \frac{\partial^2 u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha^2} \left( \frac{\partial \bar{\tau}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha} \frac{\partial^2 \bar{\tau}_\alpha}{\partial \alpha^2} \right)
\]

\[
= \rho_R \left( \frac{\partial^2 u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha^2} \left( \frac{\partial \bar{\tau}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha} \frac{\partial^2 \bar{\tau}_\alpha}{\partial \alpha^2} \right)
\]

\[
< \rho_R \left( \frac{\partial^2 u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha^2} \left( \frac{\partial \bar{\tau}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{\tau}_\alpha)}{\partial \bar{\tau}_\alpha} \frac{\partial^2 \bar{\tau}_\alpha}{\partial \alpha^2} \right)
\]

\[
(75)
\]

\[
(76)
\]

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= 0, \quad (77)

where (75) follows from \( \frac{\partial^2 u_g(x_{\alpha})}{\partial x_{\alpha}^2} = \frac{\partial^2 u_M(x_{\alpha})}{\partial x_{\alpha}^2} \) because \( u \) is quadratic; (76) because \( \frac{\partial^2 x_{\alpha}}{\partial \alpha^2} < 0 \) and \( 0 < \hat{x}_g < \hat{x}_\alpha \) implies \( \frac{\partial u_M(x_{\alpha})}{\partial x_{\alpha}} < \frac{\partial u_g(x_{\alpha})}{\partial x_{\alpha}} < 0 \); and (77) from (74). Thus, \( \frac{\partial U^E(\theta)}{\partial \alpha} \bigg|_{\alpha = \alpha} \leq 0 \) for all \( \alpha \in [0, 1] \). Proposition 5 delivers the result. \( \square \)
Appendix B

A strategy profile $\sigma = (\lambda, \pi, \varphi, \nu)$ is a stationary legislative lobbying equilibrium if it satisfies four conditions. First, for all $g \in N^G$ and $\ell \in N^L_y$, $\lambda^0_g$ places probability one on

$$\arg\max_{(y,m)} \bar{v}_\sigma(y)u_g(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_q(q) + \delta\tilde{V}_\ell(\sigma)] - m$$

s.t. $\bar{v}_\sigma(y)u_\ell(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] + m$

$$\geq$$

$$\int_X \left[ \bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \quad (78)$$

Second, for all $\ell \in N^L$ and offers $(y,m) \in W$,

$$\bar{v}_\sigma(y)u_\ell(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] + m$$

$$>$$

$$\int_X \left[ \bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \quad (79)$$

implies $\varphi_\ell(y,m) = 1$ and the opposite strict inequality implies $\varphi_\ell(y,m) = 0$. Third, for all $\ell \in N^L$,

$$\pi_\ell \left( \arg\max_{x \in X} \bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right) = 1. \quad (80)$$

Finally, for all $i \in N^V$ and $x \in X$, $u_i(x) > (1 - \delta)u_i(q) + \delta\tilde{V}_i(\sigma)$ implies $\nu_i(x) = 1$ and the opposite strict inequality implies implies $\nu_i(x) = 0$.\footnote{Thus, voting strategies are stage-undominated (Baron and Kalai, 1993; Banks and Duggan, 2006a).}

Lemma B.1 verifies that groups never offer surplus lobby payment in equilibrium. The proof is straightforward and omitted.

Lemma B.1. In every stationary legislative lobbying equilibrium, for all $\ell \in N^L$ every $(y,m) \in \text{supp}(\lambda^0_y)$ satisfies

$$\bar{v}_\sigma(y)u_\ell(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] + m$$

= 

\hspace{10em}

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\[
\int_X \left[ \mathcal{p}_\sigma(x) u_\ell(x) + [1 - \mathcal{p}_\sigma(x)](1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \pi_\ell(dx).
\] (81)

**Deferential Voting and Acceptance**

Let \( \sigma \) be a stationary strategy profile. From (12), recall \( \xi_\ell(\alpha; \sigma) = (1 - \alpha \ell) + \alpha \ell \int_W [1 - \varphi_\ell(y, m)] \lambda^\ell_y(dw) \). Define

\[
\hat{\chi}(X') = \sum_{\ell \in NL} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_{X'} \mathcal{p}_\sigma(x) \pi_\ell(dx) + \alpha \ell \int_{X' \times \mathbb{R}_+} \varphi_\ell(y, m) \mathcal{p}_\sigma(y) \lambda^\ell_y(dw) \right),
\] (82)

the probability some \( x \in X' \subseteq X \) is passed in a given period under \( \sigma \). Next, define

\[
\hat{\chi} = \sum_{\ell \in NL} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_X [1 - \mathcal{p}_\sigma(x)] \pi_\ell(dx) + \alpha \ell \int_W \varphi_\ell(y, m) [1 - \mathcal{p}_\sigma(y)] \lambda^\ell_y(dw) \right),
\] (83)

the probability of a failed proposal in a given period under \( \sigma \).

Following Banks and Duggan (2006a), each player’s continuation value can be expressed as a function of a common lottery over policy, denoted \( \chi^\sigma \). Using (82) and (83), define \( \chi^\sigma \) so that for all measurable \( X' \subseteq X \): (i) if \( q \notin X' \), then \( \chi^\sigma(X') = \frac{\hat{\chi}(X')}{1 - \delta \hat{\chi}} \), and (ii) if \( q \in X' \), then \( \chi^\sigma(X') = \frac{\hat{\chi}(X') + (1 - \delta) \hat{\chi}}{1 - \delta} \).

Set \( V^{\text{den}}(\sigma) = 1 - \delta \hat{\chi} \) and define

\[
V^{\text{num}}_i(\sigma) = \sum_{\ell \in NL} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_X \left[ \mathcal{p}_\sigma(x) u_i(x) + [1 - \mathcal{p}_\sigma(x)](1 - \delta) u_i(q) \right] \pi_\ell(dx) \right.
\]

\[
\left. + \alpha \ell \int_W \varphi(y, m) \left[ \mathcal{p}_\sigma(y) u_i(x) + [1 - \mathcal{p}_\sigma(x)](1 - \delta) u_i(q) \right] \lambda^\ell_y(dw) \right).
\]

For each \( i \in N^V \), i’s continuation value defined in (13) satisfies \( V_i(\sigma) = \frac{V^{\text{num}}_i(\sigma)}{V^{\text{det}}(\sigma)} \). Using \( \chi^\sigma \), we can express \( V_i(\sigma) \) explicitly as a lottery over policy: \( V_i(\sigma) = \int_X u_i(x) \chi^\sigma(dx) \).

The policy lottery \( \chi^\sigma \) is common to all players, but committee members may receive payment and interest groups may make payments. Define

\[
\tilde{m}_\ell(\sigma) = \rho_\ell \alpha \ell \int_W m \varphi_\ell(y, m) \lambda^\ell_y(dw),
\] (84)
which is $\ell$’s expected lobby payment in each period until passage. For $\ell \in N^L$, rearranging (14) yields

$$\tilde{V}_\ell(\sigma) = \frac{V_{\ell}^{\text{num}}(\sigma) + \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} = \int_X u_\ell(x)\chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \quad (85)$$

Similarly, for $g \in N^G$, rearranging (15) yields

$$\hat{V}_g(\sigma) = \frac{V_g^{\text{num}}(\sigma) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} = \int_X u_g(x)\chi^\sigma(dx) - \sum_{\ell \in N^L_g} \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \quad (86)$$

Finally, define

$$\tilde{U}_\ell(\sigma) = \int_X \left[ \bar{\nu}_\sigma(x)u_\ell(x) + \left(1 - \bar{\nu}_\sigma(x)\right)\left((1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)\right) \right] \pi_\ell(dx) \quad (87)$$

which is $\ell$’s expected dynamic payoff under $\sigma$, conditional on being recognized as the proposer and rejecting $g\ell$’s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium $\sigma$ such that $\chi^\sigma$ is degenerate on $q$.

**Proof.** Let $\sigma$ denote an equilibrium. To show a contradiction, assume $\chi^\sigma(q) = 1$. Thus, $V_M(\sigma) = u_M(q)$, which implies $u_M(q) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)$ and therefore $q \in A(\sigma)$. Without loss of generality, assume $q > 0$.

By assumption, there exists $\ell \in N^L$ such that $\hat{x}_\ell < q$ and $\hat{x}_{g\ell} \leq q$. Note that $u_{g\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$ is $g\ell$’s expected dynamic payoff from any offer $(y, m)$ such that: $\bar{\nu}_\sigma(y) = 1$, $\varphi_\ell(y, m) = 1$, and $\ell$ is indifferent between accepting and rejecting. Because $\tilde{U}_\ell(\sigma)$ does not depend on $y$, $\hat{y}_\ell = \arg\max_{y \in X} u_{g\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$. Furthermore, $\hat{y}_\ell < q$.

Strict concavity thus implies existence of $\varepsilon > 0$ and $y^\varepsilon < q$ such that $\bar{\nu}_\sigma(y^\varepsilon) = 1$, $\varphi_\ell(y^\varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1$, and

$$u_{g\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \tilde{U}_\ell(\sigma) - \varepsilon > u_{g\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) \quad (88)$$

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\[ \geq u_{g\ell}(q) + u_{\ell}(q) - \tilde{U}_\ell(\sigma) - \delta \left( \sum_{j \in N^L_y} \frac{m_j(\sigma)}{V_{\text{den}}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \right), \] 

(89)

where (89) follows from \( \sum_{j \in N^L_y} \frac{m_j(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{m_\ell(\sigma)}{V_{\text{den}}(\sigma)} \). Note the RHS of (88) is \( g_\ell \)'s highest expected payoff from lobbying \( \ell \) to \( q \) if \( \nu_\sigma(q) = 1 \); and (89) equals \( g_\ell \)'s highest expected payoff from lobbying \( \ell \) to any \( y' \) such that \( \nu_\sigma(y') = 0 \). Thus, \( g_\ell \) must have a profitable deviation, a contradiction. \( \square \)

**Lemma B.3.** Let \( \sigma \) denote a stationary legislative lobbying equilibrium. For all \( \ell \in N^L \) there exists \( (y, m) \in X \times \mathbb{R}_+ \) such that \( \nu_\sigma(y) = 1 \) and \( g_\ell \) strictly prefers \( (y, m) \) to any \( (y', m') \) such that \( \nu_\sigma(y') = 0 \).

**Proof.** Fix an equilibrium \( \sigma \). Lemma B.2 implies \( \chi^\sigma \) is not degenerate on \( q \). Let \( \chi^q \) denote the probability distribution degenerate on \( q \). Define the continuation distribution following rejection under \( \sigma \) as \( \chi^\sigma = (1 - \delta) \chi^q + \delta \chi^\sigma \), which is non-degenerate because \( \delta \in (0, 1) \).

For every player \( k \in N \), the expected dynamic policy payoff from a rejected policy proposal satisfies

\[ (1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx). \]

Let \( x^\sigma \) denote the mean of \( \chi \). Since \( u \) is strictly concave and \( \chi \) is non-degenerate, Jensen’s Inequality implies

\[ u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma). \] 

(90)

Consider \( \ell \in N^L \). First, assume \( \ell \) always accepts \( (y, m) \) when indifferent. The condition for \( g_\ell \) to strictly prefer offering \( (y, m) \) such that \( \nu_\sigma(y) = 1 \), rather than offering \( (y', m') \) such that \( \nu_\sigma(y') = 0 \), is

\[ u_{g\ell}(y) + u_{\ell}(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g\ell}(q) + \delta \tilde{V}_{g\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta \tilde{V}_{\ell}(\sigma) - \tilde{U}_\ell(\sigma). \]

Equivalently,

\[ u_{g\ell}(y) + u_{\ell}(y) > (1 - \delta)u_{g\ell}(q) + \delta \tilde{V}_{g\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta \tilde{V}_{\ell}(\sigma). \] 

(91)
Notice that

\[
\hat{V}_{g\ell}(\sigma) + \hat{V}_{\ell}(\sigma) = V_{g\ell}(\sigma) - \sum_{\ell' \in N^2_y} \frac{\hat{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} + V_{\ell}(\sigma) + \frac{\hat{m}_{\ell}(\sigma)}{V_{\text{den}}(\sigma)} \tag{92}
\]

\[
\leq V_{g\ell}(\sigma) - \frac{\hat{m}_{\ell}(\sigma)}{V_{\text{den}}(\sigma)} + V_{\ell}(\sigma) + \frac{\hat{m}_{\ell}(\sigma)}{V_{\text{den}}(\sigma)} \tag{93}
\]

\[
= V_{g\ell}(\sigma) + V_{\ell}(\sigma), \tag{94}
\]

where (92) follows from substituting for \(\hat{V}_{\ell}(\sigma)\) and \(\hat{V}_{g}(\sigma)\) using (85) and (86); and (93) because \(\sum_{\ell' \in N^2_y} \frac{\hat{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{\hat{m}_{\ell}(\sigma)}{V_{\text{den}}(\sigma)}\).

From (90), \(\nu_\sigma(x^\sigma) = 1\) follows because \(u_M(x^\sigma) > (1 - \delta)u_M(\bar{q}) + \delta V_M(\sigma)\). Furthermore, (90) implies \(u_{g\ell}(x^\sigma) > (1 - \delta)u_{g\ell}(q) + \delta V_{g\ell}(\sigma)\) and \(u_{\ell}(x^\sigma) > (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma)\). Then (94) implies

\[
u_{g\ell}(x^\sigma) + \nu_{\ell}(x^\sigma) > (1 - \delta)u_{g\ell}(q) + \delta V_{g\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) \geq (1 - \delta)u_{g\ell}(q) + \delta \hat{V}_{g\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta \hat{V}_{\ell}(\sigma).
\]

Thus, (91) holds.

Next, assume \(\varphi_\ell(x^\sigma, m) < 1\) for \(m\) such that \(\ell\) is indifferent between accepting \((x^\sigma, m)\) and rejecting. For sufficiently small \(\varepsilon > 0\), \(\varphi_\ell(x^\sigma, m + \varepsilon) = 1\) and the preceding argument implies \(g_\ell\) strictly prefers \((x^\sigma, m + \varepsilon)\) over any \((y', m')\) such that \(\nu_\sigma(y') = 0\). □

**Lemma B.4.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

**Proof.** Let \(\sigma\) be an equilibrium. By Duggan (2014), \(M\) is decisive. Quadratic utility and \(\hat{x}_M = 0 \neq q\) together imply \(A(\sigma) = \{x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)\}\) is a closed, non-empty interval symmetric about 0. Let \(A(\sigma) = [-\overline{x}(\sigma), \overline{x}(\sigma)]\). Then \(x \in (-\overline{x}(\sigma), \overline{x}(\sigma))\) implies \(\nu_\sigma(x) = 1\).

Fix \(\ell \in N^2\). Lemma B.2 implies \(\chi^\sigma\) is not degenerate on \(q\). Lemma B.3 implies existence of \((y, m) \in W\) such that: \(\nu_\sigma(y) = 1\) and \(g_\ell\) strictly prefers \((y, m)\) over all \((y', m')\) such that \(\nu_\sigma(y') = 0\). Thus, \(y \in A(\sigma)\) for all \((y, m) \in \text{supp}(\lambda_{g_\ell})\). Without loss of generality, assume \(\nu_\sigma(-\overline{x}(\sigma)) < 1\). It suffices to check two cases.

\begin{itemize}
  \item **Case 1:** If \(\hat{x}_\ell \leq -\overline{x}(\sigma)\) and \(u_{\ell}(-\overline{x}(\sigma)) > (1 - \delta)u_{\ell}(q) + \delta \hat{V}_{\ell}(\sigma)\), then \(x \in A(\sigma)\) for all \(x \in \text{supp}(\pi_\ell)\). Strict concavity of \(u_\ell\) and \(\nu_\sigma(-\overline{x}(\sigma)) < 1\) imply existence of
\[\varepsilon > 0\] such that \(\ell\) has a profitable deviation to \(-\bar{x}(\sigma) + \varepsilon\), a contradiction.

- **Case 2**: Assume \(\hat{y}_\ell \leq -\bar{x}(\sigma)\). Continuity, Lemma B.3, and \(\bar{v}_\sigma(-\bar{x}(\sigma)) < 1\) imply existence of \(\varepsilon, \varepsilon' > 0\) such that \(g_\ell\) has a profitable deviation to \((y', m') = (-\bar{x}(\sigma) + \varepsilon, \bar{U}_\ell(\sigma) - u_\ell(-\bar{x}(\sigma) + \varepsilon) + \varepsilon')\), a contradiction.

It follows that either \(\sigma\) must involve deferential voting, or \(\sigma\) is equivalent in outcome distribution to an equilibrium with deferential voting. \(\blacksquare\)

**Lemma B.5.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

*Proof.* Let \(\sigma\) denote an equilibrium. By Lemma B.4, we can assume \(v_\sigma(x) = 1\) iff \(x \in A(\sigma)\). Fix \(\ell \in N^L\) and define \(y^*_g = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - \bar{U}_\ell(\sigma)\), which is uniquely defined, and \(m^*_g = \bar{U}_\ell(\sigma) - u_\ell(y^*_g)\).

By Lemma B.2, \(\chi^\sigma\) is not degenerate on \(q\). For sufficiently small \(\varepsilon > 0\), Lemma B.3 implies \(g\) strictly prefers \((y^*_g, m^*_g + \varepsilon)\) over every \((y', m')\) such that \(y' \notin A(\sigma)\). Thus, if \(\pi_\ell\) is not degenerate on \(y^*_g\) and \(\varphi_\ell(y^*_g, m^*_g) < 1\), then there exists \(\varepsilon > 0\) such that \(g_\ell\) has a profitable deviation to \((y^*_g, m^*_g + \varepsilon)\), a contradiction. Thus, \(\sigma\) must satisfy either \(i\) \(\pi_\ell(y^*_g) = 1\), or \(ii\) \(\lambda^\ell_g(y^*_g, m^*_g) = 1\) and \(\varphi_\ell(y^*_g, m^*_g) = 1\), as desired. \(\blacksquare\)

A strategy profile \(\sigma\) is no-delay if \(\bar{v}_\sigma(x) = 1\) for all \(x \in \text{supp}(\pi_\ell)\) and \(\bar{v}_\sigma(y) = 1\) for all \((y, m) \in \text{supp}(\lambda^\ell_g)\).

**Lemma B.6.** Every stationary legislative lobbying equilibrium is no-delay.

*Proof.* Fix an equilibrium \(\sigma\). By Lemma B.2, \(\chi^\sigma\) is not degenerate on \(q\). Thus, Lemma B.3 implies \(g\) strictly prefers some \((y, m) \in W\) such that \(\bar{v}_\sigma(y) = 1\). Lemma B.4 implies we can assume \(\bar{v}_\sigma(x) = 1\) iff \(x \in A(\sigma)\). Lemma B.5 implies we can assume all \(\ell \in N^L\) use deferential acceptance strategies.

For each \(\ell \in N^L\), the preceding argument and Lemma B.1 imply \(\lambda^\ell_g\) puts probability one on \((y^*, m^*)\) such that \(y^* = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell; \sigma)\), which is unique. Lemmas B.4 and B.5 imply we can assume \(\bar{v}_\sigma(y^*) = 1\) and \(\varphi_\ell(y^*, m^*) = 1\).

The proof verifies \(z_\ell \notin A(\sigma)\) cannot be optimal for any \(\ell \in N^L\). To show a contradiction, assume proposing \(z_\ell \notin A(\sigma)\) is optimal for some \(\ell \in N^L\). Let \(z^* = \arg\max_{x \in A(\sigma)} u_\ell(x)\).

There are two steps. Step 1 establishes useful properties of \(\ell\)'s preferences over lotteries. Step 2 shows a contradiction.
Step 1: Recall $\chi = (1 - \delta)\chi^\delta + \delta \chi^\sigma$ and $x^\sigma$ denotes the mean of $\chi$. Jensen’s inequality implies $u_i(x^\sigma) > \int_X u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma)$ for all $i \in N$, so $x^\sigma \in \text{int}A(\sigma)$.

Next, let $\chi^{z^*}$ denote the policy lottery that is nearly equivalent to $\chi$, but transfers probability $\frac{\delta \rho \alpha}{V^\text{den}(\sigma)}$ from $y^*$ to $z^*$. Let $x^{z^*}$ denote the mean of $\chi^{z^*}$. For all $i \in N$, Jensen’s inequality implies

$$u_i(x^{z^*}) > \int_X u_i(x) \chi^{z^*}(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho \alpha u_i(y^*)}{V^\text{den}(\sigma)} + \frac{\delta \rho \alpha u_i(z^*)}{V^\text{den}(\sigma)}.$$ 

Moreover, $x^{z^*}$ is located weakly between $x^\sigma$ and $z^*$, implying $x^{z^*} \in A(\sigma)$.

Step 2: Since $z_\ell \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - u_\ell(y^*)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \tilde{m}_\ell(\sigma)}{V^\text{den}(\sigma)} - u_\ell(y^*). \tag{95}$$

Using (84), $\tilde{m}_\ell(\sigma)$ is recursively defined as

$$\tilde{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \tilde{m}_\ell(\sigma)}{V^\text{den}(\sigma)} - u_\ell(y^*) \right)$$

$$= \frac{\rho_\ell \alpha_\ell V^\text{den}(\sigma) [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^\text{den}(\sigma) - \delta \rho_\ell \alpha_\ell}. \tag{96}$$

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \tilde{V}(\sigma) \tag{97}$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \tilde{m}_\ell(\sigma)}{V^\text{den}(\sigma)} \tag{98}$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^\text{den}(\sigma) - \delta \rho_\ell \alpha_\ell}, \tag{99}$$

where (98) follows from the definition of $\tilde{V}_\ell(\sigma)$; and (99) from using (96) to substitute for $\tilde{m}_\ell(\sigma)$ and simplifying. Next, we have

$$V^\text{den}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell \tag{100}$$

$$> 0, \tag{101}$$
where (100) follows because Lemma B.3 implies that all lobby offers are accepted and passed under $\sigma$, so $V_{\text{den}}(\sigma) \geq 1 - \delta \sum_{j \in \mathcal{N}^L \setminus \ell} \rho_j (1 - \alpha_j)$; and (101) from rearranging and $\rho_\ell \alpha_\ell + \sum_{j \in \mathcal{N}^L \setminus \ell} \rho_j (1 - \alpha_j) \leq 1$. Simplifying and rearranging (97) using (99),

$$0 \leq V_{\text{den}}(\sigma) \left((1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \delta \rho_\ell \alpha_\ell (u_\ell(y^*) - u_\ell(z^*))\right)$$

$$= \int_X u_\ell(x) \chi^{z^*}(dx) - u_\ell(z^*),$$

a contradiction because $u_\ell(z^*) \geq u_\ell(x^{z^*}) > \int_X u_\ell(x) \chi^{z^*}(dx)$.

Thus, $\nu_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$. $\Box$

**Lemma B.7.** Every stationary legislative lobbying equilibrium is such that $\lambda_g$ is degenerate for all $g \in \mathcal{N}^G$ and $\pi_\ell$ is degenerate for all $\ell \in \mathcal{N}^L$.

**Proof.** Let $\sigma$ denote an stationary legislative lobbying equilibrium. By Duggan (2014), $A_M(\sigma) = A(\sigma)$, which is nonempty, compact and convex.

(i) Consider $g \in \mathcal{N}^g$ and $\ell \in \mathcal{N}^L_g$. Recall $\tilde{U}_\ell(\sigma)$ from (87). Lemmas B.1 and B.6 imply $\lambda_\ell^g$ puts probability one on $(y^*, m^*)$ satisfying $y^* = \arg\max_{y \in \mathcal{A}(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$, which is unique because the objective function is continuous and strictly concave in $y$, and $m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*)$. Thus, $\lambda_\ell^g$ is degenerate.

(ii) Consider $\ell \in \mathcal{N}^L$. Lemma B.6 implies $\pi_\ell$ puts probability one on $x^* = \arg\max_{x \in \mathcal{A}(\sigma)} u_\ell(x)$, which is unique. Thus, $\pi_\ell$ is degenerate. $\Box$

Proposition 1.2 corresponds to Part 2 of Proposition 1 in the text.

**Proposition 1.2** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

**Proof.** Consider a stationary legislative lobbying equilibrium $\sigma$. Lemma B.7 implies $\lambda$ and $\pi$ are degenerate. Lemma B.5 and (17) imply $\sigma$ is equivalent in outcome distribution to an equilibrium with all $\ell \in \mathcal{N}^L$ using deferential acceptance strategies. Lemmas B.4 - B.6 deliver the result. $\Box$
Appendix C

Consider $\ell \in N^L$. First, I define a function $\zeta^\ell$ that relates to $M$’s equilibrium voting decision. Then, Lemmas C.3 - C.6 characterize $\zeta^\ell$. Finally, Lemma 1 delivers the partitional characterization on $\hat{x}_g$ facilitating Proposition 2.

Preliminaries to define $\zeta^\ell$: Recall $\pi(0) = \pi(\hat{x}_g)$ for $\hat{x}_g = 0$. Let $\hat{D}^\ell,y = \{\hat{y}_j : |\hat{y}_j| > \pi(0), j \neq \ell\}$ and $\hat{D}^\ell,x = \{\hat{x}_j : |\hat{x}_j| > \pi(0), j \neq \ell\}$. Next, set $D^\ell,y = \{|y| : y \in \hat{D}^\ell,y\}$ and $D^\ell,x = \{|x| : x \in \hat{D}^\ell,x\}$. Define $D^\ell$ as the unique elements of $D^\ell,y \cup D^\ell,x \cup \{\pi(0)\}$. Let $K^\ell + 1 = |D^\ell|$. Denote the $k$-th element of $D^\ell$ as $d_k^\ell$. Index elements $k = 0, \ldots, K^\ell$ of $D^\ell$ in ascending order so that $d_0^\ell = \pi(0)$ and $k' > k$ implies $d_{k'}^\ell > d_k^\ell$.

For each $k$ and $j \neq \ell$, let $C_j^k = \|\hat{x}_j \in [-d_k^\ell, d_k^\ell]\|$ and $\tilde{C}_j^k = \|\hat{y}_j \in [-d_k^\ell, d_k^\ell]\|$, suppressing dependence on $\ell$. Define

$$I_j^\ell = (1 - \alpha_j)C_j^k u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^k u_M(\hat{y}_j)$$

and

$$O_j^\ell = (1 - \alpha_j)(1 - C_j^k) + \alpha_j(1 - \tilde{C}_j^k),$$

again suppressing dependence on $\ell$. Let

$$\hat{x}_k^\ell = \left(\frac{1}{\delta \rho_k} \left[ (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell) \left(1 - \delta \sum_{j \neq \ell} O_j^k \right) \right]\right)^{\frac{1}{2}}. \quad (102)$$

Because $d_0^\ell = \pi(0)$, rearranging (102) yields $\hat{x}_0^\ell = 0$.

Lemma C.1 establishes a useful identity.

**Lemma C.1.** For all $\ell \in N^l$ and each $k = 0, \ldots, K^\ell$, we have

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) = \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell) \left(1 - \delta \sum_{j \neq \ell} O_j^k \right). \quad (103)$$

**Proof.** Consider $\ell \in N^L$ and fix $k < K^\ell$. Then,

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) \left(1 - \delta \sum_{j \neq \ell} O_j^{k+1} \right)$$
where (104) follows because \( u_M(d^k_{k+1}) \sum_{j \neq \ell} \rho_j (O^k_j - O^k_j) = \sum_{j \neq \ell} \rho_j (I^k_j - I^k_j) \). \( \square \)

**Lemma C.2.** For all \( \ell \in N^L \), \( \hat{x}^\ell_k \) strictly increases in \( k \).

**Proof.** Consider \( \ell \in N^L \) and fix \( k < K^\ell \). Then,

\[
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d^k_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1})
= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d^k_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k)
\]

(106)

\[
> \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d^k_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k)
\]

(107)

where (106) follows from Lemma C.1; and (107) follows from \( 0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell) \). Thus, \( \hat{x}^\ell_k < \hat{x}^\ell_{k+1} \) follows from (102). \( \square \)

**Definition of \( \zeta^\ell_k \):** For each \( k = 0, \ldots, K^\ell \), define \( \pi^\ell_k : \mathbb{R}_+ \to \mathbb{R}_+ \) as

\[
\pi^\ell_k(x) = \left( -\frac{(1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I^k_j}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \right)^{\frac{1}{2}} \quad (108)
\]

and \( \zeta^\ell_k : \mathbb{R}_+ \to \mathbb{R} \) as

\[
\zeta^\ell_k(x) = u_M(x) - \left( (1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I^k_j + \delta u_M(\pi^\ell_k(x)) \sum_{j \neq \ell} \rho_j O^k_j \right).
\]

By construction, \( \pi^\ell_k(\hat{x}^\ell_k) = d^\ell_k \). Adopt the convention \( d^\ell_{K^\ell+1} = \infty \). Define the piecewise
function $\zeta^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$\zeta^\ell(x) = \zeta_k^\ell(x) \text{ if } x \in [d_k^\ell, d_{k+1}^\ell).$$

Lemmas C.3 - C.5 prove useful properties of $\zeta^\ell$.

**Lemma C.3.** For all $\ell \in N^L$, $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$.

**Proof.** Consider $\ell \in N^L$. First, we have

$$\zeta^\ell(0) = \zeta_0^\ell(0)$$

$$= u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho_l u_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\varpi_0^0) \sum_{j \neq \ell} \rho_j O_j^0 \right) \tag{109}$$

$$= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right) \tag{110}$$

$$> 0, \tag{111}$$

where (110) follows from $u_M(0) = 0$ and $\varpi_0^0(0) = \tilde{x}_0$.

Next, I show $\zeta^\ell(q) \leq 0$. Let $k'$ denote the largest $k$ such that $\tilde{x}_k^\ell \leq q$. There are three steps.

- **Step 1:** Because $\varpi^k(\tilde{x}_k^\ell) = d_k^\ell$, we have

$$u_M(d_k^\ell) = \frac{(1 - \delta)u_M(q) + \delta \rho_l u_M(\tilde{x}_k^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_l u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_l u_M(q) + \delta u_M(d_0^\ell)(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_l u_M(q) + \delta u_M(d_0^\ell)(1 - \rho_l - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{114}$$

where (112) follows from rearranging (108); (113) from $q \geq \tilde{x}_k^\ell$; (114) because for all $j$ the construction of $I_j^{k'}$ implies $I_j^{k'} \geq u_M(d_0^\ell)[(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}]$; and (115) because $\sum_{j \neq \ell} \rho_j[(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_l - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction.
Rearranging and simplifying (115) yields $u_M(d_{k'}^k) \geq \frac{(1-\delta+\delta \rho_e)u_M(q)}{1-\delta+\delta \rho_e} = u_M(q)$. Thus, we have

$$
\sum_{j \neq \ell} \rho_j I_j^k = \sum_{j \neq \ell} \rho_j \left[ (1-\alpha_j)C_j^k u_M(\hat{x}_j) + \alpha_j C_j^k u_M(\hat{y}_j) \right] 
$$

(116)

$$
\geq u_M(d_{k'}^k) \sum_{j \neq \ell} \rho_j \left[ (1-\alpha_j)C_j^k + \alpha_j \tilde{C}_j^k \right] 
$$

(117)

$$
u_M(d_{k'}^k)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) 
$$

(118)

$$
u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) 
$$

(119)

where (116) follows from the definition of $I_j^k$; (117) from $u_M(\hat{x}_j) \geq u_M(d_{k'}^k)$ if $C_j^k = 1$ and $u_M(\hat{y}_j) \geq u_M(d_{k'}^k)$ if $\tilde{C}_j^k = 1$; (118) because $\sum_{j \neq \ell} \rho_j [(1-\alpha_j)C_j^k + \alpha_j \tilde{C}_j^k] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction; and (119) from $u_M(d_{k'}^k) \geq u_M(q)$.

- **Step 2:** We have

$$
u_M(\overline{x}_{k'}^k(q)) = \frac{(1-\delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} 
$$

$$
\geq \frac{(1-\delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} 
$$

(120)

$$
u_M(q). 
$$

(121)

where (120) follows from Step 1 and (121) from simplifying.

- **Step 3:** To see $\zeta^\ell(q) \leq 0$, note

$$
\zeta^\ell(q) = u_M(q) - \left( (1-\delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_{k'}^k(q)) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) 
$$

$$
\leq u_M(q) - \left( (1-\delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) + \delta u_M(q) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) 
$$

(122)

$$
= 0, 
$$

(123)

where (122) follows from Steps 1 and 2.
Lemma C.4. For all $\ell \in N^L$, $\zeta^\ell$ is continuous.

Proof. Consider $\ell \in N^L$ and fix $k$. Because $\overline{x}_k^\ell(x)$ is continuous, $\zeta^\ell$ is continuous over $(\hat{\mathbf{x}}_k^\ell, \check{\mathbf{x}}_{k+1}^\ell)$. It suffices to show $\zeta_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell) = \zeta_{k+1}^\ell(\hat{\mathbf{x}}_{k+1}^\ell)$.

First, I establish $d_{k+1}^\ell = \overline{x}_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell)$. Rearranging (102) yields

$$0 = u_M(d_{k+1}^\ell) \left(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) - (1 - \delta)u_M(q) - \delta \rho_{\ell} u_M(\hat{\mathbf{x}}_{k+1}^\ell) - \delta \sum_{j \neq \ell} \rho_j I_j^{k+1}$$

$$= u_M(d_{k+1}^\ell) \left(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k \right) - (1 - \delta)u_M(q) - \delta \rho_{\ell} u_M(\hat{\mathbf{x}}_{k+1}^\ell) - \delta \sum_{j \neq \ell} \rho_j I_j^k,$$ (124)

where (124) follows from Lemma C.1. Rearranging, $u_M(d_{k+1}^\ell) = \frac{(1 - \delta)u_M(q) + \delta \rho_{\ell} u_M(\hat{\mathbf{x}}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$, which implies $d_{k+1}^\ell = \overline{x}_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell)$. Thus,

$$\zeta_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell) = u_M(\hat{\mathbf{x}}_{k+1}^\ell) - \left[(1 - \delta)u_M(q) + \delta \rho_{\ell} u_M(\hat{\mathbf{x}}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_j^k \right]$$

$$= u_M(\hat{\mathbf{x}}_{k+1}^\ell) - \left[(1 - \delta)u_M(q) + \delta \rho_{\ell} u_M(\hat{\mathbf{x}}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} + \delta u_M(\overline{x}_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_j^{k+1} \right]$$

$$= \zeta_{k+1}^\ell(\hat{\mathbf{x}}_{k+1}^\ell),$$ (125)

where (125) follows from Lemma C.1 because $d_{k+1}^\ell = \overline{x}_k^\ell(\hat{\mathbf{x}}_{k+1}^\ell)$. □

Lemma C.5. For all $\ell \in N^L$, $\zeta^\ell$ is strictly decreasing.

Proof. Consider $\ell \in N^L$. The proof shows that the derivative of $\zeta^\ell$ is strictly negative at every $x \in (\hat{\mathbf{x}}_k^\ell, \check{\mathbf{x}}_{k+1}^\ell)$ for all $k$. Continuity then implies that $\zeta^\ell$ is strictly decreasing.

Fix $k$ and consider $x \in (\hat{\mathbf{x}}_k^\ell, \check{\mathbf{x}}_{k+1}^\ell)$. Then

$$\zeta^\ell(x) = u_M(x) - \left[(1 - \delta)u_M(q) + \delta \rho_{\ell} u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right]$$

and

$$\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x \delta \rho_{\ell} - \frac{\partial u_M(\overline{x}_k^\ell(x))}{\partial \overline{x}_k^\ell(x)} \frac{\partial \overline{x}_k^\ell(x)}{\partial x} \left(\delta \sum_{j \neq \ell} \rho_j O_j^k \right)$$
By Lemma C.6, 

\[ \hat{x} = -2x + 2x\delta \rho + \frac{2x\delta \rho \left( \delta \sum_{j \neq \ell} \rho_j O^k_j \right)}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \]  \hspace{1cm} (127) 

\[ \propto \delta \rho_k + \delta \sum_{j \neq \ell} \rho_j O^k_j - 1 \]  \hspace{1cm} (128) 

\[ < 0, \]  \hspace{1cm} (129) 

where (127) follows from \( \frac{\partial u_M(x_k(x))}{\partial x_k(x)} \frac{\partial \sigma(x)}{\partial x} = -\frac{2x\delta \rho}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \); and (129) because \( \delta \in (0, 1) \) and \( \rho + \sum_{j \neq \ell} \rho_j O^k_j \leq 1 \). \( \square \)

**Lemma C.6.** For all \( \ell \in N^L \), there is a unique \( x_\ell \in (0, q) \) such that \( \zeta^\ell(x) > 0 \) for all \( x \in [0, x_\ell) \), \( \zeta^\ell(x) = 0 \), and \( \zeta^\ell(x) < 0 \) for all \( x > x_\ell \).

**Proof.** Consider \( \ell \in N^L \). Lemma C.3 implies \( \zeta^\ell(0) > 0 \) and \( \zeta^\ell(q) \leq 0 \). By Lemma C.5, \( \zeta^\ell \) is strictly decreasing. Thus, there is a unique \( x_\ell \in (0, q] \) such that \( \zeta^\ell(x) > 0 \) for all \( x \in [0, x_\ell) \) and \( \zeta^\ell(x) < 0 \) for all \( x > x_\ell \). Lemma C.4 implies \( \zeta^\ell(x_\ell) = 0 \). \( \square \)

**Lemma 1.** For all \( \ell \in N^L \), \( \hat{x}_g \in (-x_\ell, x_\ell) \) implies \( \hat{x}_g \in \text{int}A(\hat{x}_g) \). Otherwise, \( A(\hat{x}_g) = [-x_\ell, x_\ell] \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_\ell = \hat{x}_g \). There are two parts. Part 1 shows \( \hat{x}_g \in (-x_\ell, x_\ell) \) implies \( \hat{x}_g \in \text{int}A(\hat{x}_g) \). Part 2 shows \( \hat{x}_g \notin (-x_\ell, x_\ell) \) implies \( A(\hat{x}_g) = [-x_\ell, x_\ell] \).

**Part 1.** Assume \( \hat{x}_g \in (-x_\ell, x_\ell) \) and suppose \( \hat{x}_g \geq 0 \) without loss of generality. Let \( k' \) be the largest \( k \) such that \( \hat{x}_{k'} \leq \hat{x}_g \). Define the strategy profile \( \sigma' \) such that it puts probability \( \rho_{k'} \) on \( \hat{x}_g \) and for each \( j \neq \ell \) it (i) puts probability \( (1 - \alpha_j) \rho_j \) on: \( \hat{x}_j \) if \( \hat{x}_j \in [-d_{k'}^j, d_{k'}^j] \), \( \bar{x}_{k'}(\hat{x}_g) \) if \( \hat{x}_j > d_{k'}^j \), or \( -\bar{x}_{k'}(\hat{x}_g) \) if \( \hat{x}_j < -d_{k'}^j \); and (ii) puts probability \( \alpha_j \rho_j \) on: \( \hat{y}_j \) if \( \hat{y}_j \in [-d_{k'}^j, d_{k'}^j] \), \( \bar{x}_{k'}(\hat{x}_g) \) if \( \hat{y}_j > d_{k'}^j \), or \( -\bar{x}_{k'}(\hat{x}_g) \) if \( \hat{y}_j < -d_{k'}^j \). By construction, \( \bar{x}(\sigma') = \bar{x}_{k'}(\hat{x}_g) \). Furthermore, legislator proposal strategies are optimal given \( A(\sigma') = [-x_\ell, x_\ell] \).

I now check optimality for \( M \). Because \( \hat{x}_g \in [-d_{k'}^j, d_{k'}^j] \), we have \( \bar{x}(\sigma') = \bar{x}_{k'}(\hat{x}_g) \in [-d_{k'}^j, d_{k'}^j] \). Thus, \( M \) optimally accepts all offers by \( j \neq \ell \). Next, I verify \( \hat{x}_g \in \text{int}A(\sigma') \).

By Lemma C.6, \( \hat{x}_g \in (-x_\ell, x_\ell) \) implies \( \zeta(\hat{x}_g) > 0 \), which is equivalent to \( u_M(\hat{x}_g) \geq (1 - \delta) u_M(q) + \delta \rho_{\ell} u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j O^k_j \). Under \( \sigma' \), this is equivalent to \( \hat{x}_g \in \text{int}A(\sigma') \).

Thus, \( \sigma' \) is equivalent to the equilibrium \( \sigma(\hat{x}_g) \) and \( \hat{x}_g \in \text{int}A(\hat{x}_g) \), as desired.
Part 2. Assume $\hat{x}_g \notin (-\overline{x}_\ell, \overline{x}_\ell)$ and suppose $\hat{x}_g \geq 0$ without loss of generality. There are two steps. Step 1 shows $\overline{x}(\hat{x}_g) \geq \overline{x}_\ell$. Step 2 shows $\overline{x}(\hat{x}_g) \leq \overline{x}_\ell$.

- **Step 1.** Suppose $\overline{x}(\hat{x}_g) < \overline{x}_\ell$. Let $k'$ be the largest $k$ such that $\hat{x}_k' \leq \overline{x}(\hat{x}_g)$. Because $\hat{x}_g > \overline{x}_\ell > \overline{x}(\hat{x}_g)$, it follows that $\sigma(\hat{x}_g)$ puts probability $\rho_\ell$ on $\overline{x}(\hat{x}_g)$. Thus, $u_M(\overline{x}(\hat{x}_g)) = \frac{(1-\delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j l_j'}{1-\delta \sum_{j \neq \ell} \rho_j O_j}$ and rearranging yields $\zeta(\overline{x}(\hat{x}_g)) = 0$. Lemma C.6 implies $\overline{x}(\hat{x}_g) = \overline{x}_\ell$, a contradiction.

- **Step 2.** Suppose $\overline{x}(\hat{x}_g) > \overline{x}_\ell$. If $\hat{x}_g \geq \overline{x}(\hat{x}_g)$, then the argument from Step 1 shows a contradiction. Assume $\hat{x}_g < \overline{x}(\hat{x}_g)$. Let $k'$ be the largest $k$ such that $\hat{x}_k' \geq \overline{x}(\hat{x}_g)$. Then $\sigma(\hat{x}_g)$ puts probability $\rho_\ell$ on $\hat{x}_g$. Next, $M$ optimally accepts $\hat{x}_g$ under $\sigma(\hat{x}_g)$ iff $u_M(\hat{x}_g) \geq \frac{(1-\delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j l_j'}{1-\delta \sum_{j \neq \ell} \rho_j O_j}$. Rearranging, this condition is equivalent to $\zeta(\hat{x}_g) \geq 0$. Lemma C.6 implies $\hat{x}_g \leq \overline{x}_\ell$, a contradiction.
References


