Access and Influence in Legislatures

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Abstract

Access is an important prerequisite for outside influence. One prominent form is targeted access that provides interest groups with chances to lobby certain politicians while they draft proposals. I study a game-theoretic model and show how such access, merely by increasing the possibility of that influence, can itself endogenously influence several behaviors. It raises everyone’s anticipation of lobbying, thereby affecting what pivotal legislators will pass and, in turn, proposals and lobbying expenditures. The magnitude of these endogenous effects varies with polarization, and their direction depends on the group’s extremism relative to its target. They can work in the group’s favor or against it, potentially even overwhelming the direct benefit of more lobbying opportunities. For example, moderate groups crave access to relatively extreme politicians but avoid access to a range of more centrist politicians. The results build our theoretical understanding of access and have implications for various political expenditures.

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Lobbying is one of the main ways that interest groups try to influence policy, and one of the best ways to lobby is by shaping the content of bills as they are written in committee (Hall and Wayman, 1990; Powell, 2014). In order to do so, however, interest groups need typically *access*, which provides opportunities to engage and communicate with policymakers (Wright, 1996). Since politicians are busy with many other obligations and carefully allocate their scarce time, gaining access usually requires groups to develop relationships, or hire lobbyists with relationships, well ahead of time. What are the effects of those relationships? And which politicians do interest groups want to target?

Since access is critical for outside influence, it is a central topic for scholars of interest groups. Like many other forms of outside influence, however, it is notoriously difficult to study. Two important obstacles are that (i) access is difficult to observe and measure (Miller, 2021a) and (ii) interest groups and politicians are highly strategic, often allowing multiple explanations to be consistent with observed data. One way to address these obstacles is by refining our theoretical understanding of access, which can suggest new avenues for indirect evidence and shed light on empirical findings. That is the goal of this paper.

The main contribution of this paper is a theoretical analysis that isolates strategic effects of targeted access that provides chances to influence proposals. I show how such access can indirectly affect proposals and votes by other politicians who are not targeted, as well as lobbying expenditures by interest groups. I then study how an interest group’s desire to acquire access depends on (i) its own ideology, (ii) the target’s ideology and proposal power, as well as (iii) the proposal power of ideologically extreme politicians. The results complement existing theories of access, many of which consider an isolated target and thus

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1See Miller (2021a) for an overview of recent work, which has shed light on how groups can acquire access (Kalla and Broockman, 2015; Fouirnaies and Hall, 2017; McCraine, 2018; Bertrand et al., 2014; Blanes i Vidal et al., 2012), which politicians they want to target (Powell and Grimmer, 2016; Fouirnaies, 2017; Miller, 2021b; Liu, 2021), and whether access facilitates influence (Ban and You, 2019).

2See, e.g., Powell and Grimmer (2016) or Kalla and Broockman (2015) for more discussion.

3As (Miller, 2021a, p. 297) notes while reviewing empirical studies of targeted access: “though the theoretical linkages between access and other quantities of interest are sometimes unclear, formal theory can help researchers elucidate expectations and guide empirical tests.” For an exemplary empirical study of outside influence that incorporates strategic behavior, see Kang (2015).
cannot study how non-targeted politicians may react. Additionally, the analysis sheds light on longstanding puzzles about why many groups are less aggressive than expected in pursuing access, and also provides empirical implications for the relationship between access-seeking expenditures and lobbying expenditures.

To do so, I study a game-theoretic model of legislative policymaking with interest groups. The model has three key features. First, access and lobbying are linked but distinct: a group with access to a politician can potentially lobby her. Thus, the model reflects the prevailing view that access merely provides opportunities to exert influence (Powell, 2014). Second, lobbying occurs only in committee, to possibly shape proposals before they are voted on. By adopting this focus, the model isolates an important form of outside influence: interest groups participating in drafting legislation (Schattschneider, 1960; Kroeger, 2020). Third, the final key feature is an explicit political interaction with multiple strategic politicians and a strategic interest group, each of whom anticipates the possibilities of (i) revisiting failed proposals, (ii) changes in agenda control, and (iii) outside influence. Understanding targeted access in a wider political context has been highlighted as an important area for development (Baumgartner, 2010; Leech, 2010) and, moreover, it allows us to study how access interacts with elite polarization and proposal power.

A key insight of the analysis is that, simply by providing opportunities to lobby the target’s proposal, access can also endogenously affect (i) which policies would pass if proposed, (ii) what extreme politicians propose, and (iii) lobbying effort. These indirect effects arise because access causes everyone to anticipate potential influence in equilibrium, and thereby changes their expectations about future proposals that would follow rejection today. As in other models where bargaining can continue following rejected proposals (e.g., Banks and Duggan, 2006a), those expectations shape the set of policies that will pass today. Thus,

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4Leech (2010) notes that “[..] organized interests recognize the importance of legislative procedure and agenda setters” and “studies of their targeting choices, and of their lobbying activity more broadly, must consider how procedural context conditions their behavior.”

5Additionally, expanding the scope of application for legislative bargaining models has independent theoretical interest.
access can flip decisive votes on certain policies, thereby changing which policies will pass and, in turn, influencing proposals by extreme politicians who are constrained. And finally, by indirectly altering those extreme proposals, access can also indirectly affect lobbying expenditures.

I show that the nature of these indirect effects depends on whether lobbying would shift the target’s proposals towards veto players, or away from them. A key finding is that access facilitating lobbying that will shift policy inward also will induce more moderation by extremists, and vice versa. For example, if decisive voters anticipate a lower chance of more extreme proposals, then they will be more willing to continue bargaining and will accept a narrower range of proposals today, which forces extreme legislators to pass more moderate policy. Additionally, I show that such access also weakly decreases the lobbying effort that any group will exert to influence proposals. In contrast, if decisive voters anticipate a higher chance of extreme proposals, then extremists have more slack and pass more extreme policy, while lobbying effort weakly increases.

The endogenous effects of access can be good or bad for the interest group. First, if the group is not too extreme, then it always benefits from access that helps it lobby policy inward towards the center, but always dislikes the endogenous effects of access that helps it lobby outward. Thus, such groups are broadly more inclined to access a relatively extreme politician, but less inclined to access a relatively centrist one. For extreme groups, however, the indirect effects of access depend on the relative proposal power of extremists on both sides of the spectrum. If extreme politicians on the group’s side are sufficiently more likely to propose than opposing extremists, then it benefits from the endogenous effects of access that will provide chances to lobby policy outward. In contrast, it suffers from such access if opposing extremists are sufficiently more likely to propose than its aligned extremists.

I show that, from the group’s perspective, the downside of access can outweigh the upside, so that more access to certain politicians makes the group worse off. Specifically, groups that are not too extreme will forgo access to a range of more centrist legislators. I show how the
group’s expected loss from greater policy extremism can outweigh its benefit from higher chances of lobbying. This result holds even if access is free and stems from a commitment problem: whenever the group can lobby, it will always pull policy weakly towards itself. Thus, access necessarily causes everyone to anticipate that the group will potentially shift some of the target’s potential proposals in its own favor. This anticipation can also work to the group’s advantage, however, as these moderate groups always want access to a broad range of more extreme politicians because such access increases opportunities to lobby and therefore endogenously decreases expected policy extremism in the legislature, which forces extreme politicians to propose more centrist policies that are also more favorable to the group. Only the most extreme politicians are not worth targeting, since lobbying does not affect their behavior.

The analysis offers several implications for our understanding of outside influence and money in politics. First, I shed new light on the prominent view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). I show that, even if it does not have any direct influence, access can endogenously influence key votes and proposals by various politicians. Second, the results suggest potential relationships between different types of political expenditures and therefore have implications for how regulations on access-seeking behaviors might “redirect money rather than lessen it” (Powell, 2014). Third, the results speak to Tullock’s puzzle that relatively few groups contribute and even they contribute surprisingly little (Tullock, 1972; Ansolabehere et al., 2003). Finally, the results are consistent with several other empirical findings, such as that groups (i) often lobby their allies (Ainsworth, 1997; Kollman, 1997; Hojnacki and Kimball, 1998, 1999), (ii) they seek access to legislators with substantial agenda power (Powell and Grimmer, 2016; Fouirnaies, 2017), and that (iii) contributing groups are overwhelmingly centrist (Bonica, 2013, p. 301).
Related Literature

Our current theoretical understanding of access comes from models that focus on either: one politician (Austen-Smith, 1995; Lohmann, 1995; Hall and Deardorff, 2006), untargeted access in a group of politicians to influence proposals (Levy and Razin, 2013), or targeted access in a group of politicians who are voting on a fixed proposal (Awad, 2020; Schnakenberg, 2017)(Baron). I study targeted access in a group of politicians as well, but I address an important gap by focusing on access that provides chances to influence proposals. This form of access is prominent in practice and, moreover, the analysis suggests that it can have subtle effects that can (i) make it more or less desirable than previously expected, and (ii) alter incentives to influence votes. Additionally, I contribute to a theoretical literature that incorporates lobbying into models of legislative bargaining.

In seminal work, Hall and Deardorff (2006) highlight that interest groups want access to legislators who are their allies in order to assist them in pursuing their aligned interests. They study targeted access, as I do, but in contrast they focus on a single politician and therefore abstract from the possibility that targeted access has endogenous indirect effects on behavior by non-targeted politicians. I complement their analysis by explicitly modelling multiple politicians in order to allow for the possibility of such effects and show that they can lead other politicians to act more favorably or less favorably, with the effect potentially differing across politicians. Furthermore, I find that these endogenous effects can lead groups to forgo access to certain allies, as the prospect of other legislators passing less favorable proposals can outweigh the prospect of the target passing a friendlier proposal. Two additional differences are that I explicitly incorporate ideology and political institutions (e.g., proposal rights and voting), which allows me to pursue their suggestion that future work “incorporate the degree of agreement over specific policies” and explore potential tradeoffs in “a legislator’s proximity to their group’s ideal policies and the legislator’s institutional or partisan ability to get things done” (Hall and Deardorff, 2006, p. 80).

The closest analyses of strategically targeted access in a collective body with multiple
strategic politicians are Awad (2020) and Schnakenberg (2017). These papers differ from this analysis in two key ways: the informational environment and the form of lobbying that access facilitates. Specifically, each studies an incomplete information setting in which lobbying provides information about the policy environment to influence votes between two exogenous proposals in an interaction that ends after today’s vote. In contrast, I study a complete information setting in which lobbying provides resources to influence endogenous policy proposals in an interaction that continues after failed proposals. The form of lobbying that I study can be interpreted as exchanging resources for more favorable proposals (in the spirit of Grossman and Helpman (1994)), or as providing a legislative subsidy to a likeminded politician in order to influence her peers (as in Hall and Deardorff (2006)). These salient forms of lobbying are worth studying in their own right, but studying them also complements the insights from Schnakenberg (2017) and Awad (2020). Future work can study how the effects highlighted in this paper interact with the informational effects they emphasize.\textsuperscript{6}

Like this paper, Schnakenberg (2017) and Awad (2020) highlight a strategic incentive to target ally legislators, but I provide a different logic. In Schnakenberg (2017), groups seek access to allies because they are more willing to forward favorable unverifiable information to the other politicians and therefore reduce the group’s cost of persuading a majority. In Awad (2020), groups target verifiable information at moderate allies who, precisely because they are more moderate, can then provide a public cheap-talk message that is more convincing to a majority of legislators. By doing so, the group can get policies passed that would have failed if it had instead lobbied the legislature publicly. In order to influence the vote, extreme groups always need access but moderate groups may not. Thus, as in this paper, (i) targeted connections can indirectly influence key votes by non-targeted legislators in equilibrium, (ii) interest groups strategically prefer access to allies, and (iii) moderate groups have different access-seeking incentives than extreme groups.

Additionally, they both find that targeted access can indirectly affect how non-targeted

\textsuperscript{6}See Grossman and Helpman (2002) for an extensive overview of canonical informational lobbying models.
politicians behave, but through a different mechanism than in this paper. There, endogenous indirect effects require targeted politicians to actively communicate with their peers after being lobbied. Here, endogenous indirect effects do not require any action by the targeted politician: instead, access causes everyone to anticipate the potential for future lobbying, and that anticipation can endogenously affect how non-targeted politicians act today.

A key difference is that, by incorporating strategic proposals, I highlight how groups can suffer from access that expands what the legislature would pass. This consideration is not present in their analyses and generates a different implication for targeting access. I provide a logic for why groups have incentives to seek access that will narrow what can pass rather than broaden it.

I also contribute to efforts to incorporate lobbying into legislative bargaining models with strategic proposals and votes. In addition to various differences, they typically study untargeted access (e.g., Levy and Razin, 2013) or do not study access (e.g., Baron, 2019). Specifically, I extend the legislative interaction in Cho and Duggan (2003) to include ideological interest groups who can potentially transfer resources to influence proposals. I extend their equilibrium concept to account for lobbying, prove existence, and show that equilibrium behavior has a clear connection to their characterization: the distribution of equilibrium proposals with lobbying is equivalent to a slightly modified version of the model without lobbying. Moreover, I show that lobbying does not introduce delay, extending well-known no-delay properties that bargaining always ends immediately in similar legislative settings without lobbying (e.g., Banks and Duggan, 2006a).

7Grossman and Helpman (2002) discuss a model in which lobbying can affect a take-it-or-leave-it proposal and the subsequent votes. Their relatively informal analysis does not discuss access and considerations about future bargaining do not play a role.
Model

Players. The key players are a politician, denoted $\ell$, and an interest group, $g$. Additionally, there are three other politicians: a left partisan $L$, a moderate $M$, and a right partisan $R$.\footnote{In the appendix, I prove the main results in an extended model with more groups and politicians.}

Timing. Politicians bargain to set policy in the interval $X \subseteq \mathbb{R}$, which is closed and non-empty. Bargaining occurs over an infinite horizon, with periods discrete and indexed $t \in \{1, 2, \ldots \}$. A status quo policy $q \in X$ persists until policy passes. Thereafter, the passed policy remains forever and the strategic interaction ends.

More precisely, bargaining proceeds in two stages during each period $t$ prior to some proposal passing.

Proposal stage. First, the period-$t$ proposer $i_t$ is drawn from probability distribution $\rho = (\rho_\ell, \rho_L, \rho_M, \rho_R)$, where $\rho_j > 0$ is politician $j$’s recognition probability. If $i_t \neq \ell$, then $g$ is not active and $i_t$ proposes any $x_t \in X$. If $i_t = \ell$, then $g$ can lobby with probability $\alpha \in [0, 1]$, which parameterizes $g$’s access.\footnote{In the appendix, I allow groups to access multiple politicians.} If $g$ is unable to lobby, then $\ell$ simply proposes any $x_t \in X$. Otherwise, $g$ offers $\ell$ a binding contract $(y_t, m_t)$ consisting of policy $y_t \in X$ and transfer $m_t \geq 0$.\footnote{Assuming that $g$ lobbies whenever possible is without loss of generality, as $g$ can always effectively forgo lobbying by offering $\ell$’s default proposal without payment.} After observing $g$’s offer, $\ell$ decides whether to accept or reject. If $\ell$ accepts, then she proposes $x_t = y_t$ and receives $m_t$ from $g$. If $\ell$ rejects, then she can propose any $x_t \in X$ and $g$ keeps $m_t$.

Voting stage. Next, $M$ decides whether to accept the proposal. If $M$ accepts, then bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $M$ rejects, then $q$ persists and active bargaining continues in $t + 1$. This stage distills the essence of majoritarian voting in a larger interaction where $M$ is a median voter (Banks and Duggan, 2006b).\footnote{This lobbying technology is similar to Bils, Duggan and Judd (2021), who study lobbying in a model of repeated elections. They do not study access, as ideology exogenously determines which group can lobby. For other recent work that models lobbying similarly, see Martimort and Semenov (2008) and an extension in Acemoglu, Egorov and Sonin (2013).}
Information. All features are common knowledge.

Payoffs. Player $i$’s per-period policy utility from $x \in X$ is $u_i(x) = -(\hat{x}_i - x)^2$, where $\hat{x}_i$ denotes $i$’s ideal point. If $\ell$ accepts $g$’s offer $(y_t, m_t)$ and $x_t$ is the period-$t$ policy, then $g$’s period-$t$ payoff is $u_g(x_t) - m_t$ and $\ell$’s period-$t$ payoff is $u_\ell(x_t) + m_t$. Thereafter, $m_t$ does not enter per-period payoffs.

Cumulative dynamic payoffs are the sum of streams of discounted per-period payoffs, with all players sharing the common discount factor $\delta \in (0, 1)$. See the appendix for complete expressions.

To sharpen key tradeoffs, I maintain several additional assumptions that are not essential. Throughout, I normalize $\hat{x}_M = 0$. Additionally, to model $L$ and $R$ as staunchly ideological and opposing partisans, I assume $\hat{x}_L < 0 < \hat{x}_R$ and $|q| < \min\{|\hat{x}_L|, \hat{x}_R\}$.

Figure 1: A period with lobbying

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Equilibrium Concept. I study a refinement of stationary subgame perfect Nash equilibrium that builds on standard equilibrium concepts in the legislative bargaining literature.

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13If $y_t$ passes, then $x_t = y_t$. Otherwise, $x_t = q$. 

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Informally, a \textit{stationary legislative lobbying equilibrium} satisfies four conditions.\footnote{See Appendix A for a formal definition.} First, $M$ passes a proposal if and only if she weakly prefers to do so rather than reject and continue bargaining. Second, if left to their own devices, each politician proposes policy satisfying $M$ and cannot profitably deviate to any other proposal. Third, politician $\ell$ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal. Fourth, $g$’s policy offer will pass and $g$ cannot profitably deviate to any other offer. By stationarity: (i) $M$’s voting decision depends only on the current proposal; (ii) politicians other than $\ell$ propose independently of preceding play; (iii) $\ell$ accepts or rejects $g$’s offers based only on the current terms, and $\ell$’s proposals in lieu of acceptance are independent of the preceding history; and (iv) $g$’s offers are independent of previous play. Although players use strategies that are relatively straightforward behavioral rules, no player can profitably deviate to \textit{any} other strategy.

Before proceeding, I note three conditions on strategies that are without loss of generality and streamline the analysis: (i) $M$ passes proposals when indifferent; (ii) $\ell$ accepts $g$’s offer when indifferent; and (iii) players use \textit{no-delay} proposal strategies, i.e., each politician proposes passable policy and $g$ offers passable policy. In Appendix B, I define \textit{stationary mixed strategy legislative lobbying equilibrium} and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium in which politicians (i) vote in favor of proposals when indifferent and (ii) accept lobby offers when indifferent.\footnote{Standard arguments (Banks and Duggan, 2006b) imply that proposal strategies must be no delay. Although related, the no-delay property for interest groups is original to this paper. Essentially, lobbying for delay is always too expensive to be worthwhile in equilibrium. Appendix B provides the technical details.}

\section*{Model Commentary}

A core premise in theories of access is that it weakly increases opportunities to exert influence and, when such opportunities do arise, weakly increases the effectiveness of influence. The baseline model captures the first part of that premise, as access determines the probability
that the group can lobby. The model can easily be extended to capture the second part as well, e.g., by allowing access to increase \( \ell \)'s value of transfers from \( g \). Enriching the model in this way does not add substantial insight to the main results. Regardless of how these conceptions of access are combined, the direct consequence of access in the model is that it shifts the target’s expected proposal towards the group in equilibrium and thus the indirect effects are qualitatively the same.

In the baseline model, access is targeted at one politician and remains constant throughout bargaining. These assumptions streamline the analysis and can be relaxed somewhat. In the appendix, I prove the main results for an extended model in which interest groups can have access to multiple politicians. Stationary access is an analytically convenient way to capture the prevalent view that access is essentially fixed once active policymaking begins (Powell, 2014; Powell and Grimmer, 2016).

The key aspect of lobbying that the model captures is the ability to influence proposals. Groups often lobby in committee to shape the language of bills (Schlozman and Tierney, 1986; Kang and You, 2015) and the policy-for-transfer lobbying technology used here provides a tractable reduced form representation of various ways that such influence could occur (Powell, 2014). Additionally, it allows us to analyze the interaction between access expenditures and lobbying effort. This feature is important when studying whether groups want access, as I show that lobbying expenditures can increase with access and therefore reduce its appeal.

The exact interpretation of lobbying is not central in this paper, but the model accommodates two prominent forms. First, there is an *exchange* interpretation, which aligns with widespread fears that lobbying is a quid-pro-quo, but can also be interpreted more broadly (Großer et al., 2013; Powell, 2014; Baron, 2019).\(^{16}\) as drafting language (Schattschneider, 1960) or writing a model bill (Kroeger, 2020) to save politicians time or in exchange for various forms of assistance, such as future employment opportunities (Diermeier, Keane

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\(^{16}\)See Grossman and Helpman (2002) for an extensive overview and discussion.
and Merlo, 2005) or targeted charitable or donations (Bertrand, Bombardini, Fisman and Trebbi, 2020). Second, there is also a legislative subsidy, in which the group’s lobbying helps a likeminded politician influence her peers on a particular subcommittee whenever it are tasked with writing legislation (Hall and Deardorff, 2006). To streamline discussion, I use the exchange interpretation throughout the analysis.

Finally, I do not model lobbying that directly influences how politicians vote on proposals. That enables me to focus on the effects of access facilitating lobbying that influences policy content. In practice, influencing policy content is particularly appealing because it is less visible and more intimate, while consequential vote buying is relatively difficult because, legality aside, it may require groups to coordinate with several politicians, which is like “herding cats” (Milyo et al., 2000). After the analysis, I discuss how the results in this paper complement the large literature that focuses on vote buying.

**Analysis of Equilibrium Legislating and Lobbying**

In equilibrium, there is a feedback between proposals and legislative voting, as in, e.g., Banks and Duggan (2006a). To illustrate, consider a legislator recognized to propose. To pass policy, they must anticipate which policies $M$ will accept, which depends on $M$’s expectations about future policymaking, which in equilibrium are consistent with proposal strategies. A key step in the analysis is to show how access influences these expectations and thus the acceptance set, thereby affecting proposals that are constrained by the limits of what $M$ will pass.

More precisely, $M$ will pass a proposal if and only if it exceeds her reservation value of keeping $q$ for another period and continuing active bargaining. Formally, $M$’s reservation value is $(1 − δ)u_M(q) + δV^*_M$, where $V^*_M$ denotes $M$’s equilibrium continuation value immediately after rejecting a proposal.\(^{17}\) By stationarity, $V^*_M$ is the same each period, so $M$’s reservation value is constant and thus her voting behavior is the same each pe-

\(^{17}\)Appendix A contains explicit expressions of continuation values.
period. Specifically, the acceptance set is \( A^* = [-\bar{x}^*, \bar{x}^*] \), where \( \bar{x}^* \) is the positive solution to \( u_M(x) = (1 - \delta)u_M(q) + \delta V^*_M \).

Anticipating what \( M \) will pass, each politician proposes their favorite policy in \( A^* \) whenever recognized, which is also analogous to Banks and Duggan (2006a). Clearly, \( M \) will simply propose her ideal point, 0. The partisans are constrained by \( A^* \) in equilibrium, so \( L \) proposes \(-\bar{x}\) and \( R \) proposes \( \bar{x} \).\(^{18}\) Finally, \( \ell \) proposes \( z^* \), the policy in \( A^* \) closest to \( \hat{x}_\ell \), if either (i) \( g \) cannot lobby or (ii) \( \ell \) rejects \( g \)'s offer.

Finally, I characterize the interest group’s equilibrium behavior. Intuitively, \( g \) wants to shift \( \ell \)'s proposal as far towards \( \hat{x}_g \) as is worth paying for. To shift \( \ell \)'s proposal at all, however, \( g \) must compensate her for not instead rejecting and proposing \( z^* \). In equilibrium, \( g \) will always make an offer that \( \ell \) accepts, as it can always do weakly better than the trivial acceptable offer of \( z^* \) without payment. Additionally, since \( g \) knows \( \ell \)'s payoff from proposing \( z^* \), it will compensate her exactly and extract all of the surplus. Thus, from \( g \)'s perspective, there is a cost of \( u_\ell(z^*) - u_\ell(y) \) associated with each policy \( y \) that \( M \) will pass.

By stationarity, the set of policies \( M \) will pass, \( A^* \), does not depend on today’s proposal.

In principle, \( g \) could potentially benefit from lobbying for policy outside of \( A^* \) if tomorrow’s proposer is likely to be an ideological ally who will pass favorable policy for free. Yet, \( \ell \) shares those expectations about future play and therefore must be compensated accordingly in order to propose any policy outside \( A^* \). In equilibrium, the cost of buying delay is never worthwhile for \( g \) in equilibrium and therefore it never lobbies for proposals that will be rejected.\(^{19}\) Consequently, \( g \) proposes the policy in \( A^* \) that provides the best policy payoff given the associated cost: that is, \((y^*, m^*)\) consists of the policy \( y^* = \arg \max_{y \in A^*} u_g(y) + u_\ell(y) - u_\ell(z^*) \) and transfer \( m^* = u_\ell(z^*) - u_\ell(y^*) \).\(^{20}\) Thus, \( g \) successfully lobbies \( \ell \) to propose the policy in \( A^* \) that maximizes their cumulative policy utility, which is \( \bar{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2} \) since they both have quadratic policy utility.

\(^{18}\)This property follows from \( \bar{x}^* < |q| < \min\{\hat{x}_L, \hat{x}_R\} \).

\(^{19}\)See Appendix B for technical details.

\(^{20}\)Uniqueness of \( y^* \) follows because \( u_g + u_\ell \) is strictly concave and \( A^* \) is a nonempty closed interval.
Proposition 1 shows that a stationary legislative lobbying equilibrium exists and collects the preceding observations to characterize a variety of equilibrium behavior: which policies will pass and which will be rejected; which policies various politicians will propose; and which policies the interest group will lobby for and how much it will pay.

**Proposition 1.** A stationary legislative lobbying equilibrium exists and every such equilibrium has the same outcome distribution. In equilibrium,

(i) the acceptance set is $A^* = [-\bar{x}^*, \bar{x}^*]$, where $0 < \bar{x}^* < |q|$;

(ii) $M$ proposes 0, $R$ proposes $\bar{x}^*$, and $L$ proposes $-\bar{x}^*$;

(iii) if $\ell$ is not lobbied, she proposes the policy $z^* \in A^*$ closest to $\hat{x}_\ell$;

(iv) if $g$ can lobby, then it successfully lobbies $\ell$ to propose the policy $y^* \in A^*$ closest to $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ using the payment $m^* = u_\ell(z^*) - u_\ell(y^*)$.

The characterization of equilibrium lobbying implies that the model can be reinterpreted as a one-dimensional bargaining environment in which $\ell$ has recognition probability $(1 - \alpha)\rho_\ell$ and there is an additional legislator at $\hat{y}$ with recognition probability $\alpha \rho_\ell$. After modifying the legislature to include this additional proposer representing the effect of $g$’s lobbying, legislators propose acceptable bills closest to their ideal point. Applying insights from Cho and Duggan (2003) to this fictitious enlarged legislature implies that this class of equilibria has a unique distribution of equilibrium policies. Henceforth, I drop qualifiers and say *equilibrium*. Figure 2 illustrates the equilibrium acceptance set and proposals for a hypothetical legislature.
Figure 2 illustrates equilibrium proposals for a hypothetical legislature. Arrows point from legislator ideal points to proposals. The bold interval is the acceptance set, $A^*$. If legislator $\ell$ is recognized, then with probability $\alpha$ she is lobbied to propose $y^*$, the policy in $A^*$ closest to $\tilde{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$, and otherwise she proposes $z^*$, the policy in $A^*$ closest to $\hat{x}_\ell$.

Proposition 1 implies that $M$’s equilibrium continuation value is simply the weighted sum of her policy utility from equilibrium proposals, weighted by their probabilities:

$$V_M^* = \rho_M u_M(0) + \rho_L u_M(-\bar{x}^*) + \rho_R u_M(\bar{x}^*) + \rho_\ell \left( \alpha u_M(y^*) + (1 - \alpha) u_M(z^*) \right).$$

(1)

Substituting (1) into $M$’s indifference condition that defines the boundaries of the acceptance set yields Corollary 1.1, which sharpens our characterization of $\bar{x}^*$.

**Corollary 1.1.** In equilibrium, the boundaries of $A^* = [-\bar{x}^*, \bar{x}^*]$ are characterized by

$$\bar{x}^* = \left( -\frac{(1 - \delta) u_M(q) + \delta \rho_\ell \left( \alpha u_M(y^*) + (1 - \alpha) u_M(z^*) \right)}{1 - \delta (\rho_L + \rho_R)} \right)^{\frac{1}{2}}.$$  

(2)

Corollary 1.1 implies that the acceptance set expands if: the status quo ($q$) shifts away from $M$, patience ($\delta$) decreases, or total partisan recognition probability ($\rho_L + \rho_R$) increases. These effects are familiar from related models without lobbying (e.g., Banks and Duggan, 2006b). Specific to this paper, (2) also reveals that increasing access ($\alpha$) expands $A^*$ if $y^*$ is farther than $z^*$ from $M$, but shrinks $A^*$ if $y^*$ is closer than $z^*$ to $M$. Intuitively, greater access causes $M$ to put more weight on the possibility that $g$ might lobby $\ell$ in the future if
today’s proposal fails. If lobbying would make ℓ’s proposal worse for M, then the acceptance set expands because she is less inclined to reject today, and vice versa. Thus, the effect of access on \( A^* \) depends critically on how extreme the interest group is relative to the targeted politician.

Although the effect of access on \( A^* \) is original to this paper, it falls under the umbrella of a more general relationship that is familiar from related work without lobbying: the acceptance set expands as the distribution of equilibrium proposals shifts away from \( M \).

To be more precise about this general relationship, I next define a notion of changes in legislative extremism as a function of \( \alpha \) and \( \rho \). Two distinct special cases in which legislative extremism increases are (i) transferring recognition probability from \( M \) to other politicians, or (ii) increasing \( \alpha \) if \( \hat{y} \) is farther than \( \hat{x}_\ell \) from \( M \).

**Definition 1.** For any pair \((\alpha, \rho)\), let \( \Lambda(\alpha, \rho) \) be a lottery that puts probability \( \alpha \rho_\ell \) on \( |\hat{y}| \), probability \( \rho_\ell (1 - \alpha) \) on \( |\hat{x}_\ell| \), and probability \( \rho_j \) on \( |\hat{x}_j| \) for each legislator \( j \neq \ell \). Say that legislative extremism increases if changing \((\rho, \alpha)\) to \((\rho', \alpha')\) is such that: (i) for all \( x \in X \), the lottery \( \Lambda(\rho', \alpha') \) puts weakly greater probability on \( x' \) such that \( |x'| \geq |x| \) and (ii) for some \( x \in X \), the lottery \( \Lambda(\rho', \alpha') \) puts strictly greater probability on \( x' \) such that \( |x'| \geq |x| \).

Equivalently, Definition 1 uses first order stochastic dominance, a standard partial order for probability distributions, to compare distributions of unconstrained ideal proposals: legislative extremism increases if \( \Lambda(\rho', \alpha') \) first order stochastically dominates \( \Lambda(\rho, \alpha) \).

Taking stock, and generalizing our earlier observation, \( A^* \) expands as either: legislative extremism increases, \( \delta \) decreases, or \( q \) shifts away from \( M \). By changing the acceptance set, any of these changes will also shift proposals on the boundaries of \( A^* \): they always affect what \( L \) and \( R \) will propose and, moreover, they can also shift \( y^* \) or \( z^* \) if either is constrained by \( A^* \). In the second case, they can also affect \( g \)'s equilibrium lobby transfer, \( m^* = u_\ell(z^*) - u_\ell(y^*) \). Since \( y^* \) is either \( \hat{y} \) or a boundary of \( A^* \), and analogously for \( z^* \), variation in \( A^* \) is the only channel through which \( m^* \) can vary. Next, Lemma 1 shows that
\( m^* \) weakly increases as \( A^* \) expands.\(^{21}\)

**Lemma 1.** The interest groups equilibrium payment, \( m^* \), weakly increases with \( \bar{x}^* \).

Expanding \( A^* \) can increase \( m^* \) in two distinct ways, depending on whether \( y^* \) or \( z^* \) is constrained by \( A^* \). If \( y^* \) is constrained, then \( g \) gets more slack to shift \( \ell \)'s proposal farther and is willing to pay more to do so. If \( z^* \) is constrained, then \( \ell \) gets more slack to pass more favorable policy if she rejects \( g \)'s offer, and is therefore more inclined to reject, but \( g \) is willing to pay the additional amount.

Lemma 1 implies that \( m^* \) increase as legislative extremism increases, \( q \) shifts away from \( M \), or \( \delta \) decreases. Next, Proposition 2 expands on that implication by collecting the preceding observations to characterize how equilibrium voting, proposals, and expenditures each depend on legislative extremism (\( \alpha, \rho \)), the status quo (\( q \)), and patience (\( \delta \)).

**Proposition 2.** If either (i) legislative extremism increases, (ii) the status quo policy becomes more extreme, or (iii) legislator patience decreases, then:

1. the acceptance set, \( A^* \), expands;

2. proposals constrained by \( A^* \) become more extreme; and

3. the lobby payment, \( m^* \), weakly increases.

Since access (\( \alpha \)) affects legislative extremism, Proposition 2 reveals that it can have a variety of effects in equilibrium. Broadly, the direct effect of \( \alpha \) on \( g \)'s lobbying chances affects \( \ell \)'s expected proposal (target proposal effect), which can endogenously affect what will pass (voting effect), what will be proposed (extreme proposal effect), and how many resources will be devoted to lobbying (lobbying expenditure effect). Corollary 2.1 collects the consequences of increasing access.

**Corollary 2.1** (Effects of Access). If \( |\hat{y}| > |\hat{x}_\ell| \), then as \( \alpha \) increases:

\(^{21}\)See Lemma A.1 in the Appendix A.
(i) **target proposal effect** – $\ell$ is more likely to propose $y^*$ and less likely to propose $z^*$;

(ii) **voting effect** – the acceptance set, $A^*$, expands;

(iii) **extreme proposal effect** – proposals constrained by $A^*$ become more extreme; and

(iv) **lobbying expenditure effect** – the lobby payment, $m^*$, weakly increases.

If $|\hat{y}| < |\hat{x}_\ell|$, then effect (i) is analogous but effects (ii)–(iv) are reversed.

The nature of the indirect effects depends on how extreme $g$ is relative to $\ell$, as that determines whether legislative extremism will increase or decrease in access. For example, if $0 < \hat{x}_\ell < \hat{x}_g$, then increasing $\alpha$ will increase legislative extremism, so the acceptance set will expand, constrained proposals will shift farther outward, and lobbying expenditures will weakly increase.

The extreme proposal effect is not limited to the partisans, $L$ and $R$, although it indeed always shifts their proposals. It can also alter either the lobby proposal, $y^*$, or $\ell$’s non-lobby proposal, $z^*$, but it cannot alter both $y^*$ and $z^*$ simultaneously. To do so, both $y^*$ and $z^*$ would have to be constrained, which requires $M$ to be indifferent between those proposals. But then the target proposal effect does not affect $M$’s reservation value, so $A^*$ is unchanged and there is no voting effect, which implies no extreme proposal effect.

**Whom to access?**

Thus far, I have shown how access can affect several behaviors by various actors. Furthermore, I highlighted how the direction of these effects depends on relative extremism of group and target. Since groups appear to have various tools to increase their access, I now study who they want to target.

To isolate policy considerations, I allow $g$ to freely choose access. The key insights can be conveyed by studying a one-time choice of access prior to bargaining. Substantively, this

\[22\]

22The core insights are unchanged by including a standard convex cost function for access.
captures the possibility that interest groups “may make contributions in anticipation that they may need access to a legislator during a legislative term, rather than when the necessity to purchase influence arises” (Powell and Grimmer, 2016, p. 978). Specifically, I analyze how $\alpha$ affects $g$’s equilibrium value:

$$
\rho_M u_g(0) + \rho_R u_g(\bar{x}_\alpha^*) + \rho_L u_g(-\bar{x}_\alpha^*) + \rho_\ell \left[ \alpha \left( u_g(y_\alpha^*) + u_\ell(y_\alpha^*) \right) - u_\ell(z_\alpha^*) \right] + (1 - \alpha) u_g(z_\alpha^*) ,
$$

(3)

where subscripts indicate which proposals can vary with $\alpha$. Although (3) is similar to (1), it sums over $g$’s policy utility and therefore accounts for $g$’s lobbying expenditure when it lobbies $\ell$.

Inspecting (3) reveals that access can affect $g$’s welfare in two ways. First, it can change $g$’s expected lobbying gain when $\ell$ is recognized, $\rho_\ell \left[ \alpha \left( u_g(y_\alpha^*) + u_\ell(y_\alpha^*) \right) - u_\ell(z_\alpha^*) \right]$. It does so through the target proposal effect, which alters $u_g(y_\alpha^*) - u_\ell(z_\alpha^*)$, and the lobbying expenditure effect, which alters $m^* = u_\ell(z_\alpha^*) - u_\ell(y_\alpha^*)$. If $g$ is more centrist than $\ell$, then increasing $\alpha$ enables $g$ to pay weakly less for the same policy. If $g$ is more extreme than $\ell$, then increasing $\alpha$ allows $g$ to pass weakly more extreme policy, which it will pay more to do if the policy gain is worthwhile. Thus, this effect is always good for $g$.

Second, it can change $g$’s expected policy payoff when a partisan is recognized, $\rho_R u_g(\bar{x}_\alpha^*) + \rho_L u_g(-\bar{x}_\alpha^*)$, entirely through the extreme proposal effect. If both extreme proposals shift towards $\hat{x}_g$, then $g$ benefits. If both shift away, then $g$ is worse off. Finally, if one shifts closer while the other shifts away, then whether $g$ benefits will depend on the relative proposal power of $L$ and $R$, i.e., the magnitudes of $\rho_L$ and $\rho_R$. Thus, this effect can be good or bad for $g$.

Both of the preceding effects are zero if and only if lobbying does not change $\ell$’s proposal, i.e., $y_\alpha^* = z_\alpha^*$, which requires that either (i) $\hat{x}_\ell = \hat{x}_g$, or (ii) $\hat{x}_\ell$ and $\hat{y}$ are outside the acceptance
set in the same direction. In (ii), the acceptance set is $A^* = [-\overline{x}, \overline{x}]$, where

$$\overline{x} = \left( \frac{- (1 - \delta) u_M(q)}{1 - \delta (\rho_L + \rho_R + \rho_L)} \right)^{\frac{1}{\delta}}. \quad (4)$$

Although $\overline{x}$ resembles (2), it is defined in terms of primitives and, crucially, does not depend on $\hat{x}_\ell, \hat{x}_g$, or $\alpha$. Since $\overline{x}$ does not vary with $\alpha$, (ii) holds if and only if $\max\{\hat{x}_\ell, \hat{y}\} \leq -\overline{x}$ or $\overline{x} \leq \min\{\hat{x}_\ell, \hat{y}\}$. Given $\hat{x}_g$, we have $y^* = z^* = \overline{x}$ if $\hat{x}_\ell$ leans sufficiently right and $y^* = z^* = -\overline{x}$ if $\hat{x}_\ell$ leans sufficiently left. Thus, for any group, both effects are zero if $\ell$ leans far enough in either direction.

Proposition 3 shows that $g$ will not pay for access to very extreme legislators.

**Proposition 3.** The interest group strictly prefers nonzero access only if $\hat{x}_\ell \in (\chi, \chi)$, where $(-\overline{x}, \overline{x}) \subset (\chi, \chi)$.

Essentially, lobbying has no effect on very extreme legislators, so they are not worth targeting. If $\ell$ is extreme enough, then lobbying will not change her proposal, so access is inconsequential and not worth paying for. An implication of Proposition 3 is that, regardless of $\hat{x}_g$, any legislator satisfying $\hat{x}_\ell \in (-\overline{x}, \overline{x})$ may be worth targeting, as sufficiently low $\alpha$ guarantees they are unconstrained and therefore that lobbying would change their proposal.

If instead the effects of access are nonzero, then they may work together in $g$’s favor. For example, if $0 < \hat{x}_g < \hat{x}_\ell < x^*_\alpha$, then increasing access shifts extreme proposals inward towards $g$ from both sides, so $g$ clearly wants access. More broadly, this holds whenever (i) $\hat{x}_g \in A^*_\alpha$ and (ii) $y^*_\alpha$ is more centrist than $z^*_\alpha$, so that the acceptance set shrinks in $\alpha$. This implies that $g$ also benefits from access if $\ell$ is in an intermediate range on the opposite side of $M$.

Alternatively, the effects may oppose each other. For example, suppose $0 < \hat{x}_\ell < \hat{x}_g < \overline{x}^*_\alpha$. Then, the extreme proposal effect discourages access because they both shift outward away from $g$ in both directions.

In the two preceding examples, the extreme proposal effect is unambiguous because $\hat{x}_g$
is strictly inside the acceptance set, so varying \(\alpha\) either (i) shifts both partisan proposals away from \(g\), or (ii) shifts them closer. In contrast, if \(\hat{x}_g\) is not strictly inside the acceptance set, then the extreme proposal effect is ambiguous because varying \(\alpha\) makes one partisan’s proposal more favorable for \(g\), but makes the other partisan’s proposal less favorable, so the effect always depends on the relative recognition probability of \(L\) and \(R\). To distinguish these two cases, Lemma 2 characterizes when the extreme proposal effect is unambiguous by showing that moderate groups can be strictly inside the acceptance set, but extremist groups cannot. Without loss of generality, I analyze \(\hat{x}_g \geq 0\).

Lemma 2. If \(\hat{x}_g \in (-\overline{x}, \overline{x})\), then there exists \(\bar{x} \in [0, \hat{x}_g)\) such that \(\hat{x}_\ell \notin (-\overline{x}, \bar{x})\) implies \(\hat{x}_g \in \text{int}A_0^* = (-\overline{x}_0, \overline{x}_0)\). Otherwise, then \(\hat{x}_g \notin \text{int}A_\alpha^*\) for all \(\hat{x}_\ell\) and all \(\alpha\).

In light of Lemma 2, Definition 2 provides terminology that distinguishes whether \(g\) can be inside \(A^*\).

Definition 2. A player \(j\) is moderate if \(\hat{x}_j \in (-\overline{x}, \overline{x})\). Otherwise, \(j\) is extremist.

A key implication of Lemma 2 for moderate groups is that increasing access from zero has an unambiguous effect on partisan proposals if \(\ell\) is not too centrist. Thus, we can already make two broad observations. First, \(g\) wants access to more extreme legislators on its side of the spectrum because every effect is beneficial. In contrast, access to relatively centrist legislators may have harmful indirect effects that counteract the direct benefits. These observations highlight a key takeaway: \(g\)’s desire for access depends on (i) its own extremism and (ii) its extremism relative to \(\ell\).

Who do moderate groups want to access?

Proposition 4 shows that moderate groups want access to a range of more extreme legislators and an intermediate range of legislators opposite \(M\), but they will forgo access to legislators in a relatively more centrist range.
Proposition 4. If \( \hat{x}_g \in (0x) \), then there are cutpoints satisfying \(-\hat{x}_g < x' < x'' < \hat{x}_g\) such that \(g\) strictly prefers no access if \(\hat{x}_\ell \in (x'', \hat{x}_g)\) but wants access if \(\hat{x}_\ell \in (\chi, x') \cup (\hat{x}_g, \chi)\).

First, the interest group wants access to \(\ell\) if (i) she is aligned with the group, and (ii) she is more extreme than the group, but not too extreme. In this case, \(g\) benefits from every effect of increasing access. As \(g\) is more centrist, increasing \(\alpha\) improves \(M\)'s reservation value and shrinks the acceptance set. Since, \(g\) is always in the acceptance set regardless of \(\alpha\), it always benefits from the partisan proposal effect. Additionally, \(g\) pays less for the same policy when it lobbies.

Next, \(g\) also wants access if \(\ell\) is in an intermediate interval on the opposite side of the spectrum. If \(\hat{x}_\ell \in (\chi, -\bar{x}]\), then \(g\) is inside the acceptance set if \(\alpha = 0\) and that acceptance set will shrink as \(\alpha\) increases from \(\alpha = 0\). Thus, every effect of increasing access from zero works in \(g\)'s favor. And even if \(\ell\) is slightly more centrist, i.e., \(\hat{x}_\ell \in (-\bar{x}, x')\), then \(g\)'s expected gain from the the target proposal effect outweighs any expected loss from the other effects.

Next, \(g\) forgoes access if \(\ell\) is aligned with it and slightly more centrist, i.e., \(\hat{x}_\ell \in (x'', \hat{x}_g)\). Notably, existence of this range does not depend on \(\rho\), but its size will. In this case, increasing \(\alpha\) expands the acceptance set and shifts partisan proposals outward, as depicted in Figure 3. Except for possibly a range of \(\ell\) who are centrist enough, \(g\) will be strictly inside the acceptance set for \(\alpha = 0\) and therefore dislike the extreme proposal effect. Crucially, this negative extreme proposal effect dominates the other effects of access as \(\ell\) approaches \(g\) from the center. Intuitively, lobbying will not shift \(\ell\)'s proposal very much and \(g\)'s payoff is not very sensitive to those changes, so the direct benefit is small. Meanwhile, \(M\) is more sensitive to those changes, and the acceptance set expands to give extremists enough additional slack to make the negative extreme proposal effect relatively larger.  

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23The indirect effects of access on voting and proposals in this paper have connections with spatial models of dynamic bargaining (Baron, 1996; Buisseret and Bernhardt, 2017; Zápal, 2020). There, the policy in place at the end of today becomes the status quo tomorrow, so proposers weigh how today's proposal can affect what can pass tomorrow when someone else might have proposal rights. In equilibrium, legislators pass more centrist policies today in order to make centrist veto players less inclined to pass policy in the future, thus constraining the scale of policy changes by potential future proposers on the other end of the spectrum. In this paper, policymaking ends once a proposal passes, so a group considering access weighs (i) how it will
Figure 3: Forgoing access to more centrist legislators

(a)

\[ \hat{x}_L \rightarrow 0 \rightarrow \hat{x}_\ell \rightarrow \hat{x}_g \rightarrow \hat{x}_R \]

(b)

\[ \hat{x}_L \rightarrow \alpha \rightarrow 1 - \alpha \rightarrow \hat{x}_\ell \rightarrow \hat{x}_g \rightarrow \hat{x}_R \]

Figure 3 illustrates why a moderate group, \( g \), forgoes access (\( \alpha = 0 \)) if \( \hat{x}_\ell \in (x'', \hat{x}_g) \). Part (a) displays equilibrium behavior for \( \alpha = 0 \). Part (b) illustrates \( \alpha > 0 \). In each, the bold interval is the acceptance set. Increasing \( \alpha \) makes lobbying more likely, which worsens \( M \)'s expectations, and expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If \( \hat{x}_g \) and \( \hat{x}_\ell \) are close, then the loss from more extreme partisan proposals dominates and \( g \) prefers \( \alpha = 0 \).

Finally, \( g \)'s preference for access is unclear in general if \( \ell \) is in a centrist range, i.e., \( \hat{x}_\ell \in (x', x'') \). The effects of access conflict, as in the previous case, but now the overall effect depends on partisan recognition probability, specifically either their total or relative magnitude. For a stark example, \( g \) may not be in \( A_0^* \). Then, the extreme proposal effect of increasing access from zero depends on the relative magnitude of \( \rho_L \) and \( \rho_R \) because one partisan proposal becomes less favorable for \( g \) and the other more favorable.

Who do extreme groups want to access?

In general, extreme groups have ambiguous preferences for access because they are always outside the acceptance set, so the direction of the extreme proposal effect depends on the relative magnitude of \( \rho_L \) and \( \rho_R \). Proposition 5 focuses on cases in which one partisan is relatively unlikely to propose, in order to shed light on who extreme groups want to access.

**Proposition 5.** Suppose \( \hat{x}_g > \bar{x} \).

affect the target’s proposal if she is recognized, and (ii) how it will affect what happens if the target is not recognized. Since access can indirectly influence which policies can pass in equilibrium, incentives to increase or forgo access are affected by a similar desire to constrain potentially extreme proposers.
(i) If $\rho_L$ is small enough, then there exists $x' < 0$ such that $g$ wants access if $\hat{x}_\ell \in (x', \overline{x})$.

(ii) If $\rho_R$ is small enough, then there exists $x'' \geq -\overline{x}$ such that $g$ wants access if $\hat{x}_\ell \in (\underline{x}, x'')$.

For extreme groups, a sufficiently weak partisan on either side of the spectrum clarifies the overall effect of access. First, an extreme group wants access to a range of moderate legislators if its opposing extremists are unlikely to propose. Unless $\ell$ leans far enough in the opposite direction, access worsens $M$’s expectations about future policy in this case, so the acceptance set expands. This expansion benefits the extreme group because the opposing partisan is unlikely to propose. Thus, it wants access if $\ell$ is an aligned moderate, or a sufficiently centrist opponent.

Second, an extreme group aligned with weak extremists wants access to opponents who are not too extreme, and potentially also to aligned moderates who are centrist enough. In this case, the group always wants access that facilitates more centrist proposals because such connections have a positive extreme proposal effect. Additionally, they want access that facilitates more extreme proposals, as long as the favorable target proposal effect can outweigh losses from the partisan proposal effect and lobby expenditure effect.

Comparing the value of access to different politicians

Thus far, we have studied whether $g$ wants access to $\ell$. In this section, I analyze how ideology and proposal power affect $g$’s willingness to pay (WTP) for access, i.e., the marginal effect of $\alpha$ on $g$’s equilibrium value.

First, I consider how $g$’s WTP for access depends on $\ell$’s recognition probability, $\rho_\ell$. Two of the most robust empirical regularities about outside influence are that (i) legislators on important committees, especially committee chairmen, attract more contributions (Fourtinaies, 2017; Berry and Fowler, 2018), and (ii) lobbyists connected to those legislators command a premium (Blanes i Vidal et al., 2012). Interpreting these consistent empirical findings,
scholars widely take as a stylized fact that groups prioritize access to legislators who have more proposal power.

My analysis highlights that increasing a legislator’s proposal power can have competing effects on her value as a target for access. Although increasing \( \rho_\ell \) increases \( g \)’s expected lobbying benefit from access, it also amplifies the (possibly negative) indirect effect that access can have on partisan proposals. Thus, the overall effect is not always immediately clear. Yet, Proposition 6 provides a stark result: \( g \)’s WTP for access always weakly increases with \( \ell \)’s recognition probability.

**Proposition 6.** The interest group’s willingness to pay for access to legislator \( \ell \) weakly increases with \( \ell \)’s recognition probability, \( \rho_\ell \).

Proposition 6 reflects the robustness of the empirical finding that groups prioritize politicians with greater proposal power. The result does not depend on \( \ell \) or \( g \)’s policy preferences, partisan proposal power, patience, or the status quo. Depending on these other factors, increasing \( \rho_\ell \) can have competing effects, but the overall effect is always proportional to \( \rho_\ell \) whenever \( g \)’s WTP is strictly positive. Thus, \( g \)’s WTP either increases in \( \rho_\ell \) or remains at zero.

Next, I discuss how \( \hat{x}_\ell \) affects \( g \)’s WTP for access as \( \alpha \) increases from 0, which I refer to as \( g \)’s willingness to acquire (WTA) access. First, the earlier results characterizing whether \( g \) wants access (Propositions 3-5) imply that \( g \)’s WTA is zero if \( \ell \) is too extreme in either direction, but can also be zero if \( \ell \) is more centrist. Additionally, studying \( g \)’s WTA can shed light on how the group’s value of access varies with the ideology of its target. For example, a moderate group’s WTA weakly increases as \( \hat{x}_\ell \) shifts away from \( \hat{x}_g \) in either direction, but the rate of increase is asymmetric: if two legislators are equidistant from \( g \), its WTA is higher for the more extreme legislator. Like WTP, \( g \)’s WTA depends on the marginal expected lobbying gains and the marginal partisan proposal effect. As \( \hat{x}_\ell \) shifts outward from \( \hat{x}_g \), both effects increase \( g \)’s WTA, relative to an equidistant legislator skewed towards \( M \): (i) \( g \)’s lobbying profit increases, and (ii) the partisan proposal effect of access grows more
positive, since increasing $\alpha$ counteracts the negative extreme partisan proposal effect driven by $\ell$ getting more extreme.

**Discussion**

A key takeaway of the analysis is that access does not have to materialize in order to influence policy. Instead, access alone can be sufficient to influence policy due to equilibrium effects. Even by merely increasing the chances of lobbying, access can influence votes, proposals, and lobbying expenditures. This equilibrium phenomenon sheds new light on the widespread view that access is necessary for influence, but does not influence behavior on its own (e.g., Wright, 1989). First, it suggests that insider descriptions of what access-seeking behavior (e.g., contributions) ‘buys’ can be misleading, even when they are sincere. Additionally, it suggests caution for the widespread view that access is necessary for influence but does not influence behavior on its own. Observing no lobbying does not imply that a connected interest group missed out on benefiting from access.

Although some scholars have informally noted the possibility that access alone can influence behavior by non-targeted politicians (favorably or unfavorably), I formally derive a channel that flows entirely through legislative considerations. By doing so, the results reveal potential obstacles for estimating effects of access. Even if access could be randomized, equilibrium effects will prevent expectations about future proposals from being held constant by such randomization, which would be necessary to isolate the direct effect of access. Furthermore, spillover effects that arise in equilibrium would violate SUTVA. Thus, attempts to recover causal effects of access must especially clear about their estimand and how they can convincingly estimate it with their data.

The analysis also offers implications for how lobbying expenditures vary with access. First, I highlight a novel way that access-seeking expenditures can affect lobbying expendi-

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24For example, (Kalla and Broockman, 2015) suggest that other politicians might act differently with the hope of attracting donations from the group as well.
tures. By changing what some politicians would propose on their own, access can affect how much lobbying effort any group must exert to alter those proposals. This channel shows how access-seeking expenditures can affect lobbying effort even without making lobbying more efficient. An empirical implication is that observing a change in lobbying expenditures does not imply a change in lobbying ‘effectiveness.’ Second, I also highlight how access-seeking expenditures and lobbying expenditures can be complements or substitutes in equilibrium, depending on whether the group is more or less extreme than the politician it targets. An empirical implication is that observing no change in average lobbying expenditures (across group-legislator pairs) need not imply that access has no effect. Instead, some interest groups may have paid more and others less.

The analysis also offers implications for Tullock’s puzzle, the empirical regularity that surprisingly few interest groups contribute and they contribute surprisingly low amounts. (Tullock, 1972; Ansolabehere et al., 2003). Given evidence that groups can increase access in various ways (e.g., contributions, revolving door hiring) and the natural expectation that groups want more influence, why do they not spend more? Although there are other explanations based on costs or competition (e.g., Chamon and Kaplan, 2013), the results about interest group preferences over access provide a new logic that instead emphasizes legislative considerations. A key insight of this paper is that increasing your potential for influence can affect what happens if you do not get an opportunity to do so. An unfavorable effect discourages access, while a favorable effect increases the bang for the buck. Either way, these effects suggest that groups may spend less than expected on access-seeking behaviors and that they may not spend anything to target more moderate politicians.

Finally, although I aim to isolate considerations related to lobbying over policy details in committee, this paper also has implications for our theoretical understanding of direct vote buying. I highlight how access to policy formulation can endogenously strengthen or weaken incentives to buy key votes. If access expands what can pass, then groups are more willing to pay for no votes by decisive voters on proposals from opposing extremists, and vice versa.
The key difference is that models of vote buying typically consider exogenous or take-it-or-leave-it proposals (e.g., Snyder Jr., 1991; Dekel et al., 2009), whereas here politicians make strategic proposals and bargaining continues after failed proposals.

**Conclusion**

I analyze a model of legislative policymaking in which access provides interest groups with opportunities to lobby policy proposals. The model is intentionally constructed so that the effects of access are restricted to flow entirely through one channel in order to trace the effects of a standard notion of access. The equilibrium analysis here sheds new light, revealing that this notion of access has broader effects than typically appreciated, due to additional endogenous effects on voting, proposals, and lobbying. Broadly, the analysis unpacks several effects of access in legislatures. By changing each legislator’s expectations about policymaking, access endogenously alters which policies can pass. In turn, important indirect effects emerge: the potential for future lobbying can influence today’s proposal and lobbying expenditures.

Furthermore, the results shed light on how much access interest groups want with particular legislators who may be involved in writing policy. Because access to one legislator can spill over to affect behavior by other legislators, moderate groups forgo access to a range of more centrist legislators. Policy considerations drive this behavior, as such connections increase policy extremism enough to outweigh the perk of better lobbying prospects. On the other hand, these groups crave access to more extreme legislators because it facilitates lobbying and reduces policy extremism.

By developing our theoretical expectations for the consequences of a link between access and lobbying, which remain largely unexplored, I shed light on how such a link can affect policy and shape observed data. Although the channel I emphasize is prominent, other important channels are likely present in various situations. Whenever we cannot disentangle
multiple channels, we need to be aware that they may oppose, or complement, each other. Future work should study how the legislative forces highlighted here interact with vote buying, informational lobbying, and efforts to influence who gets elected.
References


Appendix A  Proofs of Main Results

Extended Model

I prove the main results in a version of the model that relaxes restrictions on the number of legislators and interest groups. There are three disjoint sets of players: \(n^V\) (finite and odd) voting legislators in \(N^V\); \(n^L \geq 3\) committee members in \(N^L\); and \(n^G \leq n^L\) interest groups in \(N^G\). Let \(N = N^V \cup N^L \cup N^G\).

Throughout, voting legislators are called voters and denoted by \(i\). To align with the main text, \(M\) denotes the median voter. I denote committee members by \(\ell\) and interest groups by \(g\). Each \(\ell \in N^L\) is associated with only one group, \(g_\ell\). Each \(g \in N^G\) can have access to multiple \(\ell \in N^L\) and this set is \(N^L_g \subseteq N^L\). Let \(\alpha_\ell \in [0,1]\) denote \(g_\ell\)'s access to \(\ell\).\(^{25}\)

Legislative bargaining occurs over an infinite number of periods \(t \in \{1,2,\ldots\}\). The policy space is a non-empty, closed interval \(X \subseteq \mathbb{R}\). Let \(\rho = (\rho_1,\ldots,\rho_{n^L}) \in \Delta([0,1])^{n^L}\) be the distribution of recognition probability.\(^{26}\) In each period \(t\), bargaining proceeds as follows. If no policy has passed before \(t\), then \(\ell\) proposes with probability \(\rho_\ell > 0\). All players observe the period-\(t\) proposer, \(\ell_t\). With probability \(1 - \alpha_\ell\), \(g_\ell\) cannot lobby and \(\ell_t\) freely proposes any \(x_t \in X\). With probability \(\alpha_\ell\), \(g_\ell\) can lobby and offers \(\ell_t\) a binding contract \((y_t,m_t) \in X \times \mathbb{R}_+\). Next, \(\ell_t\) accepts or rejects. Let \(a_t \in \{0,1\}\) denote \(\ell_t\)'s period-\(t\) acceptance decision, where \(a_t = 1\) indicates acceptance and \(a_t = 0\) if either \(\ell_t\) rejects or \(g_\ell\) is unable to lobby in \(t\). If \(a_t = 1\), then \(\ell_t\) is committed to propose \(x_t = y_t\) in \(t\) and \(g_\ell\) transfers \(m_t\) to \(\ell_t\). If \(a_t = 0\), then she can propose any \(x_t \in X\) and \(g_\ell\) keeps \(m_t\). All players observe \(x_t\). There is a simultaneous vote by \(i \in N^V\) using simple majority rule. If \(x_t\) passes, then bargaining ends with \(x_t\) enacted in \(t\) and all subsequent periods. If \(x_t\) fails, then \(q\) is enacted in \(t\) and bargaining proceeds to \(t + 1\).

Each player \(j \in N\) has quadratic policy utility with ideal point \(\hat{x}_j \in X\). As in the main text, I normalize \(\hat{x}_M = 0\) and assume \(q \neq 0\). Additionally, I assume there exists \(\ell \in N^L\) on

\(^{25}\)An independent legislator is accommodated by \(\alpha_\ell = 0\).

\(^{26}\)Where \(\Delta([0,1])^{n^L}\) denotes the \(n^L\)-dimensional unit simplex.
the same side of \( q \) as \( M \) such that: \( \alpha_\ell < 1 \) or \( g_\ell \) is on the same side of \( q \). For example, if \( q > 0 \), then some \( \ell \in N^L \) satisfies \( \hat{x}_\ell < q \) and at least one of the following holds: \( \alpha_\ell < 1 \) or \( \hat{x}_{g_\ell} \leq q \).

Players discount streams of per-period utility by common discount factor \( \delta \in (0, 1) \). For convenience, I normalize per-period payoffs by \( (1 - \delta) \). Let \( I^\ell_t \in \{0, 1\} \) equal one iff \( \ell \) is the period-\( t \) proposer and \( g_\ell \) can lobby in \( t \). Given a sequence of offers \((y_1, m_1), (y_2, m_2), \ldots \), a sequence of proposers \( \ell_1, \ell_2, \ldots \), a sequence of acceptance decisions \( a_1, a_2, \ldots \), and a sequence of independent policy proposals \( x_1, x_2, \ldots \) such that bargaining continues until \( t \), the discounted sum of per-period payoffs for \( i \in N^V \) is

\[
(1 - \delta^{t-1})u_i(q) + \delta^{t-1}\left[(1 - a_t)u_i(x_t) + a_t u_i(y_t)\right];
\]

for \( \ell \in N^\ell \),

\[
(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1}[u_\ell(q) + I^\ell_{t'}a_{t'}m_{t'}] + \delta^{t-1}\left[(1 - a_t)a_\ell(x_t) + a_t\left(u_\ell(y_t) + I^\ell_t m_t\right)\right];
\]

and for \( g \in N^g \),

\[
(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1}\left[u_g(q) - a_{t'}m_{t'} + \sum_{\ell \in N^\ell_g} I^\ell_{t'}\right] + \delta^{t-1}\left[(1 - a_t)a_g(x_t) + a_t\left(u_g(y_t) - m_t \sum_{\ell \in N^\ell_g} I^\ell_t\right)\right].
\]

Unless noted otherwise, results are proved for this more general setting. The model in the main text is a special case featuring one voter with ideal point \( \hat{x}_M \); four committee members with ideal points \( \hat{x}_L, \hat{x}_M, \hat{x}_\ell, \) and \( \hat{x}_R \); and one group at \( \hat{x}_g \) with access \( \alpha_\ell \geq 0 \) and \( \alpha_j = 0 \) for all \( j \neq \ell \).

**Strategies**

I study a refinement of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium
concept: no-delay stationary legislative lobbying equilibrium.

Let $\Delta(X)$ be the set of probability measures on $X$. Let $W = X \times \mathbb{R}_+$ denote the lobby offer space and $\Delta(W)$ denote the set of probability measures on $W$. A stationary mixed strategy for $g \in N^G$ is a probability measure $\lambda_g \in \Delta(W)^{|N^L_g|}$ over $g$’s offers $(y, m) \in W$ to each $\ell \in N^L_g$. A stationary mixed legislative strategy for $\ell \in N^L_g$ is a pair $(\pi_\ell, \varphi_\ell)$; where $\pi_\ell \in \Delta(X)$ specifies a probability measure over $\ell$’s independent proposals and $\varphi_\ell : W \rightarrow [0, 1]$ is the probability $\ell$ accepts each $(y, m) \in W$. Finally, voter $i$’s stationary mixed strategy $\nu_i : X \rightarrow [0, 1]$ specifies the probability $i$ votes for each $x \in X$.

Let $\lambda$ denote a profile of interest group strategies, $(\pi, \varphi)$ a profile of committee member strategies, and $\nu$ a profile of voter strategies. A stationary strategy profile is $\sigma = (\lambda, \pi, \varphi, \nu)$.

Under $\sigma$, let $\nu_\sigma(x)$ be the probability $x$ passes if proposed.

Let $w = (y, m) \in W$ denote an arbitrary lobby offer. Define

$$\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda^g_\ell(dw),$$

which is the probability under $\sigma$ that $\ell$ makes an independent policy proposal conditional on being recognized. Given $\sigma$, $i \in N^V$ has continuation value

$$V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\ell(y) u_i(y) + [1 - \nu_\ell(y)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \lambda^g_\ell(dw) 
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\ell(x) u_i(x) + [1 - \nu_\ell(x)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \pi_\ell(dx) \right),$$

the continuation value of $\ell \in N^L$ is

$$\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \int_W \varphi_j(y, m) \left[ \nu_\ell(y) u_\ell(y) + [1 - \nu_\ell(y)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda^g_j(dw) 
+ \xi_j(\alpha, \sigma) \int_X \left[ \nu_\ell(x) u_\ell(x) + [1 - \nu_\ell(x)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_j(dx) \right).$$
\[
+ \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y,m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m \right] \lambda_{g_\ell}(dw) \\
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right), \quad (7)
\]

and the continuation value of \( g \in N^G \) is

\[
\hat{V}_g(\sigma) = \sum_{\ell \in N^L_g} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y,m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda_{g_\ell}(dw) \\
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right) \\
+ \sum_{\ell \in N^L_g} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y,m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] - m \right] \lambda_{g_\ell}(dw) \\
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right), \quad (8)
\]

A stationary pure strategy for \( g \in N^G \) is \( (y_g, m_g) \in X^{|N^L_g|} \times \mathbb{R}^{|N^L_g|} \), where \( y_g \) is \( g \)'s profile of policy offers and \( m_g \) is \( g \)'s profile of monetary offers. A pure stationary strategy for \( \ell \in N^L \) is \( (z_\ell, a_\ell); \) where \( z_\ell \in X \) specifies \( \ell \)'s independent proposal, and \( a_\ell : X \times \mathbb{R} \to \{0, 1\} \) equals one iff \( \ell \) accepts \( g_\ell \)'s offer. Finally, for each \( i \in N^V \), \( v_i : X \to \{0, 1\} \) equals one iff \( i \) supports the proposal.

Given \( \sigma \), the set of policies that pass is constant across periods by stationarity and denoted \( A(\sigma) \). For \( \ell \in N^L \), define

\[
\tilde{U}_\ell(x; \sigma) = \begin{cases} 
    u_\ell(x) & \text{if } x \in A(\sigma) \\
    (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) & \text{else.}
\end{cases} \quad (9)
\]

Formally, \( \sigma = (y, m, z, a, v) \) is a no-delay stationary legislative lobbying equilibrium if it
satisfies five conditions. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), \((y_g^\ell, m_g^\ell)\) satisfies

\[
y_g^\ell \in \arg\max_{y \in A(\sigma)} u_{y\ell}(y) + u_\ell(y) - u_\ell(z_\ell)
\]

and

\[
m_g^\ell = u_\ell(z_\ell) - u_\ell(y_g^\ell).
\]

Second, for all \( \ell \in N^L \) and \((y, m) \in W\), \( a_\ell(y, m) = 1 \) iff

\[
\tilde{U}_\ell(y; \sigma) + m \geq \tilde{U}_\ell(z_\ell; \sigma).
\]

Third, for each \( \ell \in N^L \),

\[
z_\ell \in \arg\max_{x \in A(\sigma)} u_\ell(x).
\]

Finally, for each \( i \in N^V \), \( v_i(x) = 1 \) iff

\[
u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma).
\]

In Appendix B, I define *stationary mixed strategy legislative lobbying equilibrium* and show that all such equilibria are equivalent in outcome distribution to strategy profiles satisfying (10)-(14). Thus, the outcome distribution characterized in Proposition 1 applies even more broadly. I provide a rough outline here, as parts of the argument are noted in proving Proposition 1. First, equilibrium lobby offers always make targeted legislators indifferent. Thus, in any equilibrium each player’s continuation value can be expressed as expected utility over a policy lottery, similar to Banks and Duggan (2006a). Notably, for all voters in \( N^V \) this can be done using the same policy lottery. Therefore the outcome of equilibrium voting under majority rule always coincides with \( M \)’s preference (Duggan, 2014). Because \( q \neq 0 = \hat{x}_M \),
the policy lottery used to characterize equilibrium continuation values is not degenerate on \( q \). Then, equilibrium offers by each interest group must specify policy \( M \) will pass. Next, we can show two things that are without loss of generality in equilibrium: (i) \( M \) passes policy if indifferent, and (ii) legislators accept lobby offers if indifferent. Subsequently, I establish that every equilibrium is no-delay. To complete the equivalence in outcome distribution, I prove that legislator proposal strategies and interest group offer strategies are always degenerate in equilibrium.

**Proposition 1.** A no-delay stationary legislative lobbying equilibrium exists. Every such equilibrium has the same outcome distribution.

**Proof.** There are four parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for \( g \in N^G \) and \( \ell \in N^L \). Part 2 uses the fixed point to construct a strategy profile \( \sigma \). Part 3 verifies that \( \sigma \) satisfies (10) - (14). Part 4 shows there is a unique equilibrium outcome distribution.

**Part 1:** Let \( (y, z) = (y_1, \ldots, y_{n^L}, z_1, \ldots, z_{n^L}) \in X^{2n^L} \) and for each \( j \in N \) define

\[
r_j(y, z) = \sum_{\ell \in N^L} \rho_{\ell} \left( \alpha_{\ell} u_{\ell}(y_\ell) + (1 - \alpha_{\ell}) u_{\ell}(z_\ell) \right).
\]

(15)

Set \( A(r(y, z)) = \{ x \in X | u_M(x) \geq (1 - \delta) u_M(q) + \delta r_M(y, z) \} \), which is non-empty, compact, and convex because \( \delta \in (0, 1) \), \( q \neq 0 \), and \( u_M \) is strictly concave. Moreover, \( A(r(y, z)) \) is continuous in \( (y, z) \).

For each \( \ell \in N^L \), define

\[
\tilde{\phi}_{\ell}(y, z) = \arg \max_{y_\ell \in A(r(y, z))} u_{g_\ell}(y_\ell) + u_{\ell}(y_\ell),
\]

(16)

which is unique for all \( (y, z) \) because \( A(r(y, z)) \) is non-empty, compact and convex, and the objective function is strictly concave and continuous. Because \( A(r(y, z)) \) is continuous, the
Theorem of the Maximum implies continuity of $\tilde{\phi}_\ell(y, z)$. Next, define

$$
\tilde{\phi}_\ell(y, z) = \arg \max_{z \in A(r(y, z))} u_\ell(z),
$$

which is unique for all $(y, z)$ and continuous by the Theorem of the Maximum.

Define the mapping $\Phi : X^{2nL} \rightarrow X^{2nL}$ as $\Phi(y, z) = \prod_{\ell \in N_L} \tilde{\phi}_\ell(y, z) \times \prod_{\ell \in N_L} \phi_\ell(y, z)$, which is a product of continuous functions and thus continuous. By Brouwer’s theorem, a fixed point $(y^*, z^*) = \Phi(y^*, z^*)$ exists because $\Phi$ is a continuous function mapping a non-empty, compact, and convex set into itself.

**Part 2:** Define a stationary pure strategy profile $\sigma$ as follows. First, for all $g \in N_G$ and $\ell \in N_g^L$, set $y_g^\ell = y_\ell^*$ and $m_g^\ell = u_\ell(z_\ell^*) - u_\ell(y_\ell^*)$. Next, for $\ell \in N_L^L$, set $z_\ell = z_\ell^*$ and define

$$
a_\ell(y, m) = \begin{cases} 
1 & \text{if } u_\ell(y) + m \geq u_\ell(z_\ell^*), \text{ for } y \in A(r(y^*, z^*)) \\
1 & \text{if } (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell\alpha_\ell m_g^\ell) + m \geq u_\ell(z_\ell^*), \text{ for } y \notin A(r(y^*, z^*)) \\
0 & \text{else.}
\end{cases}
$$

Finally, for each $i \in N^V$ define $v_i$ so that $v_i(x) = 1$ if $u_i(x) \geq (1 - \delta)u_i(q) + \delta r_\ell(y^*, z^*)$ and $v_i(x) = 0$ otherwise.

**Part 3:** I check that $\sigma$ satisfies (10)-(14).

First, I verify (14) to show $A(\sigma) = A(r(y^*, z^*))$. Note that for each $g \in N_G$ and all $\ell \in N_g^L$, $y_g^\ell \in A(r(y^*, z^*))$ and $a_\ell(y_g^\ell, m_g^\ell) = 1$. Moreover, $z_\ell \in A(r(y^*, z^*))$ for all $\ell \in N_L^L$. Thus, voter $i$’s continuation value under $\sigma$ is $V_i(\sigma) = \sum_{\ell \in N_L} \rho_\ell[\alpha_\ell u_\ell(y_\ell^*) + (1 - \alpha_\ell)u_i(z_\ell^*)] = r_i(y^*, z^*)$. Thus, each voter $i$’s strategy satisfies (14). Banks and Duggan (2006b) and Duggan (2014) apply, so $A(\sigma) = A(r(y^*, z^*))$ because $M$ is decisive over lotteries.

To check (10), consider $g \in N_G$ and $\ell \in N_g^L$. Focusing on offers $\ell$ accepts is without loss of generality because $a_\ell(z_\ell, 0) = 1$. Because $A(\sigma) = A(r(y^*, z^*))$, (16) implies $\tilde{\phi}_\ell(y^*, z^*) = \ldots$
arg \max_{y\in A(\sigma)} u_{g\ell}(y) + u_\ell(y) - u_\ell(z^*_\ell). \text{ Thus, (10) holds because } \tilde{\phi}_\ell(y^*, z^*) = y^*_\ell = y^\rho_\ell. \text{ The no-delay property noted before the proof implies } y \notin A(\sigma) \text{ is not a profitable deviation for any } g \in N^G.

It is immediate that \( m^\ell_g \) satisfies (11).

To check (12), note that \( \ell \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( \tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z^*_\ell) \). Thus, \( \ell \) weakly prefers to accept any \((y, m)\) satisfying \( y \in A(r(y^*, z^*)) \) iff \( u_\ell(y) + m \geq u_\ell(z^*_\ell) \). If \( y \notin A(r(y^*, z^*)) \), then \( \ell \) weakly prefers to accept \((y, m)\) iff \((1-\delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell\alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell) \). Thus, \( a_\ell \) satisfies (12).

To check (13), note that (17) implies \( \phi_\ell(y^*, z^*) = \arg \max_{x \in A(\sigma)} u_\ell(x) \) because \( A(\sigma) = A(r(y^*, z^*)) \). Thus, (13) holds because \( \phi_\ell(y^*, z^*) = z^*_\ell = z_\ell \) for each \( \ell \in N^L \). The no-delay property implies \( x \notin A(\sigma) \) is not a profitable deviation for any \( \ell \in N^L \).

**Part 4.** Let \( \sigma \) and \( \sigma' \) be stationary legislative lobbying equilibria. It suffices to show \((y_\ell, m_\ell) = (y'_\ell, m'_\ell)\) for all \( g \in N^G \) and \( z_\ell = z'_\ell \) for all \( \ell \in N^L \). Assume \( y_\ell \neq y'_\ell \) or \( z_\ell \neq z'_\ell \) for some \( \ell \in N^L \). Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, \( A(\sigma) = A(\sigma') \). Because \( \sigma \) and \( \sigma' \) are no-delay, \( \ell \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( u_\ell(z_\ell) \) under both \( \sigma \) and \( \sigma' \). Because equilibrium lobby offers always make targeted legislators indifferent, \( m^\ell_g = u_\ell(y^\ell_g) - u_\ell(z_\ell) \). Therefore \((y_\ell, m_\ell) = (y'_\ell, m'_\ell)\) and \( z_\ell = z'_\ell \).

\( \square \)

**Comparative Statics on Lobbying Expenditures**

**Lemma A.1.** For all \( \ell \in N^L \), \( g_\ell \)'s equilibrium lobbying expenditures increase as the acceptance set expands.

**Proof.** Let \( A^* = [-\bar{x}^*, \bar{x}^*] \) denote the equilibrium acceptance set. There are two cases.

**Case 1.** Suppose \( \hat{x}_\ell \in A^* \). Then \( z_\ell = \hat{x}_\ell \). There are two subcases.

First, assume \( \hat{y}_\ell \in A^* \). Then \( y^\ell_g = \hat{y}_\ell \), and (11) implies \( m^\ell_g = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell) \). Thus, \( m^\ell_g \) is constant because \( z_\ell = \hat{x}_\ell \) and \( y^\ell_g = \hat{y}_\ell \) as \( \bar{x}^* \) increases.
Second, assume \( \hat{y}_\ell \notin A^\ast \). Since \( \hat{x}_\ell \in A^\ast \), this requires \( \hat{x}_{g_\ell} \notin [-\bar{x}^*, \bar{x}^*] \). Without loss of generality, assume \( \hat{x}_{g_\ell} > \bar{x}^* \). Thus, \( z_\ell = \hat{x}_\ell \) and \( y^g_\ell = \bar{x}^* \). Then (11) implies \( m^\ell_g = u_\ell(\hat{x}_\ell) - u_\ell(\bar{x}^*) \), which increases with \( \bar{x}^* \).

Case 2. Suppose \( \hat{x}_\ell \notin A^\ast \). Without loss of generality, assume \( \hat{x}_g \ell > z_\ell = \bar{x}^* \). There are three subcases.

First, assume \( \hat{y}_\ell < -\bar{x}^* \). Then \( y^g_\ell = -\bar{x}^* \), and (11) implies \( m^\ell_g = u_\ell(\bar{x}^*) - u_\ell(-\bar{x}^*) \). Thus, \( m^\ell_g \) increases with \( \bar{x}^* \) because \( -\bar{x}^* < \bar{x}^* < \hat{x}_\ell \).

Second, assume \( \hat{y}_\ell \in A^\ast \). Thus, \( y^g_\ell = \hat{y}_\ell \) and \( y^g_\ell \) is constant as legislative extremism increases. Then (11) implies \( m^\ell_g = u_\ell(\bar{x}^*) - u_\ell(\hat{y}_\ell) \), which increases with \( \bar{x}^* \).

Third, assume \( \hat{y}_\ell \geq \bar{x}^* \), which implies \( y^g_\ell = \bar{x}^* \). Then (11) implies \( m^\ell_g = u_\ell(\bar{x}^*) - u_\ell(\bar{x}^*) = 0 \), which is constant.

Altogether, \( m^\ell_g \) weakly increases in \( \bar{x}^* \).

Next, we show how the acceptance set changes with the distribution of equilibrium proposals. Set \( \theta = (\hat{x}, \rho, \alpha) \). Let \( \Lambda_\theta \) denote the unconstrained extremism lottery, which puts probability \( \rho_\ell \alpha_\ell \) on \( |\hat{y}_\ell| \) and probability \( \rho_\ell(1 - \alpha_\ell) \) on \( |\hat{x}_\ell| \) for each \( \ell \in N^L \). Given \( \theta \) and \( \theta' \), legislative extremism is greater under \( \theta' \) if \( \Lambda_{\theta'} \) first order stochastically dominates \( \Lambda_\theta \).

Lemma A.2. Greater legislative extremism weakly expands the equilibrium acceptance set.

Proof. Consider \( \theta \) and \( \theta' \), with legislative extremism greater under \( \theta' \). By Proposition 1.3, \( \theta \) and \( \theta' \) each induce a unique equilibrium acceptance set. Let \( \pi_\theta \) and \( \pi_{\theta'} \) denote the respective upper bounds of these sets. I show \( \pi_{\theta'} \geq \pi_\theta \).

For \( b \geq 0 \), let \( C^b_j = \mathbb{I}\{\hat{x}_j \in (-b, b)\} \) and \( \tilde{C}^b_j = \mathbb{I}\{\hat{y}_j \in (-b, b)\} \). Define \( C'^b_j \) and \( \tilde{C}'^b_j \) analogously for \( \hat{x}'_j \) and \( \hat{y}'_j \). For all \( b \geq 0 \),

\[
(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)C^b_j u_M(\hat{x}_j) + \alpha_j \tilde{C}^b_j u_M(\hat{y}_j) \right) \\
+ \delta u_M(b) \sum_{j \in N^L} \rho_j \left( (1 - \alpha_j)(1 - C^b_j) + \alpha_j(1 - \tilde{C}^b_j) \right) \tag{19}
\]
\[
\geq (1 - \delta)u_M(q) + \delta \sum_{j \in NL} \rho'_j \left((1 - \alpha'_j)C'_j b u_M(\hat{x}'_j) + \alpha'_j \tilde{C}'_j b u_M(\hat{y}'_j)\right) \\
+ \delta u_M(b) \sum_{j \in N^k} \rho'_j \left((1 - \alpha'_j)(1 - C'_j b) + \alpha'_j(1 - \tilde{C}'_j b)\right),
\]

(20)

where (20) follows because \(\mu_{\theta'} FOSD \mu_{\theta}\) and \(u_M\) is negative quadratic. The equilibrium characterization, and construction of \(C_j\) and \(\tilde{C}_j\), implies \(\pi_{\theta}^g\) is the unique \(b \geq 0\) such that \(u_M(b)\) equals (19). Analogously, \(\pi_{\theta'}\) is the unique \(b \geq 0\) such that \(u_M(b)\) equals (20). Thus, \(\pi_{\theta'} \geq \pi_{\theta}\).

\[\square\]

**Endogenous Access**

Consider \(\ell \in N^L\) and refer to \(g_\ell\) as \(g\) for convenience. Recall

\[
\hat{y}_\ell = \arg \max_{y \in X} u_g(y) + u_\ell(y) = \frac{\hat{x}_\ell + \hat{x}_\ell}{2}.
\]

(21)

The results fix \(\hat{x}_g\) and vary \(\hat{x}_\ell\). Throughout, \(\hat{x}_g > 0\), as the other case is symmetric.

Let \(\sigma(\alpha_\ell; \hat{x}_\ell)\) denote an equilibrium given \(\hat{x}_\ell\) and \(\alpha_\ell\), and denote the corresponding social acceptance set as \(A(\alpha_\ell; \hat{x}_\ell)\), with upper bound \(\pi(\alpha_\ell; \hat{x}_\ell)\). That is, \(A(\alpha_\ell; \hat{x}_\ell)\) corresponds to \(A^*_{\alpha_\ell}\) from the main text but makes explicit the dependence on \(\hat{x}_\ell\).

I first state a lemma that partitions whether \(\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)\). It plays a key role in proving Lemmas A.4 and 3.

**Lemma A.3.** For all \(\ell \in N^L\), there exists \(\pi_\ell \in (0, q]\) such that \(\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)\) if \(\hat{x}_\ell \in (-\pi_\ell, \pi_\ell)\) and otherwise \(A(0; \hat{x}_\ell) = [-\pi_\ell, \pi_\ell]\).

The proof of Lemma A.3 proceeds in a series of Lemmas that are provided in Appendix C. A rough outline of the argument is as follows. First, I define a function \(\xi^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}\) constructed so that \(\xi^\ell(x) > 0\) iff \(x \in \text{int}A(0; x)\). Then, I show that there is a unique \(\pi_\ell \in (0, q]\) such that \(\xi^\ell(x) > 0\) iff \(x \in [0, \pi_\ell]\). It follows that \(\hat{x}_\ell \in (-\pi_\ell, \pi_\ell)\) implies \(\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)\), and otherwise \(A(0; \hat{x}_\ell) = [-\pi_\ell, \pi_\ell]\).
Lemma A.4. In equilibrium, \( y_0^* \neq z_0^* \) iff \( \hat{x}_\ell \in (\chi(\hat{x}_g), \bar{\chi}(\hat{x}_g)) \).

Proof. If \( \hat{x}_\ell \in (-\bar{x}_\ell, \bar{x}_\ell) \), then Lemma A.3 implies \( z_0^* = \hat{x}_\ell \). Thus, \( \hat{x}_\ell \neq \hat{x}_g \) implies \( y_0^* \neq z_0^* \).

Otherwise, \( z_0^* \) is the boundary of \( A_0^* = [-\bar{x}, \bar{x}] \) closer to \( \hat{x}_\ell \). For \( \hat{x}_\ell \leq -\bar{x} \), we have \( y_0^* > -\bar{x} \) iff \( \hat{x}_\ell > \chi(\hat{x}_g) \). Analogously for \( \hat{x}_\ell \geq \bar{x} \), we have \( y_0^* < \bar{x} \) iff \( \hat{x}_\ell < \bar{\chi}(\hat{x}_g) \).

Lemma 3. Interest group \( g \) strictly prefers \( \alpha_\ell > 0 \) only if \( \hat{x}_\ell \in (\chi(\hat{x}_g), \bar{\chi}(\hat{x}_g)) \).

Proof. Suppose \( \hat{x}_\ell \notin (\chi(\hat{x}_g), \bar{\chi}(\hat{x}_g)) \). Lemma A.4 implies \( y_0^* = z_0^* \). Thus, \( V_\ell^{*,x} \) is constant as \( \alpha_\ell \) increases from zero and therefore \( A_\ell^{*,x} \) is constant in \( \alpha_\ell \). It follows that the equilibrium outcome distribution is constant in \( \alpha_\ell \), so \( g \) is indifferent. \( \square \)

Lemma 2. If \( \hat{x}_g \in (0, \bar{x}_\ell) \), then there exists \( x' \in [0, \hat{x}_g] \) such that \( \hat{x}_\ell \notin (-x', x') \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Otherwise, \( \hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell) \) for all \( \hat{x}_\ell \) and \( \alpha_\ell \).

Proof. Consider \( \hat{x}_g \in (0, \bar{x}_\ell) \). If \( \hat{x}_\ell = \hat{x}_g \), then Lemma A.3 implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). By symmetry, \( \hat{x}_\ell = -\hat{x}_g \) also implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Recall that \( A(0; \hat{x}_\ell) \) strictly expands as \( \hat{x}_\ell \) shifts away from 0 over \( (-\bar{x}_\ell, \bar{x}_\ell) \). Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies \( A(0; \hat{x}_\ell) \) is continuous in \( \hat{x}_\ell \). Thus, there exists \( x' \in [0, \hat{x}_g] \) such that \( \hat{x}_\ell \notin (-x', x') \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \).

To complete the proof, consider \( \hat{x}_g \geq \bar{x}_\ell \). Lemma A.3 implies \( \hat{x}_g \notin \text{int}A(0; \hat{x}_\ell) = (\bar{x}_\ell, \bar{x}_g) \) for all \( \hat{x}_\ell \geq \hat{x}_g \). Because \( A(\alpha_\ell; \hat{x}_\ell) \subseteq A(0; \hat{x}_g) \) for all \( (\alpha_\ell, \hat{x}_\ell) \), it follows that \( \hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell) \).

Next, Lemmas A.5 - A.8 establish properties used to prove Propositions 4 and 5.

Lemma A.5. Suppose \( \hat{x}_g \in (0, \bar{x}_\ell) \). There exists \( \bar{x} \in [0, \hat{x}_g] \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell) \) for all \( \alpha_\ell \in [0, 1] \).

Proof. Consider \( \hat{x}_g \in (0, \bar{x}_\ell) \) and fix \( \alpha_\ell \). By Lemma 2, there exists \( x' \in [0, \hat{x}_g] \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Then \( 0 < \hat{x}_\ell < \hat{x}_g \) implies \( A(0; \hat{x}_\ell) \subseteq A(\alpha_\ell; \hat{x}_\ell) \). \( \square \)
For each \( j \in N^L \setminus \{ \ell \} \), define \( E^{LB}_j(\alpha_\ell; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \leq -\overline{x}(\alpha_\ell; \hat{x}_\ell)\} \), \( E^{UB}_j(\alpha_\ell; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \geq \overline{x}(\alpha_\ell; \hat{x}_\ell)\} \), and \( C_j(\alpha_\ell; \hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \in \text{int}A(\alpha_\ell; \hat{x}_\ell)\} \). Define \( \tilde{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell) \), \( \tilde{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell) \), and \( \tilde{C}_j(\alpha_\ell; \hat{x}_\ell) \) analogously using \( \hat{y}_j \). Let \( I^*_g \in \{0,1\} \) indicate whether \( j \in N^L_g \).

**Assumption A.1.** There exists \( j \in N^L \setminus \{ \ell \} \) such that \( \alpha_j < 1 \) and \( \hat{x}_j \notin A(0; \hat{x}_g) \).

**Assumption A.2.** There exists \( j \in N^L \setminus \{ \ell \} \) such that \( \alpha_j > 0 \) and \( \hat{y}_j \notin A^+(0; \hat{x}_g) \).

Next, define
\[
v^g_1(\alpha_\ell; \hat{x}_\ell) = \rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right)
\] (22)
and
\[
v^g_2(\alpha_\ell; \hat{x}_\ell) = \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \tilde{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(\alpha_\ell; \hat{x}_\ell) \right] u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell)) + \left[ \alpha_j \tilde{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(\alpha_\ell; \hat{x}_\ell) \right] u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell)) + \alpha_j \left[ \tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I^*_g m^g(\alpha_\ell; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) \right)
\] (23)

**Lemma A.6.** If \( \hat{x}_\ell \neq \hat{x}_g \), then \( \frac{\partial v^g_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0 \).

**Proof.** Suppose \( \hat{x}_\ell \neq \hat{x}_g \). Then \( \frac{\partial v^g_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial}{\partial \alpha_\ell} \left( \frac{\partial^2 v^g_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell^2} \right) > 0 \), by (22) and \( \hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2} \). \( \square \)

**Lemma A.7.** Suppose \( 0 \leq \hat{x}_\ell < \hat{x}_g < \overline{x}_\ell \) and at least one of Assumption A.1 or A.2 holds. Then \( v^g_2(\alpha_\ell; \hat{x}_\ell) \) strictly decreases in \( \alpha_\ell \).

**Proof.** Fix \( \alpha_\ell \). It suffices to show that
\[
\left[ \alpha_j \tilde{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(\alpha_\ell; \hat{x}_\ell) \right] u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell)) + \left[ \alpha_j \tilde{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(\alpha_\ell; \hat{x}_\ell) \right] u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell)) + \alpha_j \left[ \tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I^*_g m^g(\alpha_\ell; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) \right)
\] (24)
Proof. Fix \( \tilde{x}_g \) such that \( \tilde{x}_g \in (0, \bar{x}_g) \) implies \( \tilde{x}_g < \bar{x}(0; \tilde{x}_g) \) by Lemma 2, so \( u_g(\bar{x}(\alpha; \tilde{x}_g)) \) and \( u_g(-\bar{x}(\alpha; \tilde{x}_g)) \) both decrease in \( \alpha \). Without loss of generality, consider \( \hat{x}_j \geq 0 \). Because \( 0 \leq \hat{x}_j < \hat{x}_g \), \( \bar{x}(\alpha; \hat{x}_j) \) increases in \( \alpha \). There are two implications. First, \( \hat{x}_g \in (0, \bar{x}) \) implies \( \hat{x}_g < \bar{x}(0; \hat{x}_j) \) by Lemma 2, so \( u_g(\bar{x}(\alpha; \hat{x}_j)) \) and \( u_g(-\bar{x}(\alpha; \hat{x}_j)) \) both decrease in \( \alpha \). Second, exactly one of the following holds: \( E^{UB}_j(\alpha; \hat{x}_j) = 1 \) for all \( \alpha \); \( C_j(\alpha; \hat{x}_j) = 1 \) for all \( \alpha \); or there is a unique \( \bar{\alpha}_j \in [0,1] \) such that \( \alpha_j \in [0, \bar{\alpha}_j] \) implies \( E^{UB}_j(\alpha; \hat{x}_j) = 1 \), and \( \alpha_j \in (\bar{\alpha}_j, 1] \) implies \( C_j(\alpha; \hat{x}_j) = 1 \). An analogous observation holds for \( E^{UB}_j(\alpha; \hat{x}_j) \) and \( C_j(\alpha; \hat{x}_j) \). Thus, both

\[
E^{LB}_j(\alpha; \hat{x}_j) u_g(-\bar{x}(\alpha; \hat{x}_j)) + E^{UB}_j(\alpha; \hat{x}_j) u_g(\bar{x}(\alpha; \hat{x}_j)) + C_j(\alpha; \hat{x}_j) u_g(\hat{x}_j) \tag{25}
\]

and

\[
\widetilde{E}^{UB}_j(\alpha; \hat{x}_j) u_g(-\bar{x}(\alpha; \hat{x}_j)) + \widetilde{E}^{UB}_j(\alpha; \hat{x}_j) u_g(\bar{x}(\alpha; \hat{x}_j)) + \widetilde{C}_j(\alpha; \hat{x}_j) u_g(\hat{y}_j) \tag{26}
\]

decrease in \( \alpha \). Furthermore, because at least one of Assumptions A.1 and A.2 holds, at least one of (25) and (26) strictly decreases for some \( j \in N^L \setminus \{\ell\} \). Lemma A.1 implies \( m_j^g(\alpha; \hat{x}_j) \) weakly increases in \( \alpha \) for all \( j \in N^L_g \). Altogether, (24) decreases in \( \alpha \) for all \( j \in N^L \setminus \{\ell\} \) and strictly decreases for some \( j \), as desired.

For \( g \in N^G \), define

\[
U^E_g(\alpha; \hat{x}_j) = v^g_1(\alpha; \hat{x}_j) + v^g_2(\alpha; \hat{x}_j). \tag{27}
\]

**Lemma A.8.** Assume \( \hat{x}_g \in (0, \bar{x}_g) \) and at least one of Assumption A.1 or A.2 holds. There exists \( x' \prec \hat{x}_g \) such that \( \hat{x}_j \in (x', \hat{x}_g) \) implies \( U^E_g(\alpha; \hat{x}_j) \) strictly decreases in \( \alpha \).

**Proof.** Fix \( \alpha_j \). I show \( \frac{\partial v^g_2(\alpha; \hat{x}_j)}{\partial \alpha_j} > \frac{\partial v^g_2(\alpha; \hat{x}_j)}{\partial \alpha_j} \) for \( \hat{x}_j \) sufficiently close to \( \hat{x}_g \).

By Lemma A.5, there exists \( \tilde{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_j \in (\tilde{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(\alpha; \hat{x}_j) \) for all \( \alpha \in [0,1] \). Fix \( \hat{x}_j \in (\tilde{x}, \hat{x}_g) \) and \( \alpha \in [0,1] \).
First, I characterize a lower bound on $|\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}|$. Define

$$
\Gamma = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_{j}^{LB}(\hat{x}_g) + (1 - \alpha_j) E_{j}^{LB}(\hat{x}_g) \right] \frac{\partial u_g(-\mathcal{F}(\hat{x}))}{\partial \mathcal{F}(\hat{x})}
+ \left[ \alpha_j \tilde{E}_{j}^{UB}(\hat{x}_g) + (1 - \alpha_j) E_{j}^{UB}(\hat{x}_g) \right] \frac{\partial u_g(\mathcal{F}(\hat{x}))}{\partial \mathcal{F}(\hat{x})}.
$$

Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\mathcal{F}(\bar{x}), \mathcal{F}(\bar{x}))$ implies $\frac{\partial u_g(\mathcal{F}(\hat{x}))}{\partial \mathcal{F}(\hat{x})} < 0$ and $\frac{\partial u_g(-\mathcal{F}(\hat{x}))}{\partial \mathcal{F}(\hat{x})} < 0$, and (ii) at least one of Assumptions A.1 and A.2 hold.

I claim $\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)} < \Gamma$, where

$$
\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)} = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}_{j}^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_{j}^{LB}(\alpha_\ell; \hat{x}_\ell) \right] \frac{\partial u_g(-\mathcal{F}(\alpha_\ell; \hat{x}_\ell))}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}
+ \left[ \alpha_j \tilde{E}_{j}^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_{j}^{UB}(\alpha_\ell; \hat{x}_\ell) \right] \frac{\partial u_g(\mathcal{F}(\alpha_\ell; \hat{x}_\ell))}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}
- I_g^j \alpha_j \frac{\partial m_g^j(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}. \tag{29}
$$

Three steps show the claim. First, note $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $\mathcal{F}(\hat{x}_g) \geq \mathcal{F}(\alpha_\ell; \hat{x}_\ell)$. Thus, we have $\tilde{E}_{j}^{UB}(\hat{x}_g) \leq \tilde{E}_{j}^{UB}(\alpha_\ell; \hat{x}_\ell)$, $E_{j}^{UB}(\hat{x}_g) \leq E_{j}^{UB}(\alpha_\ell; \hat{x}_\ell)$, $\tilde{E}_{j}^{LB}(\hat{x}_g) \leq \tilde{E}_{j}^{LB}(\alpha_\ell; \hat{x}_\ell)$, and $E_{j}^{LB}(\hat{x}_g) \leq E_{j}^{LB}(\alpha_\ell; \hat{x}_\ell)$ for all $j \neq \ell$. Second, $\hat{x}_g < \mathcal{F}(\bar{x}) < \mathcal{F}(\alpha_\ell; \hat{x}_\ell)$ implies $\frac{\partial u_g(\mathcal{F}(\alpha_\ell; \hat{x}_\ell))}{\partial \mathcal{F}(\hat{x})} < 0$ and symmetrically $\frac{\partial u_g(-\mathcal{F}(\alpha_\ell; \hat{x}_\ell))}{\partial \mathcal{F}(\hat{x})} < 0$. Third, $\frac{\partial m_g^j(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)} \geq 0$ for all $j \in N_g^L$ by Lemma A.1.

For almost all $\alpha_\ell \in [0, 1]$, $\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)} \frac{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell}$. Define $C_j(\alpha_\ell; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_f(\alpha_\ell; \hat{x}_\ell)) + \alpha_j(1 - C_j(\alpha_\ell; \hat{x}_\ell))].$ Then,

$$
\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < \Gamma \frac{\partial \mathcal{F}(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} \tag{30}
$$

$$
= \delta \rho \Gamma \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right]
= 2 \mathcal{F}(\alpha_\ell; \hat{x}_\ell) \left[ 1 - \delta \left( \sum_{j \in N_g^L} \rho_j C_j(\alpha_\ell; \hat{x}_\ell) \right) \right]. \tag{31}
$$

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\[
\begin{align*}
\frac{\delta \rho_t}{2\pi_\ell} \Gamma \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right] \\
= \frac{\delta \rho_t}{2\pi_\ell} \Gamma \left[ \frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g) \right],
\end{align*}
\]  

where (30) follows from \( \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} > 0 \) and \( 0 > \Gamma > \frac{\partial \nu(\alpha; \hat{x}_\ell)}{\partial \pi(\alpha; \hat{x}_\ell)} \); (31) from applying the implicit function theorem to \( \pi(\alpha; \hat{x}_\ell) \), which is possible for almost all \( \alpha \in [0, 1] \); (32) because \( \Gamma[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)] < 0, \pi_\ell > \pi(\alpha; \hat{x}_\ell) > 0 \), and \( \delta \sum_{j \in N\ell} \rho_j C_j^0(\alpha; \hat{x}_\ell) \in (0, 1) \); and (33) from using \( \hat{y}_\ell = \frac{x_g + \hat{x}_\ell}{2} \) and simplifying.

By Lemma A.6, \( \frac{\partial \nu(\alpha; \hat{x}_\ell)}{\partial \alpha} = \frac{\delta \rho_t}{2} (\hat{x}_g - \hat{x}_\ell)^2 \). By (30), \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < \frac{\partial \nu(\alpha; \hat{x}_\ell)}{\partial \alpha} + \Gamma \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha} \) for almost all \( \alpha \in [0, 1] \). Thus, (33) implies that \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < 0 \) if

\[
\frac{\rho_t}{2}(\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_t \Gamma}{2\pi_\ell} \left[ \frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g) \right] < 0,
\]

which holds for \( \hat{x}_\ell > \hat{x}_g \left( \frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta} \right) \). Define \( x' = \max \left\{ \hat{x}, \hat{x}_g \left( \frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta} \right) \right\} \). Note \( x' < \hat{x}_g \) because (i) \( \hat{x} < \hat{x}_g \) and (ii) \( \delta \Gamma < 0 \) implies \( \frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta} < 1 \). Thus, \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( \frac{\partial U_g^E(\alpha; \hat{x}_\ell)}{\partial \alpha} < 0 \) for almost all \( \alpha \in [0, 1] \). Continuity implies \( U_g^E(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \) for such \( \hat{x}_\ell \).

**Proposition 4** Assume \( \hat{x}_g \in (0, \pi_\ell) \) and either Assumption A.1 or A.2. There are cutpoints satisfying \( -\hat{x}_g < x' < x'' < \hat{x}_g \) such that:

(i) \( \alpha^*_x > 0 \) if \( \hat{x}_\ell \in (\chi(\hat{x}_g), x') \cup (\hat{x}_g, \chi(\hat{x}_g)) \) and

(ii) \( \alpha^*_x = 0 \) if \( \hat{x}_\ell \in (x'', \hat{x}_g) \).

**Proof. Case 1.** Consider \( \hat{x}_\ell \in [0, \hat{x}_g) \). By Lemma A.5, there exists \( \bar{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). By Lemma A.8, there exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U_g^E(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \). Let \( x'' = \max\{\bar{x}, x'\} \) and consider \( \hat{x}_\ell \in (x'', \hat{x}_g) \). Then \( z_\ell = \hat{x}_\ell \in A(\alpha; \hat{x}_\ell) \) and \( y_\ell = \hat{y}_\ell \in A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Thus, \( g \)'s ex-ante expected utility from access equals \( U_g^E(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \), so \( g \) strictly prefers \( \alpha = 0 \).
Case 2. Consider \( \hat{x}_\ell \in (\hat{x}_g, \overline{x}(\hat{x}_g)) \). It suffices to show that \( g \)'s ex ante expected utility strictly increases as \( \alpha_\ell \) increases from zero. There are two cases.

- If \( \hat{x}_\ell < \overline{x}_\ell \), then \( g \)'s ex ante expected payoff equals \( U^E_g(\alpha_\ell; \hat{x}_\ell) \) for sufficiently small \( \alpha_\ell \). By Lemma A.6, \( \frac{\partial^2 v^g_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0 \). To complete this case, I argue that \( v^g_2(\alpha_\ell; \hat{x}_\ell) \) increases for sufficiently small \( \alpha_\ell \). Under the maintained assumptions, \( \hat{x}_g \in (-\overline{x}(0; \hat{x}_\ell), \overline{x}(0; \hat{x}_\ell)) \) and \( \hat{y}_\ell \in (\hat{x}_g, \overline{x}(0; \hat{x}_\ell)) \). Thus, \( \overline{x}(\alpha_\ell; \hat{x}_\ell) \) strictly decreases for sufficiently small \( \alpha_\ell \).

- If \( \hat{x}_\ell > \overline{x}_\ell \), then \( \overline{x}(0; \hat{x}_\ell) = \hat{x}_\ell \). Thus, \( g \)'s ex ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\overline{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\overline{x}_\ell) \right) + \sum_{j \neq \ell} \rho_j \left( \alpha_j \overline{E}_{jLB}^g(0; \hat{x}_\ell) + (1 - \alpha_j) \overline{E}_{jLB}^g(0; \hat{x}_\ell) \right) u_g(-\overline{x}_\ell)
\]

\[
+ \left[ \alpha_j \overline{E}_{jUB}^g(0; \hat{x}_\ell) + (1 - \alpha_j) \overline{E}_{jUB}^g(0; \hat{x}_\ell) \right] u_g(\overline{x}_\ell)
\]

\[
+ \alpha_j \overline{C}_{j}(0; \hat{x}_\ell) u_g(\hat{y}_j) + (1 - \alpha_j) \overline{C}_{j}(0; \hat{x}_\ell) u_g(\hat{x}_j)
\]

\[
- \overline{I}_j^g \alpha_j m^j_g(0; \hat{x}_\ell)
\]

(34)

Arguments similar to Case 1 show that (34) strictly increases in \( \alpha_\ell \) at \( \alpha_\ell = 0 \).

Case 3. Consider \( \hat{x}_\ell < \overline{x}_\ell \). For \( \hat{x}_\ell \in (\overline{x}(\hat{x}_g), -\hat{x}_g) \), arguments analogous to Case 2 show that \( g \)'s ex-ante expected payoff strictly increases at \( \alpha_\ell = 0 \). Because \( g \)'s ex-ante expected payoff is continuous in \( \hat{x}_\ell \), there exist \( x' > -\hat{x}_g \) such that \( \hat{x}_\ell \in (\overline{x}(\hat{x}_g), x') \) implies \( \alpha^{*}_\ell > 0 \).

We now prove the extended version of Proposition 5.

**Proposition 5** Assume \( \hat{x}_g \geq \overline{x}_\ell \).
(i) If \( \sum_{i \in \mathcal{N}} \rho_i [1 - \alpha_i] \mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\overline{x}\} \) is sufficiently small, then there exists \( x' < 0 \) such that \( \alpha^*_\ell > 0 \) if \( \hat{x}_\ell \in (x', \overline{x}) \).

(ii) If \( \sum_{i \in \mathcal{N}} \rho_i [1 - \alpha_i] \mathbb{I}\{\hat{x}_i \geq \overline{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \geq \overline{x}\} \) is sufficiently small, then there exists \( \overline{x}' \geq -\overline{x} \) such that \( \alpha^*_\ell > 0 \) if \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{x}') \).

Proof. I prove (i), as (ii) is analogous. Consider \( \hat{x}_\ell \in [0, \overline{x}_0; \hat{x}_\ell) \) and assume \( \sum_{i \in \mathcal{N}} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\overline{x}\}] = 0 \). I show that \( g \)'s ex-ante expected payoff strictly increases at \( \alpha_\ell = 0 \).

We have \( \hat{x}_\ell \in [0, \overline{x}_0; \hat{x}_\ell) \) and \( \hat{y}_\ell > \hat{x}_\ell \). Therefore \( 0 \leq z_\ell(0; \hat{x}_\ell) = \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \). Furthermore, \( -\overline{x}(0; \hat{x}_\ell) \) is not proposed with positive probability because \( \sum_{i \in \mathcal{N}} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\overline{x}\}] = 0 \). Thus, \( g \)'s ex-ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right) \\
+ \sum_{j \neq \ell} \rho_j \left[ \alpha_j E^\ell UB_j(0; \hat{x}_\ell) + (1 - \alpha_j) E^\ell UB_j(0; \hat{x}_\ell) \right] u_g(\overline{x}(0; \hat{x}_\ell)) \\
+ \alpha_j \left[ \overline{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) - I^g_\ell m^g_\ell(0; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_j(\hat{x}_\ell). \tag{35}
\]

Three steps show (35) strictly increases at \( \alpha_\ell = 0 \).

- First, \( 0 \leq \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \) implies \( y^g_\ell(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Therefore \( u_g(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly increases and \( u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly decreases. Because \( u \) is quadratic and \( \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2} < \hat{x}_g \), it follows that \( u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) increases weakly faster than \( u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) decreases. Therefore \( u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Furthermore, \( \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \) also implies \( u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0 \). It follows that \( \alpha_\ell \left[ u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \) weakly increases at \( \alpha_\ell = 0 \).

- Second, \( \overline{x}_\ell(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \) because \( 0 \leq \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \overline{x}(0; \hat{x}_\ell) \). Since
\( \bar{x}(0; \hat{x}_\ell) \leq \hat{x}_g \), it follows that \( u_g(\bar{x}(0; \hat{x}_\ell)) \) increases at \( \alpha_\ell = 0 \).

- Third, \( m_j^g(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N_g^L \) by Lemma A.1. But it strictly increases only for \( j \in N_g^L \) such that \( \hat{y}_j > \bar{x}(0; \hat{x}_\ell) \). Thus, \( g \)'s lobbying surplus weakly increases in \( \alpha_\ell \) for all \( j \in N_g^L \).

The desired result follows because \( g \)'s ex-ante expected payoff is continuous in \( \sum_{i \in N_L} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\bar{x}\}] \).

\[ \square \]

Willingness to Pay for Access

The following applies to the model in the main text. Define \( \theta = (\hat{x}, \rho, \alpha) \). Let \( U^E_g(\theta) \) be \( g \)'s ex-ante expected utility and let \( \bar{x}_\alpha = \bar{x}(\alpha; \hat{x}_\ell) \). Define \( \frac{\partial \bar{x}_0}{\partial \alpha} = \frac{\partial \bar{x}_0}{\partial \alpha}|_{\alpha=0} \), \( \frac{\partial \bar{x}_0}{\partial \alpha} = \frac{\partial \bar{x}_0}{\partial \alpha}|_{\alpha=0} \), and \( \frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \delta_g} = \frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \delta_g}|_{\alpha=0} \).

To state the result, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, replace \( \ell \) with two legislators, \( \ell_1 \) and \( \ell_2 \), and replace \( g \) with two groups, \( g_1 \) and \( g_2 \). To isolate differences in proposal power, assume \( \hat{x}_{g_1} = \hat{x}_{g_2} \) and \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), but \( \rho_{\ell_1} \neq \rho_{\ell_2} \). These modifications do not qualitatively change the equilibrium characterization. Two identical groups avoid complications arising if one group has access to two legislators, where access to one legislator can affect offers to the other.

Proposition A.1. Consider the modified baseline model with: \( \ell_1 \) and \( \ell_2 \) such that \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), and \( g_1 \) and \( g_2 \) satisfying \( \hat{x}_{g_1} = \hat{x}_{g_2} \). For all \( \alpha \in [0,1] \), \( \rho_{\ell_2} > \rho_{\ell_1} \) implies \( \frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \).

Proof. It suffices to show that \( \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq 0 \) implies \( \frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \) for \( \alpha \in [0,1] \).

Because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \) and \( \hat{x}_{g_1} = \hat{x}_{g_2} \), we have \( y_{g_1} = y_{g_2} \) and \( z_{\ell_1} = z_{\ell_2} \). Thus, \( m_{g_1} = m_{g_2} \).

Denote \( y = y_{g_1} \), \( z = z_{\ell_1} \), and \( m = m_{g_1} \). Assume \( \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq 0 \). There are five cases.
• **Case 1:** Suppose \( z = \hat{x}_\ell \) and \( y = \hat{y} \). Then,

\[
\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} = \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) - \frac{\partial \tau_\alpha}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\tau_\alpha)}{\partial \tau_\alpha} - \rho_R \frac{\partial u_{g_1}(\tau_\alpha)}{\partial \tau_\alpha} \right)
\]

\[
= \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) + \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( \rho_L \frac{\partial u_{g_1}(-\tau_\alpha)}{\partial \tau_\alpha} + \rho_R \frac{\partial u_{g_1}(\tau_\alpha)}{\partial \tau_\alpha} \right)
\]

(36)

\[
\leq \rho_{\ell_2} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) + \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( \rho_L \frac{\partial u_{g_1}(-\tau_\alpha)}{\partial \tau_\alpha} + \rho_R \frac{\partial u_{g_1}(\tau_\alpha)}{\partial \tau_\alpha} \right)
\]

(37)

\[
= \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha}.
\]

(38)

where (36) follows from \( \frac{\partial \tau_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1}[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R) \right) \); (37) because (i) \( \rho_{\ell_2} > \rho_{\ell_1} \) and (ii) \( \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq 0 \) implies the bracketed expression in (36) is positive; and (38) because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2}, \hat{x}_{g_1} = \hat{x}_{g_2} \), and \( \frac{\partial \tau_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2}[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R) \right) \).

• **Case 2:** Suppose \( z = \tau_\alpha \) and \( y = \hat{y} \). In this case, \( \frac{\partial \tau_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1}[u_M(\hat{y}) - u_M(\tau_\alpha)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R + (1-\alpha)(\rho_{\ell_1} + \rho_{\ell_2})) \right) \) and \( \frac{\partial \tau_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2}[u_M(\hat{y}) - u_M(\tau_\alpha)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R + (1-\alpha)(\rho_{\ell_1} + \rho_{\ell_2})) \right) \). Arguments analogous to Case 1 show \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \). The argument for \( z = -\tau_\alpha \) and \( y = \hat{y} \) is symmetric.

• **Case 3:** Suppose \( z = \hat{x}_\ell \) and \( y = \tau_\alpha \). In this case, \( \frac{\partial \tau_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1}[u_M(\tau_\alpha) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2})) \right) \) and \( \frac{\partial \tau_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2}[u_M(\tau_\alpha) - u_M(\hat{x}_\ell)]}{\partial u_M[\tau_\alpha]} \left( 1 - \delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2})) \right) \). Arguments analogous to Case 1 show \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \). The argument for \( z = \hat{x}_\ell \) and \( y = -\tau_\alpha \) is symmetric.
• **Case 4:** Suppose \( z = \pi_\alpha \) and \( y = -\pi_\alpha \). In this case, 
\[
\frac{\partial \pi_\alpha}{\partial \alpha_1} = \frac{\delta \rho_1 [u_M(-\pi_\alpha) - u_M(\pi_\alpha)]}{\delta u_M(x_\alpha)[1-\delta(\rho_L+\rho_R+\rho_{\ell_1}+\rho_{\ell_2})]}
\]
and 
\[
\frac{\partial \pi_\alpha}{\partial \alpha_2} = \frac{\delta \rho_2 [u_M(-\pi_\alpha) - u_M(\pi_\alpha)]}{\delta u_M(x_\alpha)[1-\delta(\rho_L+\rho_R+\rho_{\ell_1}+\rho_{\ell_2})]}.\]
Arguments analogous to Case 1 show \( \frac{\partial U_{E_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial U_{E_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} \). The argument for \( z = -\pi_\alpha \) and \( y = \pi_\alpha \) is symmetric.

• **Case 5:** Suppose \( z = \pi_\alpha \) and \( y = \pi_\alpha \). Then, 
\[
\frac{\partial U_{E_2}(\theta)}{\partial \alpha_2}|_{\alpha_2=\alpha} = \frac{\partial U_{E_1}(\theta)}{\partial \alpha_1}|_{\alpha_1=\alpha} = 0.\]
The argument for \( z = -\pi_\alpha \) and \( y = -\pi_\alpha \) is symmetric. 

\[\square\]
Appendix B  Equivalence of Outcome Distribution

A stationary strategy profile \( \sigma = (\lambda, \pi, \varphi, \nu) \) is a stationary legislative lobbying equilibrium if it satisfies four conditions. First, for all \( g \in N_G^L \) and \( \ell \in N^L_g \), \( \lambda_g^\ell \) places probability one on

\[
\arg\max_{(y,m)} \nu_{\sigma}(y) u_g(y) + [1 - \nu_{\sigma}(y)]((1 - \delta)u_g(q) + \delta \tilde{V}_g(\sigma)) - m
\]

\[
\text{s.t. } \nu_{\sigma}(y) u_{\ell}(y) + [1 - \nu_{\sigma}(y)]((1 - \delta)u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)) + m \geq \int_X \left[ \nu_{\sigma}(x) u_{\ell}(x) + [1 - \nu_{\sigma}(x)]((1 - \delta)u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)) \right] \pi_\ell(dx). \tag{39}
\]

Second, for all \( \ell \in N^L \) and \((y,m)\) \( W \),

\[
\nu_{\sigma}(y) u_{\ell}(y) + [1 - \nu_{\sigma}(y)]((1 - \delta)u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)) + m > \int_X \left[ \nu_{\sigma}(x) u_{\ell}(x) + [1 - \nu_{\sigma}(x)]((1 - \delta)u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)) \right] \pi_\ell(dx). \tag{40}
\]

implies \( \varphi_\ell(y,m) = 1 \) and the opposite strict inequality implies \( \varphi_\ell(y,m) = 0 \). Third, for all \( \ell \in N^L \),

\[
\pi_\ell \left( \arg\max_{x \in X} \nu_{\sigma}(x) u_{\ell}(x) + [1 - \nu_{\sigma}(x)]((1 - \delta)u_{\ell}(q) + \delta \tilde{V}_\ell(\sigma)) \right) = 1. \tag{41}
\]

Finally, for all \( i \in N^V \) and \( x \in X \), \( u_i(x) > (1 - \delta)u_i(q) + \delta V_i(\sigma) \) implies \( \nu_i(x) = 1 \) and the opposite strict inequality implies \( \nu_i(x) = 0 \).\(^{27}\)

Lemma B.1 shows surplus lobby payments never happen in equilibrium.

Lemma B.1. In every stationary legislative lobbying equilibrium, for all \( \ell \in N^L \) every

\(^{27}\)Thus, voting strategies are stage-undominated (Baron and Kalai, 1993; Banks and Duggan, 2006a).
\((y, m) \in \text{supp}(\lambda_g^\ell)\) satisfies

\[
\overline{\nu}_\sigma(y) u_\ell(y) + [1 - \overline{\nu}_\sigma(y)][(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m
= 
\int_X \left[ \overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{42}
\]

The proof of Lemma B.1 is straightforward and omitted.

From (5), recall \(\xi_\ell(\alpha; \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda_g^\ell(dw)\). Define

\[
\tilde{\chi}(X') = \sum_{\ell \in N_L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_{X'} \overline{\nu}_\sigma(x) \pi_\ell(dx) + \alpha_\ell \int_{X' \times \mathbb{R}_+} \varphi_\ell(y, m) \overline{\nu}_\sigma(y) \lambda_g^\ell(dw) \right), \tag{43}
\]

the probability some \(x \in X' \subseteq X\) is passed in a given period under \(\sigma\). Next, define

\[
\check{\chi} = \sum_{\ell \in N_L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_X [1 - \overline{\nu}_\sigma(x)] \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m) [1 - \overline{\nu}_\sigma(y)] \lambda_g^\ell(dw) \right), \tag{44}
\]

the probability of a failed proposal in a given period under \(\sigma\).

Following Banks and Duggan (2006a), each player’s continuation value can be expressed as a function of a common lottery over policy, denoted \(\chi^\sigma\). Using (43) and (44), define \(\chi^\sigma\) so that for all measurable \(X' \subseteq X\): (i) if \(q \notin X'\), then \(\chi^\sigma(X') = \frac{\tilde{\chi}(X')}{1 - \delta \check{\chi}}\), and (ii) if \(q \in X'\), then \(\chi^\sigma(X') = \frac{\check{\chi}(X') + (1 - \delta) \tilde{\chi}}{1 - \delta \check{\chi}}\).

Set \(V^\text{den}(\sigma) = 1 - \delta \check{\chi}\) and define

\[
V^\text{num}_i(\sigma) = \sum_{\ell \in N_L} \rho_\ell \left( \xi_\ell(\alpha; \sigma) \int_X \left[ \overline{\nu}_\sigma(x) u_i(x) + [1 - \overline{\nu}_\sigma(x)] (1 - \delta) u_i(q) \right] \pi_\ell(dx) \\
+ \alpha_\ell \int_W \varphi(y, m) \left[ \overline{\nu}_\sigma(y) u_i(x) + [1 - \overline{\nu}_\sigma(y)] (1 - \delta) u_i(q) \right] \lambda_g^\ell(dw) \right). \tag{45}
\]

For each \(i \in N^V\), \(i\)'s continuation value defined in (6) satisfies \(V_i(\sigma) = \frac{V^\text{num}_i(\sigma)}{V^\text{den}(\sigma)}\). Then we can
express $V_i(\sigma)$ as a lottery over policy, $V_i(\sigma) = \int_X u_i(x) \chi^\sigma(dx)$.

The policy lottery $\chi^\sigma$ is common to all players, but committee members may receive payment and interest groups may make payments. Define

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \int_W m \varphi_\ell(y, m) \lambda^\ell_g(dw),$$

which is $\ell$'s expected lobby payment in each period until passage. For $\ell \in N^L$, re-arranging (7) yields

$$\hat{V}_\ell(\sigma) = \frac{V^\text{num}_\ell(\sigma) + \hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)} = \int_X u_\ell(x) \chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)}. \quad (46)$$

Similarly, for $g \in N^G$ rearranging (8) yields

$$\hat{V}_g(\sigma) = \frac{V^\text{num}_g(\sigma) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)} = \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N^L_g} \frac{\hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)}. \quad (47)$$

Finally, define

$$\tilde{U}_\ell(\sigma) = \int_X \left[ \varphi_\sigma(x) u_\ell(x) + \left( 1 - \varphi_\sigma(x) \right) \left( (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right) \right] \pi_\ell(dx),$$

which is $\ell$'s expected dynamic payoff under $\sigma$ conditional on being recognized as the proposer and rejecting $g_\ell$'s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium $\sigma$ such that $\chi^\sigma$ is degenerate on $q$.

**Proof.** Let $\sigma$ denote an equilibrium. To show a contradiction, assume $\chi^\sigma(q) = 1$. Thus, $V_M(\sigma) = u_M(q)$, which implies $u_M(q) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)$ and therefore $q \in A(\sigma)$. 

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Without loss of generality, assume $q > 0$.

By assumption, there exists $\ell \in N^L$ such that $\hat{x}_\ell < q$ and at least one of $\hat{x}_g \leq q$ or $\alpha_\ell < 1$ holds. If $\alpha_\ell < 0$, then it is straightforward to show that $\ell$ must have a profitable deviation, a contradiction.

For the other case, suppose $\hat{x}_\ell < q$, $\hat{x}_g \leq q$, and $\alpha_\ell = 1$. Note that $u_{g\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$ is $g_\ell$'s expected dynamic payoff from any offer $(y, m)$ such that $\nu_\sigma(y) = 1$, $\varphi_\ell(y, m) = 1$, and $\ell$ is indifferent between accepting and rejecting. We have $\hat{y}_\ell = \arg \max_{y \in X} u_{g\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$ and $\hat{y}_\ell < q$. Strict concavity and continuity imply existence of $\varepsilon > 0$ and $y^\varepsilon < q$ such that $\nu_\sigma(y^\varepsilon) = 1$, $\varphi_\ell(y^\varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1$, and

$$u_{g\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \tilde{U}_\ell(\sigma) - \varepsilon > u_{g\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma)$$

$$\geq u_{g\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) - \delta \left( \sum_{j \in N^L_g} \frac{\hat{m}_j(\sigma)}{\hat{V}_{den}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{\hat{V}_{den}(\sigma)} \right),$$

where (50) follows from $\sum_{j \in N^L_g} \frac{m_j(\sigma)}{\tilde{V}_{den}(\sigma)} \geq \frac{m_\ell(\sigma)}{\tilde{V}_{den}(\sigma)}$. The RHS of (49) is weakly greater than $g_\ell$'s expected payoff from lobbying $\ell$ to $q$ if $\nu_\sigma(q) = 1$; and (50) is weakly greater than $g_\ell$'s expected payoff from lobbying $\ell$ to any $y'$ such that $\nu_\sigma(y') = 0$. Thus, $g_\ell$ must have a profitable deviation, a contradiction. \qed

**Lemma B.3.** Let $\sigma$ denote a stationary legislative lobbying equilibrium. For all $\ell \in N^L$ there exists $(y, m) \in X \times \mathbb{R}_+$ such that $\nu_\sigma(y) = 1$ and $g_\ell$ strictly prefers $(y, m)$ to any $(y', m')$ such that $\nu_\sigma(y') = 0$.

**Proof.** Fix an equilibrium $\sigma$. Let $\chi^q$ denote a probability distribution degenerate on $q$. Define the continuation distribution following rejection under $\sigma$ as $\chi = (1 - \delta)\chi^q + \delta \chi^\sigma$, which is non-degenerate because $\delta \in (0, 1)$ and $\chi^\sigma(q) < 1$ by Lemma B.2.

For every player $k \in N$, the expected dynamic policy payoff from a rejected policy
proposal satisfies

$$(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).$$

Let $x^\sigma$ denote the mean of $\chi$. Since $u$ is strictly concave and $\chi$ is non-degenerate, Jensen’s Inequality implies

$$u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma). \quad (51)$$

Consider $\ell \in N^L$. First, assume $\phi_\ell(y, m) = 1$ whenever $\ell$ is indifferent. The condition for $g_\ell$ to strictly prefer $(y, m)$ such that $\nu_\sigma(y) = 1$, rather than $(y', m')$ such that $\nu_\sigma(y') = 0$, is

$$u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\sigma).$$

Equivalently,

$$u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma). \quad (52)$$

Notice that

$$\tilde{V}_{g_\ell}(\sigma) + \tilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{\ell' \in N^L_\ell} \frac{\tilde{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \quad (53)$$

$$\leq V_{g_\ell}(\sigma) - \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \quad (54)$$

$$= V_{g_\ell}(\sigma) + V_\ell(\sigma), \quad (55)$$

where (53) follows from substituting for $\tilde{V}_\ell(\sigma)$ and $\tilde{V}_{g_\ell}(\sigma)$ using (46) and (47); and (54) from

$$\sum_{\ell' \in N^L_\ell} \frac{\tilde{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}.$$

By (51), $\nu_\sigma(x^\sigma) = 1$ follows because $u_M(x^\sigma) > (1 - \delta)u_M(q) + \delta V_M(\sigma)$. Furthermore, (51) implies $u_{g_\ell}(x^\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma)$ and $u_\ell(x^\sigma) > (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma)$. Thus, (55)
implies that (52) holds because

\[ u_{g_\ell}(x^\sigma) + u_{\ell}(x^\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta V_{g_\ell}(\sigma) \]

\[ \geq (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_{\ell}(q) + \delta \tilde{V}_{\ell}(\sigma), \]

Next, assume \( \varphi_\ell(x^\sigma, m) < 1 \) for \( m \) such that \( \ell \) is indifferent between accepting \((x^\sigma, m)\) and rejecting. For sufficiently small \( \varepsilon > 0 \), \( \varphi_\ell(x^\sigma, m + \varepsilon) = 1 \) and the preceding argument implies \( g_\ell \) strictly prefers \((x^\sigma, m + \varepsilon)\) over any \((y', m')\) such that \( \nu_\sigma(y') = 0 \).

Lemma B.4. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

Proof. Let \( \sigma \) be an equilibrium. By Duggan (2014), \( M \) is decisive. Quadratic utility and \( \hat{x}_M = 0 \neq q \) together imply \( A(\sigma) = \{ x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma) \} \) is a closed, non-empty interval symmetric about 0. Let \( A(\sigma) = [\hat{x}(\sigma), \bar{x}(\sigma)] \). Then \( x \in (\hat{x}(\sigma), \bar{x}(\sigma)) \) implies \( \nu_\sigma(x) = 1 \).

Fix \( \ell \in N^L \). By Lemma B.2, \( \chi^\sigma(q) < 1 \). Lemma B.3 implies existence of \((y, m) \in W\) such that \( \nu_\sigma(y) = 1 \) and \( g_\ell \) strictly prefers \((y, m)\) over all \((y', m')\) with \( \nu_\sigma(y') = 0 \). Thus, \( y \in A(\sigma) \) for all \((y, m) \in \text{supp}(\lambda_{g_\ell})\). Without loss of generality, assume \( \nu_\sigma(\hat{x}(\sigma)) < 1 \). It suffices to check two cases.

- **Case 1:** If \( \hat{x}_\ell \leq -\bar{x}(\sigma) \) and \( u_{\ell}(\hat{x}(\sigma)) > (1 - \delta)u_{\ell}(q) + \delta \tilde{V}_{\ell}(\sigma) \), then \( x \in A(\sigma) \) for all \( x \in \text{supp}(\pi_\ell) \). Because \( u_\ell \) is strictly concave and continuous, and \( \nu_\sigma(\hat{x}(\sigma)) < 1 \), there exists \( \varepsilon > 0 \) such that \( \ell \) has a profitable deviation to \( -\bar{x}(\sigma) + \varepsilon \), a contradiction.

- **Case 2:** Assume \( \hat{y}_\ell \leq -\bar{x}(\sigma) \). Continuity, Lemma B.3, and \( \nu_\sigma(\bar{x}(\sigma)) < 1 \) imply existence of \( \varepsilon, \varepsilon' > 0 \) such that \( g_\ell \) has a profitable deviation to \((y', m') = (\bar{x}(\sigma) + \varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(\bar{x}(\sigma) + \varepsilon) + \varepsilon') \), a contradiction.

It follows that either \( \sigma \) must involve deferential voting, or \( \sigma \) is equivalent in outcome distribution to an equilibrium with deferential voting.
Lemma B.5. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

Proof. Let $\sigma$ denote an equilibrium. By Lemma B.4, we can assume $\nu_\sigma(x) = 1$ iff $x \in A(\sigma)$. Fix $\ell \in N^L$ and define $y^*_g = \arg\max_{y \in A(\sigma)} u_{\ell}(y) + u_{\ell}(z_{\ell}; \sigma) - \tilde{U}_\ell(\sigma)$, which is uniquely defined, and $m^*_g = \tilde{U}_\ell(\sigma) - u_{\ell}(y^*_g)$.

By Lemma B.2, $\chi^{\sigma}(q) < 1$. For sufficiently small $\varepsilon > 0$, Lemma B.3 implies $g$ strictly prefers $(y^*_g, m^*_g + \varepsilon)$ over every $(y', m')$ such that $y' \notin A(\sigma)$. Thus, if $\pi_\ell$ is not degenerate on $y^*_g$ and $\varphi_\ell(y^*_g, m^*_g) < 1$, then there exists $\varepsilon > 0$ such that $g_\ell$ has a profitable deviation to $(y^*_g, m^*_g + \varepsilon)$, a contradiction. Thus, $\sigma$ must satisfy either (i) $\pi_\ell(y^*_g) = 1$, or (ii) $\lambda^\ell_\sigma(y^*_g, m^*_g) = 1$ and $\varphi_\ell(y^*_g, m^*_g) = 1$, as desired.

A strategy profile $\sigma$ is no-delay if $\nu_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$ and $\nu_\sigma(y) = 1$ for all $(y, m) \in \text{supp}(\lambda^\ell_\sigma)$.

Lemma B.6. Every stationary legislative lobbying equilibrium is no-delay.

Proof. Fix an equilibrium $\sigma$. By Lemma B.2, $\chi^{\sigma}(q) < 1$. Thus, Lemma B.3 implies $g$ strictly prefers some $(y, m) \in W$ such that $\nu_\sigma(y) = 1$. Lemma B.4 implies we can assume $\nu_\sigma(x) = 1$ iff $x \in A(\sigma)$. Lemma B.5 implies we can assume all $\ell \in N^L$ use deferential acceptance strategies.

For each $\ell \in N^L$, the preceding observations and Lemma B.1 imply $\lambda^\ell_\sigma$ puts probability one on $(y^*, m^*)$ such that $y^* = \arg\max_{y \in A(\sigma)} u_{\ell}(y) + u_{\ell}(z_{\ell}; \sigma)$, which is unique. Lemmas B.4 and B.5 imply we can assume $\nu_\sigma(y^*) = 1$ and $\varphi_\ell(y^*, m^*) = 1$.

It remains to verify that $z_{\ell} \notin A(\sigma)$ cannot be optimal for any $\ell \in N^L$. To show a contradiction, assume proposing $z_{\ell} \notin A(\sigma)$ is optimal for some $\ell \in N^L$. Let $z^* = \arg\max_{x \in A(\sigma)} u_{\ell}(x)$. There are two steps. Step 1 establishes useful properties of $\ell$’s preferences over lotteries. Step 2 shows a contradiction.

Step 1: Recall the continuation lottery induced by $\sigma$, denoted $\chi = (1 - \delta)\chi^{\sigma} + \delta \chi^{\sigma}$ with
mean $x^\sigma$. Jensen’s inequality implies $u_i(x^\sigma) > \int_X u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(x)$ for all $i \in N$, so $x^\sigma \in \text{int}A(\sigma)$.

Next, let $\chi^{z^*}$ denote the policy lottery nearly equivalent to $\chi$, but shifting probability $\frac{\delta \rho_\ell \alpha_\ell}{V_{\text{den}}(\sigma)}$ from $y^*$ to $z^*$. Let $x^{z^*}$ denote the mean of $\chi^{z^*}$. For all $i \in N$, Jensen’s inequality implies

$$u_i(x^{z^*}) > \int_X u_i(x) \chi^{z^*}(dx) = (1 - \delta)u_i(q) + \delta V_i(x) - \frac{\delta \rho_\ell \alpha_\ell u_i(y^*)}{V_{\text{den}}(\sigma)} + \frac{\delta \rho_\ell \alpha_\ell u_i(z^*)}{V_{\text{den}}(\sigma)}.$$

Moreover, $x^{z^*}$ is located weakly between $x^\sigma$ and $z^*$, implying $x^{z^*} \in A(\sigma)$.

**Step 2:** Since $z_\ell \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - u_\ell(y^*)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} - u_\ell(y^*).$$

(56)

Using (45), $\tilde{m}_\ell(\sigma)$ is expressed recursively as

$$\tilde{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} - u_\ell(y^*) \right)$$

$$= \frac{\rho_\ell \alpha_\ell V_{\text{den}}(\sigma)}{V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell} \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*) \right).$$

(57)

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \tilde{V}(\sigma)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell},$$

(58)

(59)

where (59) follows from the definition of $\tilde{V}_\ell(\sigma)$ and using (57) to substitute for $\tilde{m}_\ell(\sigma)$.

Next, we have $V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell > 0$, where the first inequality follows because Lemma B.3 implies all lobby offers are accepted and passed.
under $\sigma$, so $V_{\text{den}}(\sigma) \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j)$; and the second inequality follows from $\delta[\rho_\ell \alpha_\ell + \sum_{j \in N^L} \rho_j (1 - \alpha_j)] < 1$. Rearranging and simplifying (59),

$$
0 \leq V_{\text{den}}(\sigma) \left( (1 - \delta) u_\ell(q) + \delta V_\ell(\sigma) \right) - \delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*) \left( V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \right)
$$

$$
\propto (1 - \delta) u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta \rho_\ell \alpha_\ell [u_\ell(y^*) - u_\ell(z^*)]}{V_{\text{den}}(\sigma)} - u_\ell(z^*)
$$

$$
= \int_X u_\ell(x) \chi^{z^*}(dx) - u_\ell(z^*),
$$

a contradiction because $u_\ell(z^*) \geq u_\ell(x^*) > \int_X u_\ell(x) \chi^{x^*}(dx)$.

Lemma B.7. Every stationary legislative lobbying equilibrium is such that $\lambda_g$ is degenerate for all $g \in N^G$ and $\pi_\ell$ is degenerate for all $\ell \in N^L$.

Proof. Let $\sigma$ denote an equilibrium. By Duggan (2014), $A_M(\sigma) = A(\sigma)$, which is nonempty, compact and convex.

First, consider $g \in N^g$ and $\ell \in N^L_g$. Recall $\tilde{U}_\ell(\sigma)$ from (48). Lemmas B.1 and B.6 imply $\lambda^\ell_g$ puts probability one on the unique $(y^*, m^*)$ satisfying $y^* = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$, and $m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*)$.

Second, consider $\ell \in N^L$. Lemma B.6 implies $\pi_\ell$ puts probability one on $x^* = \arg\max_{x \in A(\sigma)} u_\ell(x)$, which is unique.

Proposition 1.2. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

Proof. Follows from Lemmas B.4 - B.7.
Appendix C  Partitioning Moderates & Extremists

Consider $\ell \in N^L$. First, I define a function $\zeta^\ell$ that relates to $M$’s equilibrium voting decision. Then, Lemmas C.3 - C.6 characterize $\zeta^\ell$. Finally, Lemma 2 delivers a partitional characterization on $\hat{x}_g$ that facilitates Proposition 4.

Preliminaries to define $\zeta^\ell$. Recall $\bar{\pi}(0) = \bar{\pi}(\hat{x}_g)$ for $\hat{x}_g = 0$. Let $\hat{D}_{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \bar{\pi}(0), j \neq \ell \}$ and $\hat{D}_{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \bar{\pi}(0), j \neq \ell \}$. Next, set $D_{\ell,y} = \{|y| : y \in \hat{D}_{\ell,y} \}$ and $D_{\ell,x} = \{|x| : x \in \hat{D}_{\ell,x} \}$. Define $D^\ell$ as the unique elements of $D_{\ell,y} \cup D_{\ell,x} \cup \{\bar{\pi}(0)\}$. Let $K^\ell + 1 = |D^\ell|$. Denote the $k$-th element of $D^\ell$ as $d^\ell_k$. Index elements $k = 0, \ldots, K^\ell$ of $D^\ell$ in ascending order so that $d^\ell_0 = \bar{\pi}(0)$ and $k' > k$ implies $d^\ell_{k'} > d^\ell_k$.

For each $k$ and $j \neq \ell$, let $C^k_j = \mathbb{I}\{\hat{x}_j \in [-d^\ell_k, d^\ell_k]\}$ and $\bar{C}^k_j = \mathbb{I}\{\hat{y}_j \in [-d^\ell_k, d^\ell_k]\}$. Define

$$I^k_j = (1 - \alpha_j)C^k_j u_M(\hat{x}_j) + \alpha_j \bar{C}^k_j u_M(\hat{y}_j)$$

and

$$O^k_j = (1 - \alpha_j)(1 - C^k_j) + \alpha_j(1 - \bar{C}^k_j),$$

suppressing dependence on $\ell$. Let

$$\hat{x}^\ell_k = \left(\frac{1}{\delta \rho^\ell}\left[(1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_k) \left(1 - \delta \sum_{j \neq \ell} O^k_j\right)\right]\right)^{\frac{1}{2}}. \quad (60)$$

Because $d^\ell_0 = \bar{\pi}(0)$, rearranging (60) yields $\hat{x}^\ell_0 = 0$.

**Lemma C.1.** For all $\ell \in N^L$ and each $k = 0, \ldots, K^\ell$, we have

$$\delta \sum_{j \neq \ell} \rho_j I^{k+1}_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^{k+1}_j) = \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^k_j).$$
Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Then,

$$
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1})
$$

$$
= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j O_j^k - \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j O_j^k
$$

$$
= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k)
$$

$$
= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \quad (61)
$$

$$
= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k), \quad (62)
$$

where (61) follows because $u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})$ by construction.

**Lemma C.2.** For all $\ell \in N^L$, $\dot{x}_k^\ell$ strictly increases in $k$.

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Lemma C.1 and $0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell)$ together imply

$$
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) \quad (63)
$$

Thus, $\dot{x}_k^\ell < \dot{x}_{k+1}^\ell$ follows from (60).

**Definition of $\zeta^\ell$.** For $k = 0, \ldots, K^\ell$, define $\overline{x}_k^\ell : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$
\overline{x}_k^\ell(x) = \left( -\frac{(1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \right)^{\frac{1}{2}} \quad (64)
$$

and $\zeta_k^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$
\zeta_k^\ell(x) = u_M(x) - \left( (1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right).
$$

66
By construction, \( \bar{x}^\ell_k(x^\ell_k) = d_k^\ell \) for all \( k \). Adopt the convention \( d_{K^r+1}^\ell = \infty \). Define the piecewise function \( \zeta^\ell : \mathbb{R}_+ \rightarrow \mathbb{R} \) as

\[
\zeta^\ell(x) = \zeta_k^\ell(x) \quad \text{if} \ x \in [d_k^\ell, d_{k+1}^\ell).
\]

**Lemma C.3.** For all \( \ell \in N^L \), \( \zeta^\ell(0) > 0 \) and \( \zeta^\ell(q) \leq 0 \).

**Proof.** Consider \( \ell \in N^L \). First, we have

\[
\zeta^\ell(0) = \zeta^\ell_0(0) = u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(0) + \delta \sum_{j \neq \ell} \rho_j I^0_j + \delta u_M(\bar{x}^\ell_0(0)) \sum_{j \neq \ell} \rho_j O^0_j \right)
\]

\[
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^0_j + \delta u_M(d^\ell_0) \sum_{j \neq \ell} \rho_j O^0_j \right) \quad (65)
\]

\[> 0,\]

where (65) follows from \( u_M(0) = 0 \) and \( \bar{x}^\ell_0(0) = \bar{x}_0 \).

Next, I show \( \zeta^\ell(q) \leq 0 \). Let \( k' \) denote the largest \( k \) such that \( \bar{x}_k^\ell \leq q \).

- **Step 1:** Because \( \bar{x}^{k'}_{k'}(\bar{x}_k^\ell) = d_{k'}^\ell \), we have

\[
u_M(d^\ell_{k'}) = \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(\bar{x}_k^\ell) + \delta \sum_{j \neq \ell} \rho_j I^k_j}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \quad (66)
\]

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^k_j}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \quad (67)
\]

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(d^\ell_{k'}) \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \bar{C}_j^{k'}]}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j} \quad (68)
\]

\[
= \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(d^\ell_{k'}) (1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^k_j)}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_j}, \quad (69)
\]

where (66) follows from rearranging (64); (67) from \( \bar{x}_k^\ell \leq q \); (68) because for all \( j \) the construction of \( I^k_j \) implies \( I^k_j \geq u_M(d^\ell_{k'}) [(1 - \alpha_j)C_j^{k'} + \alpha_j \bar{C}_j^{k'}] \); and (69) because

\[
\sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \bar{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^k_j \quad \text{by construction.}
\]
Rearranging and simplifying (69) yields \( u_M(d'_k) \geq \frac{(1-\delta+\delta\rho_{\ell})u_M(q)}{1-\delta+\delta\rho_{\ell}} = u_M(q) \). Thus,

\[
\sum_{j \neq \ell} \rho_j I_j^{k'} = \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^{k'} u_M(\hat{y}_j) \right] \tag{70}
\]

\[
\geq u_M(d'_k) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'} \right] \tag{71}
\]

\[
= u_M(d'_k)(1 - \rho_{\ell} - \sum_{j \neq \ell} \rho_j O_j^{k'}) \tag{72}
\]

\[
\geq u_M(q)(1 - \rho_{\ell} - \sum_{j \neq \ell} \rho_j O_j^{k'}) \tag{73}
\]

where (70) follows from the definition of \( I_j^{k'} \); (71) from \( u_M(\hat{x}_j) \geq u_M(d'_k) \) if \( C_j^{k'} = 1 \) and \( u_M(\hat{y}_j) \geq u_M(d'_k) \) if \( \tilde{C}_j^{k'} = 1 \); (72) because \( \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_{\ell} - \sum_{j \neq \ell} \rho_j O_j^{k'} \) by construction; and (73) from \( u_M(d'_k) \geq u_M(q) \).

- **Step 2:** We have

\[
u_M(x_{k'}(q)) = \frac{(1 - \delta)u_M(q) + \delta\rho_{\ell}u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta\rho_{\ell}u_M(q) + \delta u_M(q)(1 - \rho_{\ell} - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{74}
\]

\[
= u_M(q), \tag{75}
\]

where (74) follows from Step 1 and (75) from simplifying.

- **Step 3:** To see \( \zeta^{\ell}(q) \leq 0 \), note

\[
\zeta^{\ell}(q) = u_M(q) - \left( (1 - \delta)u_M(q) + \delta\rho_{\ell}u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'} + \delta u_M(x_{k'}(q)) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \leq u_M(q) - \left( (1 - \delta)u_M(q) + \delta\rho_{\ell}u_M(q) + \delta u_M(q)(1 - \rho_{\ell} - \sum_{j \neq \ell} \rho_j O_j^{k'}) + \delta u_M(q) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \tag{76}
\]

\[
= 0, \tag{77}
\]
where (76) follows from Steps 1 and 2.

Lemma C.4. For all $\ell \in N^L$, $\zeta^\ell$ is continuous.

Proof. Consider $\ell \in N^L$ and fix $k$. Because $\pi^\ell_k(x)$ is continuous, $\zeta^\ell$ is continuous over $(\tilde{x}^\ell_k, \tilde{x}^\ell_{k+1})$. It suffices to show $\zeta^\ell_k(\tilde{x}^\ell_{k+1}) = \zeta^\ell_{k+1}(\tilde{x}^\ell_{k+1})$.

First, I establish $d^\ell_{k+1} = \pi^\ell_k(\tilde{x}^\ell_{k+1})$. Rearranging (60) for $k + 1$ yields

$$0 = u_M(d^\ell_{k+1}) \left( 1 - \Delta \sum_{j \neq \ell} \rho_j O^k_j \right) - (1 - \Delta) u_M(q) - \Delta \rho_k u_M(\tilde{x}^\ell_{k+1}) - \Delta \sum_{j \neq \ell} \rho_j I^k_j \label{eq:78}$$

where (78) follows from Lemma C.1. Thus, $u_M(d^\ell_{k+1}) = \frac{(1 - \Delta) u_M(q) + \Delta \rho_k u_M(\tilde{x}^\ell_{k+1}) + \Delta \sum_{j \neq \ell} \rho_j I^k_j}{1 - \Delta \sum_{j \neq \ell} \rho_j O^k_j}$, so $d^\ell_{k+1} = \pi^\ell_k(\tilde{x}^\ell_{k+1})$. Then,

$$\zeta^\ell_k(\tilde{x}^\ell_{k+1}) = u_M(\tilde{x}^\ell_{k+1}) - \left( (1 - \Delta) u_M(q) + \Delta \rho_k u_M(\tilde{x}^\ell_{k+1}) + \Delta \sum_{j \neq \ell} \rho_j I^k_j + \Delta u_M(\pi^\ell_k(\tilde{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O^k_j \right) \label{eq:79}$$

$$= u_M(\tilde{x}^\ell_{k+1}) - \left( (1 - \Delta) u_M(q) + \Delta \rho_k u_M(\tilde{x}^\ell_{k+1}) + \Delta \sum_{j \neq \ell} \rho_j I^{k+1}_j + \Delta u_M(\pi^\ell_{k+1}(\tilde{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O^{k+1}_j \right)$$

$$= \zeta^\ell_{k+1}(\tilde{x}^\ell_{k+1}), \label{eq:80}$$

where (79) follows from Lemma C.1 because $d^\ell_{k+1} = \pi^\ell_k(\tilde{x}^\ell_{k+1})$.

Lemma C.5. For all $\ell \in N^L$, $\zeta^\ell$ is strictly decreasing.

Proof. Consider $\ell \in N^L$ and fix $k$. The proof shows that the derivative of $\zeta^\ell$ is strictly negative at every $x \in (\tilde{x}^\ell_k, \tilde{x}^\ell_{k+1})$. Continuity then implies that $\zeta^\ell$ is strictly decreasing.
Consider $x \in (\hat{x}_k^\ell, \hat{x}_{k+1}^\ell)$. Then

$$
\zeta^\ell(x) = u_M(x) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\bar{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right)
$$

and

$$
\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x \delta \rho_\ell + \frac{2x \delta \rho_\ell (\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}
$$

$$
\propto \delta \rho_\ell + \delta \sum_{j \neq \ell} \rho_j O_j^k - 1
$$

$$
< 0,
$$

where (81) follows from $\frac{\partial u_M(\bar{x}_k^\ell(x))}{\partial x} \frac{\partial x}{\partial x} = -\frac{2x \delta \rho_\ell}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$; and (83) because $\delta \in (0, 1)$ and $\rho_\ell + \sum_{j \neq \ell} \rho_j O_j^k \leq 1$. \hfill \Box

Lemma C.6. For all $\ell \in N^L$, there is a unique $\bar{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}_\ell)$, $\zeta^\ell(\bar{x}_\ell) = 0$, and $\zeta^\ell(x) < 0$ for all $x > \bar{x}_\ell$.

Proof. Consider $\ell \in N^L$. Lemma C.3 implies $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. By Lemma C.5, $\zeta^\ell$ is strictly decreasing. Thus, there is a unique $\bar{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}_\ell)$ and $\zeta^\ell(x) < 0$ for all $x > \bar{x}_\ell$. Lemma C.4 implies $\zeta^\ell(\bar{x}_\ell) = 0$. \hfill \Box

Lemma 2. For all $\ell \in N^L$, $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Otherwise, $A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell]$.

Proof. Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_\ell = \hat{x}_g$.

Part 1. First, suppose $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ and assume $\hat{x}_g \geq 0$ without loss of generality. I show $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Let $k'$ be the largest $k$ such that $\hat{x}_g \leq \hat{x}_{k'}$. Define the strategy profile $\sigma'$ such that it puts probability $\rho_\ell$ on $\hat{x}_g$ and for each $j \neq \ell$ it (i) puts probability $(1 - \alpha_j)\rho_j$ on: $\hat{x}_j$ if $\hat{x}_j \in [-d^k_{k'}, d^k_{k'}]$, $\bar{x}_k^\ell(\hat{x}_g)$ if $\hat{x}_j > d^k_{k'}$, or $-\bar{x}_k^\ell(\hat{x}_g)$ if $\hat{x}_j < -d^k_{k'}$; and (ii) puts probability
Lemma C.6, \( \hat{\sigma} \)] in two steps. Step 1 shows\( \hat{x}_g \not\in (\overline{\pi}_g, \overline{\pi}_g) \) and suppose \( \hat{x}_g \geq 0 \) without loss of generality. I verify \( A(\hat{x}_g) = [\overline{\pi}_g, \overline{\pi}_g] \) in two steps. Step 1 shows \( \overline{\pi}(\hat{x}_g) \geq \overline{\pi}_g \). Step 2 shows \( \overline{\pi}(\hat{x}_g) \leq \overline{\pi}_g \).

**Step 1.** Suppose \( \overline{\pi}(\hat{x}_g) < \overline{\pi}_g \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_{k'} \leq \overline{\pi}(\hat{x}_g) \). Because \( \hat{x}_g \geq \overline{\pi}_g > \overline{\pi}(\hat{x}_g) \), it follows that \( \sigma(\hat{x}_g) \) puts probability \( \rho_{k'} \) on \( \overline{\pi}(\hat{x}_g) \). Thus, \( u_M(\overline{\pi}(\hat{x}_g)) = \frac{(1-\delta)u_M(\hat{x}_g) + \delta \rho_{k'} u_M(\hat{x}_g) + \delta \sum_{j \neq k'} \rho_j T_{j,k'}'}{1-\delta \sum_{j \neq k'} \rho_j O_{j,k'}} \) and rearranging yields \( \zeta(\overline{\pi}(\hat{x}_g)) = 0 \). Lemma C.6 implies \( \overline{\pi}(\hat{x}_g) = \overline{\pi}_g \), a contradiction.

**Step 2.** Suppose \( \overline{\pi}(\hat{x}_g) > \overline{\pi}_g \). If \( \hat{x}_g \geq \overline{\pi}(\hat{x}_g) \), then the argument from Step 1 shows a contradiction. Assume \( \hat{x}_g < \overline{\pi}(\hat{x}_g) \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_{k'} \leq \overline{\pi}(\hat{x}_g) \). Then \( \sigma(\hat{x}_g) \) puts probability \( \rho_{k'} \) on \( \hat{x}_g \). Next, \( M \) optimally accepts \( \hat{x}_g \) under \( \sigma(\hat{x}_g) \) iff \( u_M(\hat{x}_g) \geq \frac{(1-\delta)u_M(\hat{x}_g) + \delta \rho_{k'} u_M(\hat{x}_g) + \delta \sum_{j \neq k'} \rho_j T_{j,k'}'}{1-\delta \sum_{j \neq k'} \rho_j O_{j,k'}} \). Rearranging, this condition is equivalent to \( \zeta(\hat{x}_g) \geq 0 \). By Lemma C.6, this requires \( \hat{x}_g \leq \overline{\pi}_g \), a contradiction.