Access in Legislatures: Who? And How Long?

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Abstract

Which legislators and interest groups form connections that facilitate lobbying? And how durable are those connections? I study a model where access provides opportunities to lobby policy proposals by particular legislators, but can vary in durability. In equilibrium, durable access acts as an indirect form of vote buying/selling by changing each legislator’s expectations about continued policymaking. Thus, it can also indirectly alter policy proposals by legislators not connected to the group. These indirect effects lead non-extremist groups to prioritize temporary access to a range of more centrist legislators, as durable access would polarize proposals enough to outweigh the perk of better lobbying prospects. In contrast, because of beneficial indirect effects these groups prioritize durable access to more extreme co-partisans and intermediate opponents. The results have implications for various tools that groups use to gain access, including campaign contributions and revolving door hiring.

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Special interests and lobbying are unpopular. In recent surveys (Pew Center, 2018 and 2019), over half of US respondents viewed interest groups and lobbyists as a “very big problem” (Bombardini and Trebbi, forthcoming). This view stems from the observation that interest groups lobby key politicians for favorable policies that may push aside majority interests.

An important condition for successful lobbying is access, which provides crucial opportunities for strategic interaction and attention from politicians. Although some fixed factors like geographic connections may facilitate access, groups can also try to increase access using various means including campaign contributions (Fourinaies and Hall, 2017) and revolving door hiring (Blanes i Vidal et al., 2012).\textsuperscript{1} Intuitively, groups will strategically choose which legislators to target and how much access to seek. And groups can also choose whether to seek short-term access or pursue more durable relationships.

To further understand how lobbying influences policymaking, we need to answer a fundamental question: what kinds of connections do interest groups want to form with various legislators? More precisely, will groups connect with relatively extreme politicians, or instead prioritize access to moderate ones? And furthermore, how durable are those connections and how quickly do they solidify?

I study these questions by analyzing a game-theoretic model that introduces strategic, access-seeking interest groups into a legislative bargaining setting. Access allows groups to potentially influence proposals by lobbying. More access makes lobbying more likely. I show that groups seek temporary access to a broad range of legislators, but are more selective with durable access. In equilibrium, durable access to one legislator can spillover to change behavior by other legislators. Moderate groups forgo durable connections with a range of more centrist legislators to avoid indirectly facilitating more extreme proposals. Crucially, this holds true even if groups can increase access for free. The results thus refine the standard intuition that groups always crave more access, and have important normative and empirical implications.

The model has three key features. First, lobbying requires access, reflecting the widespread view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). Second, lobbying allows groups to influence policy by providing resources to shape proposals before they reach the floor, in the spirit of Grossman

\textsuperscript{1}Also see Kalla and Broockman (2015) and Grimmer and Powell (2016) for evidence on contributions. For revolving door hiring, also see Bertrand et al. (2014) and McCrain (2018).
and Helpman (1994). Third, forward-looking legislators anticipate the possibilities of (i) outside influence, (ii) unpredictable changes in agenda control, and (iii) revisiting failed proposals. The rich legislative environment allows heterogeneity in ideology and access durability, but expanding the scope of application for legislative bargaining models also has independent theoretical interest.

First, to isolate the effect of lobbying on policymaking, I analyze a model in which access is durable and exogenous. In equilibrium, groups pull policy in their favored direction whenever access leads to lobbying. However, their influence may be constrained by the need to satisfy a legislative majority. As in other models where bargaining can continue after rejected proposals, the set of policies that a majority will accept today depends on legislator expectations about future proposals that would follow rejection. Unique to this paper, durable access alters these expectations by changing the prospect of future lobbying.

Essentially, in addition to providing potential lobbying opportunities, durable access acts as an indirect form of vote buying/selling on marginal votes. Forward-looking legislators anticipate future lobbying behavior that would follow rejected proposals. Durable access therefore affects each legislator’s reservation value, which is generated endogenously by equilibrium expectations. This effect can change which policies pass. In turn, durable access can change the equilibrium proposals of any legislator constrained by majority approval. From a group’s perspective, this indirect effect can be good or bad, and the magnitude depends on various legislative conditions.

To study the types of connections that form, I extend the baseline model so that groups form connections before policymaking. They choose who to access, and for how long. Specifically, groups can seek temporary, durable, or delayed access. Temporary access only creates opportunities for lobbying in the current period, but durable access ensures such opportunities are available throughout policymaking. Delayed access only provides opportunities in future periods.

The main takeaways are organized around two related questions. First, who do groups want to access? The second question builds on the first to ask: if a group can benefit from multiple kinds of access (i.e., durable, temporary, or delayed) to a particular legislator, which does it prioritize?

For the first question, groups are least selective with temporary access, which avoids indirect effects and simply provides a direct benefit from more lobbying opportunities.

\[ \text{See Grossman and Helpman (2002) for an extensive overview.} \]
today. Groups want short-term connections with any legislator who is not so extreme that lobbying is inconsequential. And there is a set of moderate legislators with whom any group wants temporary access.

Groups are more selective with durable access. The discrepancy arises because it has two effects, as discussed above. First, it provides more lobbying opportunities throughout policymaking, benefiting the group. This extended window for lobbying also produces the second effect: changing policy proposals even when the group does not lobby. This indirect effect, which is absent for temporary access, can harm the group. Thus, groups want durable access to only a subset of those legislators sought for temporary access. For example, moderate groups forgo durable connections with a range of more centrist legislators to avoid indirectly inducing more extreme proposals. When helpful, however, the indirect effect leads groups to prioritize durable access.

Groups are most selective with delayed access, targeting only a subset of legislators sought for durable access. Yet, because indirect effects can help, groups indeed can want delayed access even though it never materializes on the equilibrium path. For example, moderate groups target more extreme co-partisans to indirectly constrain extreme proposals.

Taken together, these results show that groups do not always crave more access. Even if gaining access is free, interest groups prefer not to form durable connections with certain legislators because it may have harmful indirect effects on strategic behavior by other politicians.

For the second question, groups broadly prioritize temporary access to more moderate legislators because durable access has harmful indirect effects. In contrast, they prioritize durable access to relatively extreme copartisans and an intermediate range of opponents because of beneficial indirect effects. Groups never prioritize delayed access.

The analysis sheds light on two additional questions. For each, the answer depends on access durability.

First, do legislators want to price discriminate when giving access? For temporary access, no. Otherwise, yes. This distinction emerges because such price discrimination is entirely driven by indirect effects. From any target legislator’s perspective, the direct effect of access is always zero because equilibrium lobbying exactly compensates for what she would have proposed independently. In contrast, the indirect effect varies with interest group ideology and is typically not zero. For example, when selling durable or delayed access, moderate legislators favor more centrist groups on their side.
of the aisle over more extreme aligned groups.

Second, do different groups ever want to work together? Fully analyzing this question exceeds this paper’s scope, but the main analysis suggests that even groups on opposite sides of a legislator can have strong incentives to cooperate. Specifically, some groups prefer conceding durable or delayed access to an opposing group purely for policy reasons. Although conceding access implies lost lobbying opportunities, this cost is sometimes outweighed by favorable indirect effects through partisan moderation. In contrast, groups never concede temporary access to opposing groups for policy reasons alone: without an indirect effect, conceding only reduces the group’s lobbying opportunities.

The analysis aligns with several stylized facts about access and, more broadly, money in politics. These include the regularities that interest groups: (i) lobby their allies but not their strongest opponents, (ii) sometimes rely on industry associations or coalitions of groups, and (iii) seek access to legislators with substantial agenda power. Moreover, the theory provides implications that further unpack these regularities. For example, the analysis suggests that relatively extreme allies are more likely to be lobbied than relatively moderate allies because groups will prioritize these connections and legislators are more inclined to form them.

The results suggest additional implications. First, to the extent that campaign contributions provide less durable access than revolving door hiring, groups will contribute broadly but hire selectively. Second, observable behaviors might provide indirect evidence about the durability of access gained through different channels, e.g. contributions versus revolving door hiring. Finally, the results on delayed access provide a logic for why groups pursue “hopeless” connections that never appear to facilitate lobbying in practice.

Related Literature

A large literature in formal theory analyzes interest group influence on policy proposals (see Helpman and Persson (2001) and Baron (2006)). These works typically assume that lobbies have exogenous access to legislators. I contribute to this literature by

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3For a recent overview of empirical regularities in lobbying, Bombardini and Trebbi (forthcoming).
4A second literature studies vote buying in legislatures, typically with distributive policies or public goods (e.g., Snyder Jr., 1991; Dal Bó, 2007; Dekel, Jackson and Wolinsky, 2009). In these papers, however, proposals are typically exogenous.
allowing for *endogenous access*, to study which connections groups pursue to facilitate lobbying on policy proposals. The key innovation here is to analyze deliberate variations in *access durability*, and thus investigate whether and when groups prefer short-term connections or long-lasting relationships with legislators.

Although no existing model varies access durability, Levy and Razin (2013) study a setting closer to this paper. They analyze a dynamic model with a one-dimensional policy space and endogenous status quo. In each period, a continuum of groups compete in an all-pay auction for *temporary agenda control*.\(^5\) They characterize conditions for policies to moderate over time. They do not address which connections form, as proposing politicians are not explicitly modeled and thus are implicitly homogeneous in ideological preferences. Additional important differences in my analysis are that lobbying is explicit and bargaining ends when a proposal passes.

I model lobbying explicitly, motivated by the observation that studying access and lobbying in tandem is important to understand political influence (Powell, 2014). Specifically, I follow the quid-pro-quo tradition of Grossman and Helpman (1994) in the vein of, e.g., Martimort and Semenov (2008) and an extension in Açemoglu, Egorov and Sonin (2013). Lobbying has been modeled in other ways, such as groups providing useful information (Austen-Smith, 1995; Bennedsen and Feldmann, 2002)\(^6\) or services to politicians and voters (Hall and Deardorff, 2006). Here, I choose to focus on the cynical form of lobbying as quid-pro-quo because it aligns with a widespread view and underlies public concern about special interests. Moreover, beyond their literal interpretation, quid-pro-quo offers also serve as a tractable reduced form for efforts to shape policy content.

Scholars have also studied access acquisition in static, informational lobbying environments (Austen-Smith, 1995; Lohmann, 1995; Cotton, 2012). In Schnakenberg (2017), groups can buy costly access in a legislature.\(^7\) Access allows groups to provide information that may influence a legislative vote over an *exogenous* policy proposal in a static setting. This is in sharp contrast with this paper, where lobbying takes place in a dynamic, complete information setting and durable access influences *endogenous* policy proposals by changing politicians’ expectations about future bargaining outcomes. Crucially, this implies that groups sometimes optimally forgo free access, a

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\(^5\)See also Yildirim (2007) and Ali (2015) for sequential bargaining models allowing players to purchase temporary agenda access.

\(^6\)Also see, e.g., Schnakenberg (2015) and Awad (2020).

\(^7\)In Schnakenberg and Turner (2019), politicians choose whether to allow access, to signal to voters.
result unique to this paper.

The logic of forgoing durable access connects to moderation results in spatial models of dynamic bargaining with endogenous status quo (Baron, 1996; Buisseret and Bernhardt, 2017; Zápal, 2020). There, to constrain future proposers, legislators prefer proposing more centrist policies today. By not exploiting the full power of their temporary agenda control, they constrain the scale of policy changes if future proposers have substantially different preferences. Here, policymaking ends once a proposal passes, but incentives to forgo durable access arise from a similar desire to constrain potentially distant proposers.

Model of Legislative Bargaining with Lobbying

I introduce the legislative interaction with exogenous access, focusing on durable access that persists throughout policymaking. I show that it can influence policy proposals through several channels. Overall, this approach sets the stage to subsequently study access acquisition, including temporary and delayed. To illustrate the main results, I present a streamlined setting in the main text. The appendix presents a model allowing for more groups and legislators. I discuss several features in the subsequent Model Discussion section.

Players. The legislature consists of a left partisan $L$, a moderate $M$, a right partisan $R$, and a generic legislator $\ell$. There is also an interest group, denoted $g$.

Sequence of Play. Legislators bargain to set policy in the closed and non-empty interval $X \subseteq \mathbb{R}$. Bargaining occurs over an infinite horizon, with periods discrete and indexed $t \in \{1, 2, \ldots \}$. A status quo policy $q \in X$ persists until policy passes. Thereafter, the passed policy remains forever and the strategic interaction ends.

In each period $t$ before a proposal passes, bargaining proceeds in two stages as follows.

Proposal stage. First, the period-$t$ proposer $i_t$ is drawn from probability distribution $\rho = (\rho_\ell, \rho_L, \rho_M, \rho_R)$, where $\rho_j > 0$ is legislator $j$’s recognition probability. If $i_t \neq \ell$, then $g$ is not active and $i_t$ proposes any $x_t \in X$. If $i_t = \ell$, then $g$ can lobby with probability $\alpha \in [0, 1]$, which parameterizes $g$’s access. Without lobbying, $\ell$ simply proposes any $x_t \in X$. With lobbying, $g$ offers $\ell$ a binding contract $(y_t, m_t)$ consisting of policy $y_t \in X$ and transfer $m_t \geq 0$. After observing $g$’s offer, $\ell$ decides whether to accept or reject.
If $\ell$ accepts, then she proposes $x_t = y_t$ and receives $m_t$ from $g$. If $\ell$ rejects, then she can propose any $x_t \in X$ and $g$ keeps $m_t$.

**Voting stage.** Next, $M$ decides whether to accept the proposal. If $M$ accepts, then the proposal passes and bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $M$ rejects, then $q$ persists and active bargaining continues in $t+1$.

This stage distills the essence of a larger majoritarian legislature where $M$ is a decisive median legislator (Banks and Duggan, 2006b). I establish the main results for such a setting in the appendix. Additionally, I abstract from direct vote buying, primarily to isolate considerations related to lobbying over policy details in committee. But substantively, the median ideology is a robust statistic in large legislatures and meaningful vote buying likely requires coordinating deals with several legislators.

**Information.** All features are common knowledge.

**Payoffs.** Each player $i$ has policy preferences represented by quadratic loss function $u_i$ and therefore has associated ideal point $\hat{x}_i \in X$. Thus, $i$’s per-period policy utility from $x \in X$ is $u_i(x) = -(\hat{x}_i - x)^2$. Throughout, I normalize $\hat{x}_M = 0$ and maintain $\hat{x}_\ell \neq \hat{x}_g$. Additionally, to model $L$ and $R$ as staunchly ideological and opposing partisans, I assume $\hat{x}_L < 0 < \hat{x}_R$ and $|q| < \min\{||\hat{x}_L|, \hat{x}_R\}$. These assumptions are not essential, but sharpen key tradeoffs.

If $\ell$ accepts $g$’s offer $(y_t, m_t)$ and $x_t$ is enacted in $t$, then $g$’s period-$t$ payoff is $u_g(x_t) - m_t$ and $\ell$’s period payoff is $u_\ell(x_t) + m_t$.

Cumulative dynamic payoffs are the discounted sum of streams of per-period payoffs. See the appendix for complete expressions. All players have common discount factor $\delta \in (0, 1)$.

**Model Discussion**

I now discuss several features of the baseline model.

**Access.** The baseline setup represents $g$’s access to $\ell$ as $\alpha \in [0, 1]$, the probability $g$ can lobby $\ell$ whenever recognized. Substantively, $\alpha$ can be interpreted as the probability $g$’s lobbyists get a meeting with $\ell$. Alternatively, it could be the proportion of legislators associated with $g$ in an otherwise homogeneous bloc sharing ideal point $\hat{x}_\ell$. The analysis is also similar if access is binary, e.g., $\alpha \in \{0, 1\}$, or represented by a multiplier on $\ell$’s value of transfers.

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8Note that $x_t = q$ if $y_t$ does not pass.
Figure 1: A period with lobbying

Figure 1 illustrates the within-period interaction if $\ell$ proposes and $g$ can lobby. It includes stage payoffs following rejection, and cumulative stage payoffs following acceptance. If $\ell$ does not propose or $g$ cannot lobby, the within-period interaction is analogous to Figure 1 after $\ell$ rejects $g$’s offer.

Additionally, $g$ only has access to $\ell$. I relax this assumption in the appendix, but it reflects that exogenous factors can prevent access. The group simply may not be able to afford access to many different legislators, but other barriers to widespread access include limited geographic ties (Wright, 1989) or strong constituent opposition to the group’s mission or tactics (Stratmann, 1992). Existing models with limited access include Austen-Smith (1995); Lohmann (1995) and Cotton (2012).

Lobbying. The quid-pro-quo offers can be interpreted as a tractable reduced form for various efforts to modify policy content (Powell, 2014). Lobbying in committee to shape the language of bills is a prominent form of outside influence (Kang and You, 2015). For example, groups draft legislation (Schlozman and Tierney, 1986) and frequently present legislators with model bills (Kroeger, 2020). To get these provisions into bills, groups can help legislators in various ways: future employment opportunities (Diermeier, Keane and Merlo, 2005), targeted charitable donations (Bertrand, Bombardini, Fisman and Trebbi, 2020), or facilitating other tasks such as constituent service and fundraising (Hall and Deardorff, 2006).
Analysis of Equilibrium Legislating and Lobbying

I study a refinement of stationary subgame perfect Nash equilibrium following standard equilibrium concepts in the legislative bargaining literature, e.g., Banks and Duggan (2006a). Here, stationarity implies: g’s offers are independent of previous play; ℓ accepts or rejects g’s offers based only on the current terms, and ℓ’s proposals in lieu of acceptance are independent of the preceding history; legislators other than ℓ propose independently of preceding play; and M’s voting decision depends only on the current proposal.

Informally, a stationary legislative lobbying equilibrium requires four conditions. First, g’s policy offer will pass and g cannot profitably deviate to any other offer. Second, legislator ℓ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal. Third, without lobbying, each legislator proposes policy satisfying M and cannot profitably deviate to a different proposal. Fourth, M passes a proposal if and only if she weakly prefers it relative to rejecting and continuing bargaining.

This equilibrium concept is less restrictive than it appears. First, it is without loss of generality to focus on no-delay proposal strategies, i.e., each legislator proposes passable policy and g offers passable policy. Second, although players use straightforward behavioral rules, no player can profitably deviate to any other strategy. Third, it is without loss of generality to assume M passes proposals when indifferent and ℓ accepts g’s offer when indifferent. Finally, imposing that g lobbies whenever possible is innocuous because g can effectively forgo lobbying by offering ℓ’s default proposal without payment.

Equilibrium Behavior

In equilibrium, there is a feedback between proposals and legislative voting, as in, e.g., Banks and Duggan (2006a). To illustrate, consider a legislator recognized to propose. To pass policy, the proposer must anticipate which policies M will accept. That acceptance set depends on M’s expectations about future policymaking, which in

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9See Appendix A for a formal definition.
10In Appendix B, I define stationary mixed strategy legislative lobbying equilibrium and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium in which legislators support proposals when indifferent and accept lobby offers when indifferent.
equilibrium are consistent with proposal strategies. In this section, I show how durable access influences these expectations and thus the acceptance set, thereby affecting equilibrium proposals of ℓ and the partisan legislators, L and R.

To characterize equilibrium behavior more precisely, I proceed in three steps. First, I outline the qualitative features of the equilibrium acceptance set, denoted $A^*$. This outline allows a qualitative description of equilibrium proposals. Finally, I sharpen the characterization using insights from steps 1 and 2.

**Structure of Equilibrium Acceptance Set:** In equilibrium, $M$ passes all proposals exceeding her reservation value. That is, the equilibrium acceptance set is:

$$A^* = \{ x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M^* \},$$

where $V_M^*$ denotes $M$’s equilibrium continuation value immediately after rejecting a proposal. By stationarity, $V_M^*$ is constant over time. Because $\hat{x}_M = 0$ and $u_M$ is quadratic, $A^*$ is a closed interval symmetric about 0, with boundaries where $M$ is indifferent between approving and rejecting. Formally, the upper bound of $A^*$, denoted $\bar{x}^*$, is the positive solution to

$$u_M(x) = (1 - \delta)u_M(q) + \delta V_M^*.$$  

(1)

Symmetry implies $A^* = [-\bar{x}^*, \bar{x}^*]$.

**Qualitative Description of Equilibrium Proposals:** Given $A^*$, each legislator proposes her favorite passable policy whenever recognized: $M$ offers $\hat{x}_M = 0$, legislator $L$ offers $-\bar{x}^*$ and $R$ offers $\bar{x}^*$. The partisans, $L$ and $R$, are always constrained in equilibrium because their ideal policies will not pass. If $\ell$ is recognized and does not accept a lobby offer, either because $g$ cannot lobby or because $\ell$ rejects $g$’s offer, then she proposes $z^* = \arg \max_{x \in A^*} u_\ell(x)$.

Thus far, legislative behavior parallels Banks and Duggan (2006a).

I now describe equilibrium lobbying. There are three broad properties to note. First, $g$ always makes a lobbying offer $\ell$ accepts, as it can always offer $\ell$’s independent proposal without payment, i.e., $(z^*, 0)$. Second, $g$ will not give a surplus transfer and thus makes $\ell$ indifferent between accepting and rejecting. Third, $g$ never lobbies $\ell$ to propose policy outside $A^*$. In principle, $g$ could potentially benefit from paying

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11Appendix A contains explicit expressions for continuation values.

12This property follows from $\bar{x}^* < |q| < \min\{ |\hat{x}_L|, |\hat{x}_R| \}$. 

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for unpassable policy today if tomorrow’s proposer is likely to be an ideological ally who will pass very favorable policy for cheap. But such lobbying is never optimal in equilibrium: $\ell$ shares $g$’s expectations about future proposals, which determine her equilibrium price for proposing unpassable policy and this price is always too high for $g$.\textsuperscript{13}

The three preceding properties imply that $g$’s equilibrium offer is $(y^*, m^*)$ such that

$$y^* \in \arg \max_{y \in A^*} u_g(y) + u_\ell(y) - u_\ell(z^*),$$

(2)

and $m^* = u_\ell(z^*) - u_\ell(y^*)$. Thus, $g$’s policy offer is

$$y^* = \arg \max_{y \in A^*} u_g(y) + u_\ell(y),$$

(3)

where equality follows from uniqueness because $u_g + u_\ell$ is strictly concave and $A^*$ is compact, convex, and nonempty. Because $u_g$ and $u_\ell$ are quadratic, $g$’s optimal unconstrained policy offer is $\hat{y} = \frac{\delta + \delta g}{2}$. If $\hat{y} \in A^*$, then $y^* = \hat{y}$. Otherwise, strict concavity implies that $y^*$ equals the boundary of $A^*$ closest to $\hat{y}$.

\textit{Sharpening the Equilibrium Characterization:} We can sharpen the characterization of $M$’s equilibrium continuation value:

$$V_{\alpha, M}^* = \rho_M u_M(0) + \rho_L u_M(-\bar{\pi}_\alpha^*) + \rho_R u_M(\bar{\pi}_\alpha^*) + \rho_\ell \left( \alpha u_M(y_\alpha^*) + (1 - \alpha) u_M(z_\alpha^*) \right),$$

(4)

where dependence on $\alpha$ is now explicit. Substituting (4) into (1) and rearranging yields explicit boundaries for $A^*_\alpha$:

$$\bar{\pi}_\alpha^* = \left( -\frac{(1 - \delta)u_M(q) + \delta \rho_\ell \left( \alpha u_M(y_\alpha^*) + (1 - \alpha) u_M(z_\alpha^*) \right)}{1 - \delta (\rho_L + \rho_R)} \right)^\frac{1}{\delta}.$$  

(5)

Expression (5) reveals how durable access can affect the acceptance set. If $M$ is forward looking, i.e. $\delta > 0$, then $A^*_\alpha$ depends on $\alpha$ if either of the following is in $(-\bar{\pi}_\alpha^*, \bar{\pi}_\alpha^*)$: $g$’s policy offer, $y_\alpha^*$, or $\ell$’s independent proposal, $z_\alpha^*$. Durable access can therefore influence proposals by $L$ and $R$, even though they are not lobbied, because they are constrained

\textsuperscript{13} See Appendix B for technical details.
by $A^*_\alpha$.

Although complicated by lobbying, the model can be reinterpreted as a one-dimensional bargaining environment where $\ell$ has recognition probability $(1 - \alpha)\rho_\ell$ and there is an additional legislator at $\hat{y}$ with recognition probability $\alpha \rho_\ell$. After modifying the legislature to include this additional proposer representing the effect of $g$’s lobbying, legislators propose acceptable bills closest to their ideal point. Applying insights from Cho and Duggan (2003) to this fictitious enlarged legislature implies a unique equilibrium outcome distribution.

Proposition 1 collects the preceding observations and establishes three properties. First, a stationary legislative lobbying equilibrium exists. Second, this class of equilibria has a unique outcome distribution. Third, equilibrium behavior has a sharp characterization: the acceptance set is a closed interval symmetric around $\hat{x}_M = 0$; $L$ proposes the leftmost passable policy, $R$ proposes the rightmost, and $M$ proposes 0; without lobbying, $\ell$ proposes the passable policy closest to $\hat{x}_\ell$; and when lobbying, $g$ successfully lobbies $\ell$ to propose the passable policy closest to $\hat{y}$. Because of the uniqueness property, henceforth I drop qualifiers and say equilibrium. Figure 2 illustrates Proposition 1 for a hypothetical legislature.

**Proposition 1.** A stationary legislative lobbying equilibrium exists and every such equilibrium has the same outcome distribution. In equilibrium,

(i) the acceptance set is $A^*_\alpha = [-\overline{x}_\alpha, \overline{x}_\alpha]$, where $0 < \overline{x}_\alpha < |q|$;

(ii) $M$ proposes 0, $R$ proposes $\overline{x}_\alpha$, and $L$ proposes $-\overline{x}_\alpha$;

(iii) when lobbied, $\ell$ proposes the policy in $A^*_\alpha$ closest to $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$; and

(iv) otherwise, $\ell$ proposes the policy in $A^*_\alpha$ closest to $\hat{x}_\ell$.

Seek Access? And for How Long?

I now endogenize access, focusing primarily on how $g$’s preferences over access depend on durability and $\ell$’s ideology. The key results can be conveyed by studying a one-time choice of access prior to bargaining. Later on, I discuss additional insights that emerge if $g$ can repeatedly choose access. I analyze $\hat{x}_g \geq 0$ without loss of generality. And on a
Figure 2: Equilibrium characterization

Figure 2 illustrates equilibrium behavior for a hypothetical legislature. The bold interval is the acceptance set, \( A^*_\alpha \). Arrows point from legislator ideal points to proposals. If legislator \( \ell \) is recognized, then with probability \( \alpha \) she is lobbied and proposes the acceptable policy closest to \( \hat{y} = \hat{x}_g + \hat{x}_\ell \), and otherwise she proposes the acceptable policy closest to \( \hat{x}_\ell \).

technical note, the uniqueness property in Proposition 1 avoids additional equilibrium selection.

To vary the durability and speed of access, I consider three distinct possibilities: temporary, durable, and delayed. I model each starkly to highlight the key forces and leverage the baseline analysis. The results are not knife-edge, however, and later on I discuss intermediate cases.

**Definition 1.** The interest group has:

(i) temporary access \( \alpha_{\text{temp}} = \alpha \) if it can lobby with probability \( \alpha \) in \( t = 1 \) and probability 0 in \( t = 2, \ldots, \infty \);

(ii) durable access \( \alpha_{\text{dur}} = \alpha \) if it can lobby with probability \( \alpha \) in \( t = 1, \ldots, \infty \); and

(iii) delayed access \( \alpha_{\text{del}} = \alpha \) if it can lobby with probability 0 in \( t = 1 \) and probability \( \alpha \) in \( t = 2, \ldots, \infty \).

Temporary access may provide an immediate lobbying opportunity, but certainly not thereafter. The other two kinds of access allow future lobbying opportunities if policymaking is prolonged. Durable access corresponds to the baseline model, determining \( g \)'s chances of lobbying \( \ell \) throughout active bargaining. Delayed access certainly does not provide an immediate lobbying opportunity, but can provide them later on. For simplicity, I model it as the opposite of temporary access, i.e., durable access without the possibility of immediate lobbying.

I begin by studying the extensive margin of access. Does \( g \) want access to \( \ell \), i.e., \( \alpha > 0 \)? And if so, which kind?
To isolate policy considerations and abstract from specific cost functions, I allow $g$ to freely choose access. Specifically, I simply analyze $g$’s ex-ante equilibrium payoff. The qualitative insights are unchanged by standard cost functions. In practice, the cost of acquiring access depends on various factors, including the connections of the group’s lobbyists (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kang and You, 2015), constituent interests in the legislator’s district (Stratmann, 1992), or the group’s number of affiliated voters (Bombardini and Trebbi, 2011).\footnote{La Raja and Schaffner (2015) emphasize that contributions translate into influence differently for different pairs of interest groups and legislators.}

A basic observation is that, regardless of durability, $\alpha$ access affects $g$’s ex-ante payoff only if lobbying alters $\ell$’s proposal, i.e., $y_\alpha^* \neq z_\alpha^*$. On the extensive margin, the key necessary condition is therefore $y_0^* \neq z_0^*$, as $y_\alpha^*$ is constant in $\alpha$ otherwise. Define

$$\overline{\pi} = \left( - \frac{(1 - \delta) u_M(q)}{1 - \delta (\rho_L + \rho_R + \rho \ell)} \right)^{\frac{1}{2}},$$

which does not depend on $\hat{x}_\ell, \hat{x}_g$, or $\alpha$ and, moreover, is not an equilibrium object despite the resemblance to (5). Given $\hat{x}_g, \hat{x}_\ell$ and $\overline{\pi}$, we can precisely characterize whether $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2} \in (-\pi, \pi)$, which fully characterizes whether $y_0^* \neq z_0^*$, using

$$\underline{\chi}(\hat{x}_g) = -2\overline{\pi} - \hat{x}_g \quad \text{and} \quad \overline{\chi}(\hat{x}_g) = \max\{\overline{\pi}, 2\overline{\pi} - \hat{x}_g\}. \quad (7)$$

These cutpoints always satisfy $\underline{\chi}(\hat{x}_g) \leq -\overline{\pi} < \overline{\pi} \leq \overline{\chi}(\hat{x}_g)$.

**Lemma 1.** In equilibrium, $y_0^* \neq z_0^*$ if and only if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), \overline{\chi}(\hat{x}_g))$.

Lemma 1 immediately yields a general necessary condition: $g$ strictly prefers nonzero access of some kind only if $\ell$ is not too extreme.

**Lemma 2.** The interest group strictly prefers nonzero access only if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), \overline{\chi}(\hat{x}_g))$.

Lemma 2 implies $g$ will not pay for access to very extreme legislators. Intuitively, accessing these legislators is never worthwhile because they are so extreme that non-trivial lobbying is too expensive. In contrast, moderate legislators $\hat{x}_\ell \in (-\pi, \pi) \subset (\underline{\chi}(\hat{x}_g), \overline{\chi}(\hat{x}_g))$ are always potential targets. Regardless of $\hat{x}_g$, these legislators are unconstrained if $g$ has low access, so lobbying is always non-trivial as $\alpha$ increases from 0. Thus, any group may want access to moderate legislators.
Temporary Access

With temporary access, $M$’s continuation value from rejecting any first-period proposal equals $V_{M,0}^*$ from the baseline. The acceptance set does not vary with $\alpha_{\text{temp}}$ and always equals $A_0^* = [-\pi_0^*, \pi_0^*]$. Thus, $g$’s ex-ante payoff from $\alpha_{\text{temp}} = \alpha$ is:

$$
\rho_M u_g(0) + \rho_R u_g(\pi_0^*) + \rho_L u_g(-\pi_0^*) + \rho_t \left[ \alpha \left( u_g(y_0^*) + u_\ell(y_0^*) - u_\ell(z_0^*) \right) + (1 - \alpha) u_g(z_0^*) \right],
$$

(8)

where $g$’s equilibrium lobbying utility is $u_g(y_0^*) - m_0^* = u_g(y_0^*) + u_\ell(y_0^*) - u_\ell(z_0^*)$.

Proposition 2 sharpens Lemma 2 for temporary access by fully characterizing the set of $\hat{x}_\ell$ for which $g$ strictly prefers $\alpha\text{temp}>0$.

**Proposition 2.** The interest group strictly prefers nonzero temporary access if and only if $\hat{x}_\ell \in (\chi(\hat{x}_g), \chi(\hat{x}_g))$.

Whenever lobbying is nontrivial, $g$ wants temporary access. Because temporary access disappears after the first-period, it does not alter any proposals by changing the acceptance set. Thus, there are no indirect effects. As seen in (8), temporary access only affects $g$’s probability of receiving the lobbying surplus, $u_g(y_0^*) + u_\ell(y_0^*) - u_\ell(z_0^*)$, at the expense of $\ell$ proposing $z_0^*$. Naturally, this direct effect is strictly positive whenever lobbying is nontrivial. Formally, the marginal effect of $\alpha$ in (8) is $u_g(y_0^*) + u_\ell(y_0^*) - u_\ell(z_0^*) - u_g(z_0^*)$, which is strictly positive if $y_0^* \neq z_0^*$ and zero otherwise.

Durable Access and Delayed Access

Like temporary access, durable access has a direct effect through $g$’s lobbying probability in $t = 1$. Unlike temporary access, however, durable access can also have indirect effects because $\pi_\alpha^*$ can vary with $\alpha$. Intuitively, increasing durable access acts as an indirect form of vote buying/selling by flipping $M$’s vote on policies near the boundaries of the acceptance set. Formally, if $\alpha_{\text{dur}} = \alpha$, then $M$’s continuation value from rejecting first-period proposals equals $V_{M,0}^{*,\alpha}$ from the baseline. The acceptance set is thus equivalent to $A_\alpha^* = [-\pi_\alpha^*, \pi_\alpha^*]$, which can depend on $\alpha$.

Thus, $g$’s ex-ante payoff from $\alpha_{\text{dur}} = \alpha$ is:

$$
\rho_M u_g(0) + \rho_R u_g(\pi_\alpha^*) + \rho_L u_g(-\pi_\alpha^*) + \rho_t \left[ \alpha \left( u_g(y_\alpha^*) + u_\ell(y_\alpha^*) - u_\ell(z_\alpha^*) \right) + (1 - \alpha) u_g(z_\alpha^*) \right],
$$

(9)
Durable access has a beneficial direct effect, like temporary access. But it also has indirect effects that can be good or bad, as there are several channels for \( x^* \) to affect (9): (i) proposals by \( L, R \); and (ii) lobbying profits, through changes in \( y^* \) or \( z^* \). For (i), \( L \) and \( R \) modify their proposals, which changes \( \rho_R u_g(x^*_{\alpha}) + \rho_L u_g(-x^*_{\alpha}) \). This effect can be good or bad for \( g \) depending on how \( x^*_{\alpha} \) and \( -x^*_{\alpha} \) shift relative to \( \hat{x}_g \). If both shift towards \( \hat{x}_g \), then \( g \) benefits, and vice versa if both shift away. If one shifts towards \( \hat{x}_g \) and the other shifts away, then the relative magnitudes of \( \rho_L \) and \( \rho_R \) determine whether \( g \) benefits. For (ii), changing \( y^*_{\alpha} \) or \( z^*_{\alpha} \) can alter \( g \)'s lobbying profit, \( u_g(y^*_{\alpha}) + u_\ell(y^*_{\alpha}) - u_\ell(z^*_{\alpha}) - u_g(z^*_{\alpha}) \). But this profit weakly increases with \( \alpha_{\text{dur}} \). If \( g \) is more centrist than \( \ell \), then \( g \) pays weakly less for the same policy. Otherwise, \( g \) pays weakly more for weakly more favorable policy, but the policy gain always dominates.

A key takeaway is that \( g \)'s desire for durable access depends on relative extremism. We can already make two broad observations. First, \( g \) wants durable access to more extreme legislators on its side of the spectrum because every effect is beneficial. In contrast, durable access to relatively centrist legislators may have harmful indirect effects that counteract the direct benefits.

Delayed access does not change \( g \)'s lobbying probability in \( t = 1 \), but does change this probability in \( t \geq 2 \). Thus, it affects the acceptance set equivalently to durable access. Formally, \( g \)'s ex-ante payoff from \( \alpha_{\text{del}} = \alpha \) is:

\[
\rho_M u_g(0) + \rho_R u_g(x^*_{\alpha}) + \rho_L u_g(-x^*_{\alpha}) + \rho_\ell u_g(z^*_{\alpha}).
\] (10)

In equilibrium, delayed access never actually provides \( g \) with a lobbying opportunity because bargaining ends immediately. Thus, it has no direct effect. It can, however, have indirect effects by changing the proposals of \( L, R \), and \( \ell \). The effect on partisan proposals equals that of durable access because the acceptance set changes identically. Yet, the overall indirect effect can differ because \( g \) cannot lobby immediately: if \( \ell \) proposes first, then delayed access can indirectly change her proposal but not \( g \)'s lobbying surplus. This discrepancy is small at low levels of durable/delayed access, and vanishes as they go to zero.

With a handle on key forces and incentives, I now characterize when \( g \) strictly prefers nonzero access of either kind. To facilitate the analysis, I first partition \( g \)'s
ideology into two cases using $\bar{\mathcal{r}}$ from (6).

**Definition 2.** Interest group $g$ is moderate if $\hat{x}_g < \bar{\mathcal{r}}$. Otherwise, $g$ is extremist.

Next, Lemma 3 shows that Definition 2 fully characterizes whether $g$ can be strictly inside $A_0^*$: moderates can, but extremists cannot. Crucially, this property distinguishes whether durable and delayed access can have unambiguous indirect effects on $g$.

**Lemma 3.** If $g$ is moderate, then there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (x', x')$ implies $\hat{x}_g \in \text{int}A_0^* = (-\bar{\mathcal{r}}_0^*, \bar{\mathcal{r}}_0^*)$. If $g$ is extreme, then $\hat{x}_g \notin \text{int}A_0^*$ for all $\hat{x}_\ell$ and all $\alpha$.

Lemma 3 works as follows. First, if $\hat{x}_\ell = 0$, then (5) and (6) imply $0 < \bar{\mathcal{r}}_0^* < \bar{\mathcal{r}}$. Moreover, $\bar{\mathcal{r}}_0^*$ increases as $\hat{x}_\ell$ shifts away from 0, but $|\hat{x}_\ell|$ eventually surpasses $\bar{\mathcal{r}}_0^*$ at $\bar{\mathcal{r}}$. Thus, $\bar{\mathcal{r}}_0^* = \bar{\mathcal{r}}$ for all $\hat{x}_\ell$ more extreme than $\bar{\mathcal{r}}$. Because moderate groups satisfy $\hat{x}_g \in (-\bar{\mathcal{r}}, \bar{\mathcal{r}})$, we know $\hat{x}_\ell = \hat{x}_g$ implies $\hat{x}_g \in (-\bar{\mathcal{r}}_0^*, \bar{\mathcal{r}}_0^*)$. Continuity ensures this also holds for slightly more centrist $\hat{x}_\ell$. In contrast, extreme groups are always outside $(-\bar{\mathcal{r}}_\alpha^*, \bar{\mathcal{r}}_\alpha^*)$ regardless of $\hat{x}_\ell$ and $\alpha$.

**Moderate groups.** Proposition 3 provides a cutpoint characterization of regions in which a moderate group does and does not want each type of access. Qualitatively, the characterization is similar for both durable and delayed access: moderate groups want to connect with a range of more extreme legislators and an intermediate range of legislators opposite $M$, but they forgo access to legislators in a relatively more centrist range. As noted previously, however, $g$ is more selective with delayed access. Recall from Lemma 2 that $g$ is indifferent if $\hat{x}_\ell \notin (\chi(\hat{x}_g), \chi(\hat{x}_g))$.

**Proposition 3.** Suppose interest group $g$ is moderate. There exist cutpoints satisfying $-\hat{x}_g < x'_{del} < x'_{dur} < x''_{det} < x''_{dur} < \hat{x}_g$ such that $g$ strictly prefers:

(i) no delayed access if $\hat{x}_\ell \in (x''_{del}, \hat{x}_g)$ and nonzero delayed access if $\hat{x}_\ell \in (\chi(\hat{x}_g), x'_{det}) \cup (\hat{x}_g, \chi(\hat{x}_g))$; and

(ii) no durable access if $\hat{x}_\ell \in (x''_{dur}, \hat{x}_g)$ and nonzero durable access if $\hat{x}_\ell \in (\chi(\hat{x}_g), x'_{dur}) \cup (\hat{x}_g, \chi(\hat{x}_g))$.

To discuss Proposition 3, I focus on durable access because it has direct and indirect effects. The logic for delayed access works entirely through indirect effects similar to those of durable access.

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16See Lemma A.1 in Appendix A.
First, moderate groups forgo durable access to an interval of more centrist legislators, \( \hat{x}_\ell \in (x''_{\text{dur}}, \hat{x}_g) \). In this case, \( M \) prefers \( \ell \)'s independent proposal, \( z_{\alpha}^* \), to \( g \)'s policy offer, \( y_{\alpha}^* \). Increasing \( \alpha_{\text{dur}} \) makes \( M \) more pessimistic about future policymaking and expands the acceptance set. If \( \hat{x}_\ell \) and \( \hat{x}_g \) are close, then \( \hat{x}_g \) is strictly inside the acceptance set for \( \alpha_{\text{dur}} = 0 \), i.e., \( \hat{x}_g < \pi_0^* \). Because the partisans \( L \) and \( R \) are constrained, their proposals shift outward and away from \( \hat{x}_g \). If \( \ell \) and \( g \) are close enough, the indirect effect is unambiguously bad for \( g \) and dominates the direct gain from better lobbying prospects. Specifically, \( g \)'s marginal benefit from increasing \( \alpha_{\text{dur}} \) is less than the marginal loss from more extreme partisan proposals. This relationship holds even though these marginal effects are small for \( \hat{x}_\ell \) close to \( \hat{x}_g \). Figure 3 illustrates the nature of these effects.

![Figure 3: Forgoing durable access to more centrist legislators](image)

Figure 3 illustrates why a moderate group, \( g \), forgoes durable access (\( \alpha_{\text{dur}} = 0 \)) to legislator \( \ell \) if \( \hat{x}_\ell \in (x''_{\text{dur}}, \hat{x}_g) \). Part (a) displays equilibrium behavior for \( \alpha_{\text{dur}} = 0 \). Part (b) illustrates \( \alpha_{\text{dur}} = \alpha > 0 \). In each, the bold interval is the acceptance set. Increasing \( \alpha \) makes lobbying more likely, which worsens \( M \)'s expectations, and expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If \( \hat{x}_g \) and \( \hat{x}_\ell \) are close, then the loss from more extreme partisan proposals dominates and \( g \) prefers \( \alpha_{\text{dur}} = 0 \).

Second, moderate groups crave access to an interval of more extreme co-partisans, \( \hat{x}_\ell \in (\hat{x}_g, \chi(\hat{x}_g)) \). In these pairs, \( \hat{x}_g \in (-\pi_\alpha^*, \pi_\alpha^*) \) for all \( \alpha \in [0, 1] \). Because \( g \) is more centrist, increasing \( \alpha \) improves \( M \)'s continuation value and shrinks the acceptance set, which pulls partisan proposals towards \( \hat{x}_g \). Every effect benefits \( g \).

Incentives are similar for access to an intermediate interval of legislators on the other side of the spectrum. Without access, \( g \) is inside the acceptance set if \( \hat{x}_\ell \in \left(\chi(\hat{x}_g), x'_{\text{del}}\right) \). Increasing delayed or durable access pulls all proposals weakly inward, so
the indirect and direct effects both benefit $g$. And for $\hat{x}_t \in (x'_{\text{del}}, x'_{\text{dur}})$ the direct gains from increasing $\alpha_{\text{dur}}$ outweigh any indirect losses.

Finally, $g$’s preference for durable access is unclear in general if $\hat{x}_t \in (x'_{\text{dur}}, x''_{\text{dur}})$. In this centrist range, the direct and indirect effects conflict ambiguously. For a stark example, $\hat{x}_g$ may not be in $A^*_g$. Because the acceptance set boundaries always shift in opposite directions, greater access makes one partisan proposal less favorable for $g$ and the other more favorable. Clear conclusions therefore require conditions on $\rho_R$ and $\rho_L$, as the overall indirect effect depends on their relative magnitudes.

Extreme groups. In general, extremist groups have ambiguous preferences for durable access. Extreme groups are always outside $A^*_g$, so the indirect effect of increasing durable access depends on the relative magnitude $\rho_L$ and $\rho_R$. For example, if the opposite partisan has low agenda power, then an extreme group benefits from expanding the acceptance set. Thus, it wants durable/delayed access to all aligned moderates, and moderates opposite $M$ who are sufficiently centrist.

Prioritizing Access

So far, we have studied whether $g$ wants each kind of access in isolation. But how does $g$ prioritize if it wants more than one kind of access? Intuitively, I now study the following question: which kind of access does $g$ want to buy with its next dollar? More precisely, I compare how $g$’s ex-ante payoff changes as $\alpha$ increases from zero, which I refer to as $g$’s willingness to acquire access (WTA).

Definition 3. The interest group, $g$, prioritizes a particular kind of access if and only if it uniquely maximizes $g$’s WTA.

A general observation from Propositions 2 and 3 is that $g$ seeks delayed access more selectively than temporary or durable access. Delayed access lacks the benefit of potential immediate lobbying opportunities and also has indirect effects weakly inferior to those of durable access. Thus, whenever $g$ wants delayed access, it prefers durable access. As $g$ wants durable access only if it also wants temporary access, it follows that $g$ is most selective with delayed access and never prioritizes it.

Proposition 4. The interest group never prioritizes delayed access.

Next, comparing durable access against temporary access using Propositions 2 and 3 reveals that $g$ will pay for temporary access to a broader range of legislators than
it will pay for durable access. Specifically, \( g \) wants durable access to a subset of the \( \hat{x}_\ell \) for which it wants temporary access. On the extensive margin, indirect effects produce a discrepancy between durable and temporary access only by discouraging durability. Proposition 5 fully characterizes when \( g \) prioritizes durable access, using the observation that \( WTA_{dur} = WTA_{temp} + WTA_{del} \).

**Proposition 5.** The interest group prioritizes durable access if and only if its willingness to acquire delayed access is strictly positive.

Together, Lemma 2 and Propositions 2-5 shed light on which kind of access \( g \) prioritizes. First, Lemma 2 implies that \( g \) prioritizes some form of access only if \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \). And by Proposition 4, \( g \) only prioritizes temporary or durable access. Then, Proposition 3 and Proposition 5 yield sufficient conditions and necessary conditions on how \( g \) prioritizes. These conditions take a partitional form characterizing the respective sets of \( \hat{x}_\ell \) for which \( g \) prioritizes temporary and durable access.

Proposition 6 characterizes how moderate groups prioritize over access and Figure 4 illustrates.

**Proposition 6.** Suppose \( g \) is moderate. There exist cutpoints satisfying \(-\overline{x} < x' < x'' < \overline{x}\) such that \( g \) prioritizes:

\[(i)\] temporary access if \( \hat{x}_\ell \in (x'', \hat{x}_g) \) and only if \( \hat{x}_\ell \in (x', \hat{x}_g) \); and

\[(ii)\] durable access if \( \hat{x}_\ell \in (\chi(\hat{x}_g), x') \cup (\hat{x}_g, \overline{\chi}(\hat{x}_g)) \) and only if \( \hat{x}_\ell \in (\chi(\hat{x}_g), x'') \cup (\hat{x}_g, \overline{\chi}(\hat{x}_g)) \).

Figure 4: Moderate group prioritizing access

Figure 4 illustrates how a moderate interest group prioritizes access given \( \ell \)'s ideal point, \( \hat{x}_\ell \).

Next, Proposition 7 shows how extreme groups want to prioritize access if either partisan has low agenda power. Part (i) suggests that extreme groups facing moderate
opposition will prioritize durable access to all legislators who are not too extreme. In contrast, part (ii) suggests that extreme groups prioritize durable access to moderate opponents if aligned partisans have low agenda power.

**Proposition 7.** Suppose the interest group, \( g \), is extremist.

(i) If \( \rho_L \) is sufficiently small, then there exists \( x' < 0 \) such that \( g \) prioritizes durable access if \( \hat{x}_\ell \in (x', \pi) \).

(ii) If \( \rho_R \) is sufficiently small, then there exists \( x'' \geq -\pi \) such that \( g \) prioritizes durable access if \( \hat{x}_\ell \in (\chi(\hat{x}_g), x'') \).

In Proposition 7(i), a sufficiently weak opposing partisan clarifies the indirect effect of durable access: more extremism helps \( g \) and less extremism hurts. Unless \( \hat{x}_\ell \) is sufficiently far across 0, durable access increases extremism by worsening \( M \)'s expectations about future policy and \( g \) benefits because the opposing partisan is unlikely to propose. Thus, \( g \) clearly wants access to aligned moderates, and even opposing legislators close enough to zero. Proposition 7(ii) has a symmetric logic.

**Effects of Target Legislator Characteristics**

Having studied how \( g \) prioritizes, I now discuss how \( g \)'s value for access depends on \( \ell \)'s ideology and proposal power, \( \rho_\ell \).

First, I discuss how \( g \)'s WTA varies with \( \hat{x}_\ell \). Because Proposition 2 implies WTA is strictly positive only if \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \), it is non-monotonic in \( |\hat{x}_g - \hat{x}_\ell| \) quite generally.

For temporary access, the relationship flows entirely through \( g \)'s direct gain from potential lobbying today. Therefore WTA is strictly positive and increasing in \( |\hat{x}_g - \hat{x}_\ell| \) over \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \). Moreover, if \( g \) is moderate, then this relationship is symmetric for \( \ell \) close to \( g \). For \( \hat{x}_\ell \) far enough that lobbying is constrained, however, WTA eventually starts to decrease in \( |\hat{x}_g - \hat{x}_\ell| \) and equals to zero for \( \hat{x}_\ell \notin (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \) because lobbying is inconsequential.

For durable access, WTA depends on the effect of potential lobbying today and also indirect effects. Similar to temporary access, WTA is non-monotonic in \( |\hat{x}_g - \hat{x}_\ell| \) and zero for \( \hat{x}_\ell \notin (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \). But in contrast, WTA can be zero for \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g)) \) and also need not increase in \( |\hat{x}_g - \hat{x}_\ell| \) over this range. For moderate groups, however,
WTA does weakly increases in $|\hat{x}_g - \hat{x}_\ell|$ if $\hat{x}_\ell$ is close enough to $\hat{x}_g$, but the relationship is asymmetric: if two legislators are equidistant from $g$, it favors the more extreme one. Two forces increase $g$’s WTA as $\hat{x}_\ell$ shifts outward from $\hat{x}_g$. First, $g$’s lobbying profit increases. Second, increasing $\alpha$ induces greater partisan moderation because $M$ gains more from $g$’s policy offer relative to $\ell$’s non-lobby proposal. Thus, $g$ gains more by inducing greater partisan moderation. Both effects increase $g$’s WTA relative to an equidistant legislator skewed towards $M$.

For proposal power, $g$’s willingness to pay (WTP) for any amount of any kind of access increases with $\rho_{\ell}$ regardless of $\hat{x}_\ell$ and $\hat{x}_g$.\textsuperscript{17} For temporary access, $\rho_{\ell}$ straightforwardly increases the marginal ex-ante gain by increasing the probability of enjoying the lobbying surplus. For durable access, however, $\rho_{\ell}$ can have competing effects: increasing the direct benefit but amplifying the (possibly negative) indirect effect. Although the overall effect is not immediately clear, it is proportional to $\rho_{\ell}$ whenever $g$’s WTP is strictly positive. Thus, $g$’s WTP weakly increases in $\rho_{\ell}$. This stark result matches the empirically robust regularities that (i) legislators on important committees, especially committee chairmen, attract more contributions (Fouirnaies, 2017; Berry and Fowler, 2018), and (ii) lobbyists connected to those legislators command a premium (Blanes i Vidal et al., 2012).

### Conceding Access

The analysis has implications for access competition between multiple groups. Although, I abstract from competition to isolate how legislative considerations shape access-seeking,\textsuperscript{18} and a full analysis is outside the scope of this paper, we can make several observations.

For slightly more centrist legislators, moderate groups would concede and even actively support durable access by a range of even more centrist groups. This incentive can arise purely for policy reasons: beneficial indirect effects of partisan moderation. To illustrate more precisely, say $g$ \textit{concedes access} to another group $g'$ if $g$ strictly prefers to let $g'$ optimally acquire durable access. Suppose legislator $\ell$ is slightly more centrist than a moderate group, $g$. By Proposition 3, $g$ forgoes durable access to $\ell$, and $g'$ seeks durable access if slightly more centrist than $\ell$. If $g'$ is close enough to $\ell$,

---

\textsuperscript{17}See Lemma A.1 in Appendix A.

\textsuperscript{18}Competition over temporary agenda power has been studied elsewhere (e.g., Levy and Razin, 2013).
then $g$ concedes durable access to $g'$.

These forces suggest that competition for durable access is more likely to feature centrist groups against extreme groups, with intermediate groups on the sideline or even providing resources to the centrists. The forces are also consistent with groups frequently lobbying unopposed (Baumgartner and Leech, 2001). Existing explanations for this regularity include collective action problems, free-riding incentives, and entry costs. I provide another possibility: equilibrium consequences of durable access.

Another implication is that groups may prefer to delegate to lobbyists who are more centrist, and possibly even on the opposite side of the aisle. This implication derives from the logic of conceding access. Substantively, to the extent that outside lobbyists have more autonomy than in-house lobbyists, the model suggests a new rationale for why wealthy groups sometimes hire outsiders.$^{19}$ Alternatively, this rationale could partially explain why some industries lobby through trade associations funded by diverse interests.

**Legislator Welfare and Selling Access**

With a grasp on $g$’s perspective, I now discuss legislator welfare and implications for price discrimination when selling access.

Whenever $g$ lobbies $\ell$ in equilibrium, it exactly compensates $\ell$ for modifying her proposal. Thus, temporary access does not change $\ell$'s ex-ante payoff. In contrast, durable access does have an effect by indirectly changing partisan proposals and the direction depends on $\ell$’s extremism relative to $g$. For example, more durable access makes $\ell$ better off if $g$ is slightly more centrist. It acts as a commitment device on $\ell$’s proposals, indirectly constraining partisan proposals. But $\ell$ is always weakly worse off giving durable access to a more extreme aligned group because extremism increases.

Although I do not model price discrimination explicitly, these observations suggest that it is likely more pervasive for durable access and typically favors centrists.

Together with the results on prioritizing access, the preceding observations offer a logic for the regularity that interest groups frequently target allies. Additionally, the logic here suggests an important asymmetry: moderate groups will disproportionately lobby more extreme allies. Specifically, they avoid durable connections to nearby more centrist legislators, but are especially likely to form such connections with nearby more

$^{19}$See Hirsch et al. (2020) for an informational rationale.
extreme legislators because they both benefit. This asymmetry is distinct from, e.g., Awad (2020), where groups prioritize moderate allies for informational reasons.

Next, to study how $g$’s access to $\ell$ affects other legislators, I focus on $M$’s expected payoff, which captures the spirit of majority welfare in this ordered setting (Banks and Duggan, 2006b). Clearly, $M$’s welfare increases with temporary access if lobbying shifts $\ell$’s proposal inward and otherwise decreases. Durable access has the same directional effect, but amplifies the magnitude because the indirect effect on $M$’s welfare always pushes in the same direction as the direct effect. Specifically, durably increasing the probability of more centrist $\ell$ proposals also pulls partisan proposals inward, and vice versa.

The analysis of endogenous access has welfare implications for a wide range of group-legislator pairs. Under broad conditions, the legislative interaction discourages durable access reducing $M$’s welfare, and instead encourages durable access favoring $M$. These incentives arise even though $g$ does not intrinsically value $M$’s welfare.

**What if groups can choose access again later?**

Thus far, I have focused on a one-time choice of access before bargaining and shown that durability has important equilibrium effects. If access acquired today will not completely disappear tomorrow, then it indirectly influences how others act today. Otherwise, it does not affect expectations about future policymaking and indirect effects are absent.

If $g$ can choose temporary access each period, then $g$’s preference over $\alpha_{\text{temp}}$ in stationary equilibria is unchanged: $\alpha_{\text{temp}} = 1$ if and only if $\hat{x}_\ell \in (\chi(\hat{x}_g), \overline{\chi}(\hat{x}_g))$.\textsuperscript{20} Despite equivalent access seeking, however, $g$’s ex-ante payoff differs from the one-shot case: instead, it now equals $g$’s ex-ante payoff from maximum durable access, i.e., $\alpha_{\text{dur}} = 1$. The main analysis thus implies moderate groups would forgo the option to repeatedly acquire temporary access to a range of more centrist legislators. Instead, $g$ would prefer to contract against future access by specifying immediate, one-shot access without possible future access. This arrangement is equivalent to the one-shot, temporary access studied earlier and provides only a direct benefit. Yet, under other conditions, $g$ can benefit from the indirect effects of durable access and then always wants more regardless of persistence or contracting.

\textsuperscript{20}A one-shot deviation in temporary access does not affect continuation values in stationary strategy profiles, so the acceptance set is unaffected and $g$ has a profitable deviation if $\alpha_{\text{temp}} < 1$. 
Conclusion

I analyze a model of legislative policymaking where access provides interest groups with opportunities to influence policy proposals. To study which connections form, the interest group chooses how much access to acquire to particular legislators, and also its durability and speed.

Broadly, the analysis unpacks a neglected byproduct of durable access in legislatures: the prospect of future lobbying can spill over to influence today’s proposal by other legislators. In equilibrium, durable access acts as an indirect form of vote buying/selling. By endogenously changing each legislator’s expectations about policymaking, it alters which policies can pass. In turn, it can indirectly alter policy proposals by legislators who are not lobbied.

Due to these indirect effects, moderate groups prioritize temporary access to a range of more centrist legislators. Policy considerations drive this behavior, as durable access to these legislators increases policy extremism enough to outweigh the perk of better lobbying prospects. On the other hand, these groups prioritize durable access to more extreme legislators because it facilitates lobbying and reduces policy extremism.

Substantively, revolving door hiring likely provides more durable access than campaign contributions. If so, Proposition 6 suggests that a broad range of moderate groups will hire staffers of legislators who are relatively more extreme but not too extreme. These connections benefit both group and legislator regardless of price, and are therefore likely to form. On the other hand, groups benefit from temporary access to a broad swath of legislators. These shorter-lived connections benefit the group and do not harm the legislator. Altogether, the model suggests that groups will (i) hire more selectively than they contribute, and (ii) target hiring but not contributions by relative extremism.

Of course, it is an empirical question whether access via revolving door hiring is more durable than access via contributions. The preceding implications therefore suggest a potential approach to measure relative durability: compare the ideological distribution of contributions or hiring. Where the data allows, this approach could also compare hiring patterns of in-house lobbyists versus contract lobbyists.
References


Appendix A  Proofs of Main Results

Extended Model

I prove the main results in a version of the model that relaxes restrictions on the number of legislators and interest groups. There are three disjoint sets of players: \( n^V \) (finite and odd) voting legislators in \( N^V \); \( n^L \geq 3 \) committee members in \( N^L \); and \( n^G \leq n^L \) interest groups in \( N^G \). Let \( N = N^V \cup N^L \cup N^G \).

Throughout, voting legislators are called voters and denoted by \( i \). To align with the main text, \( M \) denotes the median voter. I denote committee members by \( \ell \) and interest groups by \( g \). Each \( \ell \in N^L \) is associated with only one group, \( g_\ell \). Each \( g \in N^G \) can have access to multiple \( \ell \in N^L \) and this set is \( N^G_\ell \subseteq N^L \). Let \( \alpha_\ell \in [0,1] \) denote \( g_\ell \)’s access to \( \ell \).

Legislative bargaining occurs over an infinite number of periods \( t \in \{1,2,\ldots\} \). The policy space is a non-empty, closed interval \( X \subseteq \mathbb{R} \). Let \( \rho = (\rho_1,\ldots,\rho_{n^L}) \in \Delta([0,1])^{n^L} \) be the distribution of recognition probability. In each period \( t \), bargaining proceeds as follows. If no policy has passed before \( t \), then \( \ell \) proposes with probability \( \rho_\ell > 0 \). All players observe the period-\( t \) proposer, \( \ell_t \). With probability \( 1 - \alpha_\ell \), \( g_\ell \) cannot lobby and \( \ell_t \) freely proposes any \( x_t \in X \). With probability \( \alpha_\ell \), \( g_\ell \) can lobby and offers \( \ell_t \) a binding contract \( (y_t, m_t) \in X \times \mathbb{R}_+ \). Next, \( \ell_t \) accepts or rejects. Let \( a_t \in \{0,1\} \) denote \( \ell_t \)’s period-\( t \) acceptance decision, where \( a_t = 1 \) indicates acceptance and \( a_t = 0 \) if either \( \ell_t \) rejects or \( g_\ell \) is unable to lobby in \( t \). If \( \ell_t \) accepts, then \( \ell_t \) is committed to propose \( x_t = y_t \) in \( t \) and \( g_\ell \) transfers \( m_t \) to \( \ell_t \). If \( \ell_t \) rejects, then she can propose any \( x_t \in X \) and \( g_\ell \) keeps \( m_t \). All players observe \( x_t \). There is a simultaneous vote by \( i \in N^V \) using simple majority rule. If \( x_t \) passes, then bargaining ends with \( x_t \) enacted in \( t \) and all subsequent periods. If \( x_t \) fails, then \( q \) is enacted in \( t \) and bargaining proceeds to \( t + 1 \).

Each player \( j \in N \) has quadratic policy utility with ideal point \( \hat{x}_j \in X \). As in the main text, I normalize \( \hat{x}_M = 0 \) and assume \( q \neq 0 \). Additionally, I assume there exists \( \ell \in N^L \) on the same side of \( q \) as \( M \) such that: \( \alpha_\ell < 1 \) or \( g_\ell \) is on the same side of \( q \). For example, if \( q > 0 \), then some \( \ell \in N^L \) satisfies \( \hat{x}_\ell < q \) and at least one of the following holds: \( \alpha_\ell < 1 \) or \( \hat{x}_g_\ell \leq q \).

Players discount streams of per-period utility by common discount factor \( \delta \in (0,1) \). For convenience, I normalize per-period payoffs by \( (1 - \delta) \). Let \( I_t^\ell \in \{0,1\} \) equal

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21 An independent legislator is accommodated by \( \alpha_\ell = 0 \).
22 Where \( \Delta([0,1])^{n^L} \) denotes the \( n^L \)-dimensional unit simplex.
one iff \( \ell \) is the period-\( t \) proposer and \( g_\ell \) can lobby in \( t \). Given a sequence of offers \((y_1, m_1), (y_2, m_2), \ldots \), a sequence of proposers \( \ell_1, \ell_2, \ldots \), a sequence of acceptance decisions \( a_1, a_2, \ldots \), and a sequence of independent policy proposals \( x_1, x_2, \ldots \) such that bargaining continues until \( t \), the discounted sum of per-period payoffs for \( i \in N^V \) is

\[
(1 - \delta^{t-1}) u_i(q) + \delta^{t-1} \left[ (1 - a_t) u_i(x_t) + a_t u_i(y_t) \right];
\]

for \( \ell \in N^\ell \),

\[
(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1} [u_\ell(q) + I_\ell a_t m_{t'}] + \delta^{t-1} \left[ (1 - a_t) u_\ell(x_t) + a_t \left( u_\ell(y_t) + I_\ell m_t \right) \right];
\]

and for \( g \in N^g \),

\[
(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1} \left[ u_g(q) - a_t m_{t'} \sum_{\ell \in N^L_g} I_\ell \right] + \delta^{t-1} \left[ (1 - a_t) u_g(x_t) + a_t \left( u_g(y_t) - m_t \sum_{\ell \in N^L_g} I_\ell \right) \right].
\]

Unless noted otherwise, results are proved for this more general setting. The model in the main text is a special case featuring one voter with ideal point \( \hat{x}_M \); four committee members with ideal points \( \hat{x}_L, \hat{x}_M, \hat{x}_\ell, \) and \( \hat{x}_R \); and one group at \( \hat{x}_g \) with access \( \alpha_\ell \geq 0 \) and \( \alpha_j = 0 \) for all \( j \neq \ell \).

Strategies

I study a refinement of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: no-delay stationary legislative lobbying equilibrium.\(^{23}\)

Let \( \Delta(X) \) be the set of probability measures on \( X \). Let \( W = X \times \mathbb{R}_+ \) denote the lobby offer space and \( \Delta(W) \) denote the set of probability measures on \( W \). A stationary mixed strategy for \( g \in N^G \) is a probability measure \( \lambda_g \in \Delta(W)^{|N^L_g|} \) over \( g \)'s offers \((y, m) \in W \) to each \( \ell \in N^L_g \). A stationary mixed legislative strategy for \( \ell \in N^L_g \) is a pair \((\pi_\ell, \varphi_\ell)\); where \( \pi_\ell \in \Delta(X) \) specifies a probability measure over \( \ell \)'s independent proposals and \( \varphi_\ell : W \to [0, 1] \) is the probability \( \ell \) accepts each \((y, m) \in W \). Finally,

\(^{23}\)More generally, we can define stationary mixed strategy legislative lobbying equilibria and show that they are all equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium. Details available upon request.
voter $i$’s stationary mixed strategy $\nu_i : X \to [0, 1]$ specifies the probability $i$ votes for each $x \in X$.

Let $\lambda$ denote a profile of interest group strategies, $(\pi, \varphi)$ a profile of committee member strategies, and $\nu$ a profile of voter strategies. A stationary strategy profile is $\sigma = (\lambda, \pi, \varphi, \nu)$. Under $\sigma$, let $\tau_\sigma(x)$ be the probability $x$ passes if proposed.

Let $w = (y, m) \in W$ denote an arbitrary lobby offer. Define

$$\xi(\alpha, \sigma) = (1 - \alpha) + \alpha \int_W \left[ 1 - \varphi(y, m) \right] \lambda_g^\ell(dw),$$

which is the probability under $\sigma$ that $\ell$ makes an independent policy proposal conditional on being recognized. Given $\sigma$, $i \in N^\nu$ has continuation value

$$V_i(\sigma) = \sum_{\ell \in N^\ell} \rho_\ell \left( \alpha_\ell \int_W \varphi(y, m) \left[ \bar{v}_\sigma(y) u_i(y) + [1 - \bar{v}_\sigma(y)] (1 - \delta) u_i(q) + \delta V_i(\sigma) \right] \lambda_{g^\ell}^\ell(dw) \right.$$

$$+ \xi(\alpha, \sigma) \int_X \left[ \bar{v}_\sigma(x) u_i(x) + [1 - \bar{v}_\sigma(x)] (1 - \delta) u_i(q) + \delta V_i(\sigma) \right] \pi_i(dx) \right),$$

the continuation value of $\ell \in N^\ell$ is

$$\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \int_W \varphi_j(y, m) \left[ \bar{v}_\sigma(y) u_\ell(y) + [1 - \bar{v}_\sigma(y)] (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \lambda_{g^\ell}^\ell(dw) \right.$$

$$+ \xi_j(\alpha, \sigma) \int_X \left[ \bar{v}_\sigma(x) u_\ell(x) + [1 - \bar{v}_\sigma(x)] (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \pi_j(dx) \right)$$

$$+ \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \bar{v}_\sigma(y) u_\ell(y) + [1 - \bar{v}_\sigma(y)] (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \lambda_{g^\ell}^\ell(dw) \right.$$ 

$$+ \xi_\ell(\alpha, \sigma) \int_X \left[ \bar{v}_\sigma(x) u_\ell(x) + [1 - \bar{v}_\sigma(x)] (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right] \pi_\ell(dx) \right),$$

and the continuation value of $g \in N^G$ is

$$\tilde{V}_g(\sigma) = \sum_{\ell \in N^G} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \bar{v}_\sigma(y) u_g(y) + [1 - \bar{v}_\sigma(y)] (1 - \delta) u_g(q) + \delta \tilde{V}_g(\sigma) \right] \lambda_{g^\ell}^\ell(dw) \right.$$
\[ + \xi(\alpha, \sigma) \int_X \left[ \nu \sigma(x) u_g(x) + (1 - \nu \sigma(x)) [(1 - \delta) u_g(q) + \delta \tilde{V}(\sigma)] \right] \pi_\ell(dx) \]

\[ + \sum_{\ell \in N_g} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu \sigma(y) u_g(y) + (1 - \nu \sigma(y)) [(1 - \delta) u_g(q) + \delta \tilde{V}(\sigma)] - m \right] \lambda_\ell(dy) \]

\[ + \xi(\alpha, \sigma) \int_X \left[ \nu \sigma(x) u_g(x) + (1 - \nu \sigma(x)) [(1 - \delta) u_g(q) + \delta \tilde{V}(\sigma)] \right] \pi_\ell(dx) \right), \]

(14)

A stationary pure strategy for \( g \in N^G \) is \((y_g, m_g) \in X^{[N_g^L]} \times \mathbb{R}^{[N_g^L]}\), where \( y_g \) is \( g \)'s profile of policy offers and \( m_g \) is \( g \)'s profile of monetary offers. A pure stationary strategy for \( \ell \in N^L \) is \((z_\ell, a_\ell)\); where \( z_\ell \in X \) specifies \( \ell \)'s independent proposal, and \( a_\ell : X \times \mathbb{R} \rightarrow \{0, 1\} \) equals one iff \( \ell \) accepts \( g_\ell \)'s offer. Finally, for each \( i \in N^V \), \( \nu_i : X \rightarrow \{0, 1\} \) equals one iff \( i \) supports the proposal.

Given \( \sigma \), the set of policies that pass is constant across periods by stationarity and denoted \( A(\sigma) \). For \( \ell \in N^L \), define

\[ \tilde{U}_\ell(x; \sigma) = \begin{cases} 
  u_\ell(x) & \text{if } x \in A(\sigma) \\
  (1 - \delta) u_\ell(q) + \delta \tilde{V}(\sigma) & \text{else.}
\end{cases} \]

(15)

Formally, \( \sigma = (y, m, z, a, v) \) is a no-delay stationary legislative lobbying equilibrium if it satisfies five conditions. First, for all \( g \in N^G \) and \( \ell \in N_g^L \), \((y_\ell^g, m_\ell^g)\) satisfies

\[ y_\ell^g \in \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) \]

(16)

and

\[ m_\ell^g = u_\ell(z_\ell) - u_\ell(y_\ell^g). \]

(17)

Second, for all \( \ell \in N^L \) and \((y, m) \in W \), \( a_\ell(y, m) = 1 \) iff

\[ \tilde{U}_\ell(y; \sigma) + m \geq \tilde{U}_\ell(z_\ell; \sigma). \]

(18)
Third, for each $\ell \in N^L$,
\begin{equation}
    z_\ell \in \arg \max_{x \in A(\sigma)} u_\ell(x).
\end{equation}

Finally, for each $i \in N^V$, $v_i(x) = 1$ iff
\begin{equation}
    u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma).
\end{equation}

In Appendix B, I define stationary mixed strategy legislative lobbying equilibrium and show that all such equilibria are equivalent in outcome distribution to strategy profiles satisfying (16)-(20). Thus, the outcome distribution characterized in Proposition 1 applies even more broadly. I provide a rough outline here, as parts of the argument are noted in proving Proposition 1. First, equilibrium lobby offers always make targeted legislators indifferent. Thus, in any equilibrium each player’s continuation value can be expressed as expected utility over a policy lottery, similar to Banks and Duggan (2006a). Notably, for all voters in $N^V$ this can be done using the same policy lottery. Therefore the outcome of equilibrium voting under majority rule always coincides with $M$’s preference (Duggan, 2014). Because $q \neq 0 = \hat{x}_M$, the policy lottery used to characterize equilibrium continuation values is not degenerate on $q$. Then, equilibrium offers by each interest group must specify policy $M$ will pass. Next, we can show two things that are without loss of generality in equilibrium: (i) $M$ passes policy if indifferent, and (ii) legislators accept lobby offers if indifferent. Subsequently, I establish that every equilibrium is no-delay. To complete the equivalence in outcome distribution, I prove that legislator proposal strategies and interest group offer strategies are always degenerate in equilibrium.

**Proposition 1.** A no-delay stationary legislative lobbying equilibrium exists. Every such equilibrium has the same outcome distribution.

**Proof.** There are four parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for $g \in N^G$ and $\ell \in N^L$. Part 2 uses the fixed point to construct a strategy profile $\sigma$. Part 3 verifies that $\sigma$ satisfies (16) - (20). Part 4 shows there is a unique equilibrium outcome distribution.
Part 1: Let \((y,z) = (y_1, \ldots, y_n, z_1, \ldots, z_n) \in X^{2nL}\) and for each \(j \in N\) define
\[
r_j(y,z) = \sum_{\ell \in N_L} \rho_{\ell} \left( \alpha_{\ell} u_{\ell}(y_\ell) + (1 - \alpha_{\ell}) u_{\ell}(z_\ell) \right).
\]
Set \(A(r(y,z)) = \{x \in X | u_M(x) \geq (1 - \delta) u_M(q) + \delta r_M(y,z)\}\), which is non-empty, compact, and convex because \(\delta \in (0,1), q \neq 0,\) and \(u_M\) is strictly concave. Moreover, \(A(r(y,z))\) is continuous in \((y,z)\).

For each \(\ell \in N_L\), define
\[
\tilde{\phi}_{\ell}(y,z) = \arg \max_{y_\ell \in A(r(y,z))} u_{g_{\ell}}(y_\ell) + u_\ell(y_\ell),
\]
which is unique for all \((y,z)\) because \(A(r(y,z))\) is non-empty, compact and convex, and the objective function is strictly concave and continuous. Because \(A(r(y,z))\) is continuous, the Theorem of the Maximum implies continuity of \(\tilde{\phi}_{\ell}(y,z)\). Next, define
\[
\phi_{\ell}(y,z) = \arg \max_{z_\ell \in A(r(y,z))} u_\ell(z_\ell),
\]
which is unique for all \((y,z)\) and continuous by the Theorem of the Maximum.

Define the mapping \(\Phi : X^{2nL} \to X^{2nL}\) as \(\Phi(y,z) = \prod_{\ell \in N_L} \tilde{\phi}_{\ell}(y,z) \times \prod_{\ell \in N_L} \phi_{\ell}(y,z)\), which is a product of continuous functions and thus continuous. By Brouwer’s theorem, a fixed point \((y^*, z^*) = \Phi(y^*, z^*)\) exists because \(\Phi\) is a continuous function mapping a non-empty, compact, and convex set into itself.

Part 2: Define a stationary pure strategy profile \(\sigma\) as follows. First, for all \(g \in N^G\) and \(\ell \in N^L_g\), set \(y_\ell^g = y_\ell^*\) and \(m_\ell^g = u_\ell(z_\ell^*) - u_\ell(y_\ell^*)\). Next, for \(\ell \in N^L\), set \(z_\ell = z_\ell^*\) and define
\[
a_{\ell}(y,m) = \begin{cases} 
1 & \text{if } u_{\ell}(y) + m \geq u_{\ell}(z_\ell^*), \text{ for } y \in A(r(y^*, z^*)) \\
1 & \text{if } (1 - \delta) u_{\ell}(q) + \delta r_{\ell}(y^*, z^*) + \rho_{\ell} \alpha_{\ell} m_{\ell}^g + m \geq u_{\ell}(z_\ell^*), \text{ for } y \notin A(r(y^*, z^*)) \\
0 & \text{else.}
\end{cases}
\]

Finally, for each \(i \in N^V\) define \(v_i\) so that \(v_i(x) = 1\) if \(u_i(x) \geq (1 - \delta) u_i(q) + \delta r_{\ell}(y^*, z^*)\) and \(v_i(x) = 0\) otherwise.

Part 3: I check that \(\sigma\) satisfies (16)-(20).
First, I verify (20) to show \( A(\sigma) = A(r(y^*, z^*)) \). Note that for each \( g \in N^G \) and all \( \ell \in N^L_g \); \( y^\ell_g \in A(r(y^*, z^*)) \) and \( a_\ell(y^\ell_g, m^\ell_g) = 1 \). Moreover, \( z_\ell \in A(r(y^*, z^*)) \) for all \( \ell \in N^L \). Thus, voter \( i \)'s continuation value under \( \sigma \) is \( V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell [\alpha_\ell u_i(y^\ell_{z_\ell}) + (1 - \alpha_\ell)u_i(z^*_\ell)] = r_i(y^*, z^*) \). Thus, each voter \( i \)'s strategy satisfies (20). Banks and Duggan (2006b) and Duggan (2014) apply, so \( A(\sigma) = A(r(y^*, z^*)) \) because \( M \) is decisive over lotteries.

To check (16), consider \( g \in N^G \) and \( \ell \in N^L_g \). Focusing on offers \( \ell \) accepts is without loss of generality because \( a_\ell(z_\ell, 0) = 1 \). Because \( A(\sigma) = A(r(y^*, z^*)) \), (22) implies \( \tilde{\phi}_\ell(y^*, z^*) = \arg \max_{y \in A(\sigma)} u_\ell(y) + u_\ell(y) - u_\ell(z^*_\ell) \). Thus, (16) holds because \( \tilde{\phi}_\ell(y^*, z^*) = y^\ell_{z_\ell} = y^\ell_{z_\ell} \). The no-delay property noted before the proof implies \( y \not\in A(\sigma) \) is not a profitable deviation for any \( g \in N^G \).

It is immediate that \( m^\ell_g \) satisfies (17).

To check (18), note that \( \ell \)'s expected dynamic payoff from rejecting \( y_\ell \)'s offer is \( \tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z^*_\ell) \). Thus, \( \ell \) weakly prefers to accept any \((y, m)\) satisfying \( y \in A(r(y^*, z^*)) \) iff \( u_\ell(y) + m \geq u_\ell(z^*_\ell) \). If \( y \not\in A(r(y^*, z^*)) \), then \( \ell \) weakly prefers to accept \((y, m)\) iff \( (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell) \). Thus, \( a_\ell \) satisfies (18).

To check (19), note that (23) implies \( \phi_\ell(y^*, z^*) = \arg \max_{x \in A(\sigma)} u_\ell(x) \) because \( A(\sigma) = A(r(y^*, z^*)) \). Thus, (19) holds because \( \phi_\ell(y^*, z^*) = z^*_\ell = z_\ell \) for each \( \ell \in N^L \). The no-delay property implies \( x \not\in A(\sigma) \) is not a profitable deviation for any \( \ell \in N^L \).

**Part 4.** Let \( \sigma \) and \( \sigma' \) be stationary legislative lobbying equilibria. It suffices to show \((y_\ell, m_\ell) = (y'_\ell, m'_\ell)\) for all \( g \in N^G \) and \( z_\ell = z'_\ell \) for all \( \ell \in N^L \). Assume \( y_\ell \neq y'_\ell \) or \( z_\ell \neq z'_\ell \) for some \( \ell \in N^L \). Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, \( A(\sigma) = A(\sigma') \). Because \( \sigma \) and \( \sigma' \) are no-delay, \( \ell \)'s expected dynamic payoff from rejecting \( y_\ell \)'s offer is \( u_\ell(z_\ell) \) under both \( \sigma \) and \( \sigma' \). Because equilibrium lobby offers always make targeted legislators indifferent, \( m^\ell_g = u_\ell(y^\ell_g) - u_\ell(z_\ell) \). Therefore \((y_\ell, m_\ell) = (y'_\ell, m'_\ell)\) and \( z_\ell = z'_\ell \).

\( \square \)

**Endogenous Access**

Consider \( \ell \in N^L \) and refer to \( g_\ell \) as \( g \) for convenience. Recall

\[
\hat{y}_\ell = \arg \max_{y \in X} u_\ell(y) + u_\ell(y) = \frac{\hat{x}_\ell + \hat{x}_\ell}{2},
\]

(25)
The results fix \( \hat{x}_g \) and vary \( \hat{x}_\ell \). Throughout, \( \hat{x}_g > 0 \), as the other case is symmetric.

Let \( \sigma(\alpha_\ell; \hat{x}_\ell) \) denote an equilibrium given \( \hat{x}_\ell \) and \( \alpha_\ell \), and denote the corresponding social acceptance set as \( A(\alpha_\ell; \hat{x}_\ell) \), with upper bound \( \overline{x}(\alpha_\ell; \hat{x}_\ell) \). That is, \( A(\alpha_\ell; \hat{x}_\ell) \) corresponds to \( A^*_\alpha \) from the main text but makes explicit the dependence on \( \hat{x}_\ell \).

I first state a lemma that partitions whether \( \hat{x}_\ell \) is in \( \text{int}A(0; \hat{x}_\ell) \). It plays a key role in proving Lemmas 1 and 2.

**Lemma A.1.** For all \( \ell \in N^L \), there exists \( \overline{x}_\ell \in (0, q] \) such that \( \hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell) \) if \( \hat{x}_\ell \in (\overline{x}_\ell, \overline{x}_\ell) \) and otherwise \( A(0; \hat{x}_\ell) = [\overline{x}_\ell, \overline{x}_\ell] \).

The proof of Lemma A.1 proceeds in a series of Lemmas that are provided in Appendix C. A rough outline of the argument is as follows. First, I define a function \( \xi^\ell : \mathbb{R}_+ \to \mathbb{R} \) constructed so that \( \xi^\ell(x) > 0 \) iff \( x \in \text{int}A(0; x) \). Then, I show that there is a unique \( \overline{x}_\ell \in (0, q] \) such that \( \xi^\ell(x) > 0 \) iff \( x \in [0, \overline{x}_\ell) \). It follows that \( \hat{x}_\ell \in (\overline{x}_\ell, \overline{x}_\ell) \) implies \( \hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell) \), and otherwise \( A(0; \hat{x}_\ell) = [\overline{x}_\ell, \overline{x}_\ell] \).

**Lemma 1.** In equilibrium, \( y_0^* \neq z_0^* \) iff \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{x}(\hat{x}_g)) \).

**Proof.** If \( \hat{x}_\ell \in (\overline{x}_\ell, \overline{x}_\ell) \), then Lemma A.1 implies \( z_0^* = \hat{x}_\ell \). Thus, \( \hat{x}_\ell \neq \hat{x}_g \) implies \( y_0^* \neq z_0^* \).

Otherwise, \( z_0^* \) is the boundary of \( A_0^* = [\overline{x}, \overline{x}] \) closer to \( \hat{x}_\ell \). For \( \hat{x}_\ell \leq \overline{x} \), we have \( y_0^* > -\overline{x} \) iff \( \hat{x}_\ell > \chi(\hat{x}_g) \). Analogously for \( \hat{x}_\ell \geq \overline{x} \), we have \( y_0^* < \overline{x} \) iff \( \hat{x}_\ell < \overline{x}(\hat{x}_g) \).

**Lemma 2.** Interest group \( g \) strictly prefers \( \alpha_\ell > 0 \) only if \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{x}(\hat{x}_g)) \).

**Proposition 2.** Interest group \( g \) strictly prefers \( \alpha_\ell^{\text{temp}} > 0 \) iff \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{x}(\hat{x}_g)) \).

**Proof.** Lemma 2 implies that we can focus on \( \hat{x}_\ell \in (\chi(\hat{x}_g), \overline{x}(\hat{x}_g)) \). Lemma 1 implies \( y_0^* \neq z_0^* \). The marginal effect of increasing \( \alpha_\ell^{\text{temp}} \) from zero on \( g \)'s ex-ante payoff is \( u_g(y_0^*) - u_g(z_0^*) + u_\ell(y_0^*) - u_\ell(z_0^*) > 0 \), where the strict inequality follows because \( g \) strictly prefers \((y_0^*, m_0^*)\) to the lobby offer \((z_0^*, 0)\), so \( u_g(y_0^*) - m_0^* = u_g(y^*) - [u_\ell(z_0^*) - u_\ell(y_0^*)] > u_g(z_0^*) \). Thus, \( \alpha_\ell^{\text{temp}} = 0 \) is not optimal.

**Lemma 3.** If \( \hat{x}_g \in (0, \overline{x}_\ell) \), then there exists \( x' \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \notin (-x', x') \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Otherwise, \( \hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell) \) for all \( \hat{x}_\ell \) and \( \alpha_\ell \).
Proof. Consider \( \hat{x}_g \in (0, \overline{x}_\ell) \). If \( \hat{x}_\ell = \hat{x}_g \), then Lemma A.1 implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). By symmetry, \( \hat{x}_\ell = -\hat{x}_g \) also implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Recall that \( A(0; \hat{x}_\ell) \) strictly expands as \( \hat{x}_\ell \) shifts away from 0 over \((-\overline{x}_\ell, \overline{x}_\ell)\). Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies \( A(0; \hat{x}_\ell) \) is continuous in \( \hat{x}_\ell \). Thus, there exists \( x' \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \notin (-x', x') \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \).

To complete the proof, consider \( \hat{x}_g \geq \overline{x}_\ell \). Lemma A.1 implies \( \hat{x}_g \notin \text{int}A(0; \hat{x}_\ell) = (\overline{x}_\ell, \overline{x}_\ell) \) for all \( \hat{x}_\ell \geq \hat{x}_g \). Because \( A(\alpha_\ell; \hat{x}_\ell) \subset A(0; \hat{x}_g) \) for all \( (\alpha_\ell, \hat{x}_\ell) \), it follows that \( \hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell) \).

Next, Lemmas A.2 - A.6 establish properties used to prove Propositions 3 and 7.

**Lemma A.2.** Suppose \( \hat{x}_g \in (0, \overline{x}_\ell) \). There exists \( \bar{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(\alpha_\ell^{\text{dur}}; \hat{x}_\ell) \) for all \( \alpha_\ell^{\text{dur}} \in [0, 1] \).

**Proof.** Consider \( \hat{x}_g \in (0, \overline{x}_\ell) \) and let \( \alpha_\ell^{\text{dur}} = \alpha_\ell \). By Lemma 3, there exists \( x' \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(0; \hat{x}_\ell) \). Then \( 0 < \hat{x}_\ell < \hat{x}_g \) implies \( A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell) \).

**Lemma A.3.** For all \( \ell \in N^L \), \( g_\ell \)’s equilibrium lobbying expenditures increase as the acceptance set expands.

**Proof.** Let \( A^* = [-\overline{x}^*, \overline{x}^*] \) denote the equilibrium acceptance set. There are two cases.

**Case 1.** Suppose \( \hat{x}_\ell \in A^* \). Then \( z_\ell = \hat{x}_\ell \). There are two subcases.

First, assume \( \hat{y}_\ell \notin A^* \). Then \( y^\ell_g = \hat{y}_\ell \), and (17) implies \( m^\ell_g = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell) \). Thus, \( m^\ell_g \) is constant because \( z_\ell = \hat{x}_\ell \) and \( y^\ell_g = \hat{y}_\ell \) as \( \overline{x}^* \) increases.

Second, assume \( \hat{y}_\ell \notin A^* \). Since \( \hat{x}_\ell \in A^* \), this requires \( \hat{x}_\ell \notin [-\overline{x}^*, \overline{x}^*] \). Without loss of generality, assume \( \hat{x}_\ell \geq \overline{x}^* \). Thus, \( z_\ell = \hat{x}_\ell \) and \( y^\ell_g = \overline{x}^* \). Then (17) implies \( m^\ell_g = u_\ell(\hat{x}_\ell) - u_\ell(\overline{x}^*) \), which increases with \( \overline{x}^* \).

**Case 2.** Suppose \( \hat{x}_\ell \notin A^* \). Without loss of generality, assume \( \hat{x}_\ell > z_\ell = \overline{x}^* \). There are three subcases.

First, assume \( \hat{y}_\ell < -\overline{x}^* \). Then \( y^\ell_g = -\overline{x}^* \), and (17) implies \( m^\ell_g = u_\ell(\overline{x}^*) - u_\ell(-\overline{x}^*) \). Thus, \( m^\ell_g \) increases with \( \overline{x}^* \) because \( -\overline{x}^* < \overline{x}^* < \hat{x}_\ell \).

Second, assume \( \hat{y}_\ell \in A^* \). Thus, \( y^\ell_g = \hat{y}_\ell \) and \( y^\ell_g \) is constant as legislative extremism increases. Then (17) implies \( m^\ell_g = u_\ell(\overline{x}^*) - u_\ell(\hat{y}_\ell) \), which increases with \( \overline{x}^* \).

Third, assume \( \hat{y}_\ell \geq \overline{x}^* \), which implies \( y^\ell_g = \overline{x}^* \). Then (17) implies \( m^\ell_g = u_\ell(\overline{x}^*) - u_\ell(\overline{x}^*) = 0 \), which is constant.

Altogether, \( m^\ell_g \) weakly increases in \( \overline{x}^* \).
For each $j \in N^L\setminus\{\ell\}$, define $E^{LB}_j(\alpha_\ell; \hat{x}_\ell) = \I\{\hat{x}_j \leq -\bar{x}(\alpha_\ell; \hat{x}_\ell)\}$, $E^{UB}_j(\alpha_\ell; \hat{x}_\ell) = \I\{\hat{x}_j \geq \bar{x}(\alpha_\ell; \hat{x}_\ell)\}$, and $C_j(\alpha_\ell; \hat{x}_\ell) = \I\{\hat{x}_j \in \text{int}\text{A}(\alpha_\ell; \hat{x}_\ell)\}$. Define $\bar{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell)$, $\bar{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell)$, and $\tilde{C}_j(\alpha_\ell; \hat{x}_\ell)$ analogously using $\hat{y}_j$. Let $I_g^\ell \in \{0, 1\}$ indicate whether $j \in N_g^L$.

**Assumption A.1.** There exists $j \in N^L\setminus\{\ell\}$ such that $\alpha_j < 1$ and $\hat{x}_j \notin \text{A}(0; \hat{x}_g)$.

**Assumption A.2.** There exists $j \in N^L\setminus\{\ell\}$ such that $\alpha_j > 0$ and $\hat{y}_j \notin \text{A}^*(0; \hat{x}_g)$.

Next, define

$$v^\ell_1(\alpha_\ell; \hat{x}_\ell) = \rho_\ell \left(\alpha_\ell \left[u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell)\right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell)\right) \tag{26}$$

and

$$v^\ell_2(\alpha_\ell; \hat{x}_\ell) = \sum_{\substack{j \neq \ell}} \rho_j \left[\alpha_j \bar{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(\alpha_\ell; \hat{x}_\ell)\right] u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \left[\alpha_j \bar{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(\alpha_\ell; \hat{x}_\ell)\right] u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \alpha_j \left[\tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha_\ell; \hat{x}_\ell)\right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{x}_\ell) \right). \tag{27}$$

**Lemma A.4.** If $\hat{x}_\ell \neq \hat{x}_g$, then $\frac{\partial v^\ell_1(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$.

*Proof.* Suppose $\hat{x}_\ell \neq \hat{x}_g$. Then $\frac{\partial v^\ell_1(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = v_0^\ell \frac{(\hat{x}_g - \hat{x}_\ell)^2}{2} > 0$, by (26) and $\hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2}$.

**Lemma A.5.** Suppose $0 \leq \hat{x}_\ell < \hat{x}_g < \bar{x}_\ell$ and at least one of Assumption A.1 or A.2 holds. Then $v^\ell_2(\alpha_\ell^\text{dur}; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell^\text{dur}$.

*Proof.* Let $\alpha_\ell^\text{dur} = \alpha_\ell$. It suffices to show that

$$\left[\alpha_j \bar{E}^{LB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(\alpha_\ell; \hat{x}_\ell)\right] u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \left[\alpha_j \bar{E}^{UB}_j(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(\alpha_\ell; \hat{x}_\ell)\right] u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \alpha_j \left[\tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha_\ell; \hat{x}_\ell)\right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{x}_\ell) \right]$$

decreases in $\alpha_\ell$ for all $j \in N^L\setminus\{\ell\}$ and strictly decreases for some $j$. 

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Without loss of generality, consider $\hat{x}_j \geq 0$. Because $0 \leq \hat{x}_\ell < \hat{x}_g$, $\pi(\alpha; \hat{x}_\ell)$ increases in $\alpha$. There are two implications. First, $\hat{x}_g \in (0, \pi)$ implies $\hat{x}_g < \pi(0; \hat{x}_\ell)$ by Lemma 3, so $u_g(\pi(\alpha; \hat{x}_\ell))$ and $u_g(-\pi(\alpha; \hat{x}_\ell))$ both decrease in $\alpha$. Second, exactly one of the following holds: $E^{UB}_j(\alpha; \hat{x}_\ell) = 1$ for all $\alpha$, $C_j(\alpha; \hat{x}_\ell) = 1$ for all $\alpha$, or there is a unique $\pi'_{\ell} \in [0, 1]$ such that $\alpha \in [0, \pi'_{\ell})$ implies $E^{UB}_j(\alpha; \hat{x}_\ell) = 1$, and $\alpha \in (\pi'_{\ell}, 1]$ implies $C_j(\alpha; \hat{x}_\ell) = 1$. An analogous observation holds for $\tilde{E}^{UB}_j(\alpha; \hat{x}_\ell)$ and $\tilde{C}_j(\alpha; \hat{x}_\ell)$. Thus, both

$$E^{LB}_j(\alpha; \hat{x}_\ell) u_g(-\pi(\alpha; \hat{x}_\ell)) + E^{UB}_j(\alpha; \hat{x}_\ell) u_g(\pi(\alpha; \hat{x}_\ell)) + C_j(\alpha; \hat{x}_\ell) u_g(\hat{x}_g) \quad (29)$$

and

$$\tilde{E}^{LB}_j(\alpha; \hat{x}_\ell) u_g(-\pi(\alpha; \hat{x}_\ell)) + \tilde{E}^{UB}_j(\alpha; \hat{x}_\ell) u_g(\pi(\alpha; \hat{x}_\ell)) + \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_g) \quad (30)$$

decrease in $\alpha$. Furthermore, because at least one of Assumptions A.1 and A.2 holds, at least one of (29) and (30) strictly decreases for some $j \in N^L \setminus \{\ell\}$. Lemma A.3 implies $m'_j(\alpha; \hat{x}_\ell)$ weakly increases in $\alpha$ for all $j \in N^L_g$. Altogether, (28) decreases in $\alpha$ for all $j \in N^L \setminus \{\ell\}$ and strictly decreases for some $j$, as desired. $\square$

For $g \in N^G$, define

$$U^E_g(\alpha; \hat{x}_\ell) = v^E_1(\alpha; \hat{x}_\ell) + v^E_2(\alpha; \hat{x}_\ell). \quad (31)$$

**Lemma A.6.** Assume $\hat{x}_g \in (0, \pi_{\ell})$ and at least one of Assumption A.1 or A.2 holds. There exists $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $U^E_g(\alpha^\text{dur}; \hat{x}_\ell)$ strictly decreases in $\alpha^{\text{dur}}$.

**Proof.** Let $\alpha^{\text{dur}} = \alpha$. I show $|\partial_{\alpha}\tilde{E}_j(\alpha; \hat{x}_\ell)| > \partial_{\alpha}\pi(\alpha; \hat{x}_\ell)$ for $\hat{x}_\ell$ sufficiently close to $\hat{x}_g$.

By Lemma A.2, there exists $\bar{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $\bar{x} \in \text{int}A(\alpha; \hat{x}_\ell)$ for all $\alpha \in [0, 1]$. Fix $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ and $\alpha \in [0, 1]$.

First, I characterize a lower bound on $|\partial_{\alpha}\pi(\alpha; \hat{x}_\ell)|$. Define

$$\Gamma = \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \tilde{E}^{LB}_j(\hat{x}_g) + (1 - \alpha_j) E^{LB}_j(\hat{x}_g) \right] \frac{\partial u_g(-\pi(\bar{x}))}{\partial \pi(\bar{x})} + \left[ \alpha_j \tilde{E}^{UB}_j(\hat{x}_g) + (1 - \alpha_j) E^{UB}_j(\hat{x}_g) \right] \frac{\partial u_g(\pi(\bar{x}))}{\partial \pi(\bar{x})} \right), \quad (32)$$

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Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\bar{x}(\hat{x}), \bar{x}(\hat{x}))$ implies $\frac{\partial u_g(\bar{x}(\hat{x}))}{\partial \bar{x}(\hat{x})} < 0$ and $\frac{\partial u_g(-\bar{x}(\hat{x}))}{\partial -\bar{x}(\hat{x})} < 0$, and (ii) at least one of Assumptions A.1 and A.2 hold.

I claim $\frac{\partial v_2(\alpha; \hat{\ell})}{\partial \bar{x}(\alpha; \hat{\ell})} < \Gamma$, where

$$
\frac{\partial v_2(\alpha; \hat{\ell})}{\partial \bar{x}(\alpha; \hat{\ell})} = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}^{LB}_j(\alpha_i; \hat{x}_\ell) + (1 - \alpha_j) E^{LB}_j(\alpha_i; \hat{x}_\ell) \right] \frac{\partial u_g(-\bar{x}(\alpha_i; \hat{x}_\ell))}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)} \\
+ \left[ \alpha_j \tilde{E}^{UB}_j(\alpha_i; \hat{x}_\ell) + (1 - \alpha_j) E^{UB}_j(\alpha_i; \hat{x}_\ell) \right] \frac{\partial u_g(\bar{x}(\alpha_i; \hat{x}_\ell))}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)} \\
- I^j_\ell \alpha_j \frac{\partial m^j_\ell(\alpha_i; \hat{x}_\ell)}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)}.
$$

(33)

Three steps show the claim. First, note $\hat{x}_\ell \in (\hat{x}, \hat{x}_g)$ implies $\bar{x}(\hat{x}_g) \geq \bar{x}(\alpha_i; \hat{x}_\ell)$. Thus, we have $\tilde{E}^{UB}_j(\hat{x}_g) \leq \tilde{E}^{UB}_j(\alpha_i; \hat{x}_\ell)$, $E^{UB}_j(\hat{x}_g) \leq E^{UB}_j(\alpha_i; \hat{x}_\ell)$, $\tilde{E}^{LB}_j(\hat{x}_g) \leq \tilde{E}^{LB}_j(\alpha_i; \hat{x}_\ell)$, and $E^{LB}_j(\hat{x}_g) \leq E^{LB}_j(\alpha_i; \hat{x}_\ell)$ for all $j \neq \ell$. Second, $\hat{x}_g < \bar{x}(\hat{x}) < \bar{x}(\alpha_i; \hat{x}_\ell)$ implies $\frac{\partial u_g(\bar{x}(\alpha_i; \hat{x}_\ell))}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)} < \frac{\partial u_g(-\bar{x}(\alpha_i; \hat{x}_\ell))}{\partial -\bar{x}(\alpha_i; \hat{x}_\ell)} < 0$ and symmetrically $\frac{\partial u_g(-\bar{x}(\alpha_i; \hat{x}_\ell))}{\partial -\bar{x}(\alpha_i; \hat{x}_\ell)} < \frac{\partial u_g(-\bar{x}(\hat{x}))}{\partial -\bar{x}(\hat{x})} < 0$. Third, $\frac{\partial m^j_\ell(\alpha_i; \hat{x}_\ell)}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)} \geq 0$ for all $j \in N^g_\ell$ by Lemma A.3.

For almost all $\alpha_i \in [0, 1]$, $\frac{\partial v_2(\alpha; \hat{\ell})}{\partial \alpha_i} = \frac{\partial v_2(\alpha; \hat{\ell})}{\partial \bar{x}(\alpha; \hat{\ell})} \frac{\partial \bar{x}(\alpha; \hat{\ell})}{\partial \alpha_i}$. Define $C_{j}(\alpha_i; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_j(\alpha_i; \hat{x}_\ell)) + \alpha_j(1 - \tilde{C}_j(\alpha_i; \hat{x}_\ell))]$. Then,

$$
\frac{\partial v_2(\alpha; \hat{\ell})}{\partial \alpha_i} < \Gamma \frac{\partial \bar{x}(\alpha_i; \hat{x}_\ell)}{\partial \alpha_i}
$$

(34)

$$
= \delta \rho_i \Gamma \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_i) \right]
$$

(35)

$$
= \frac{2\pi(\alpha_i; \hat{x}_\ell)}{1 - \delta \left( \sum_{j \in N^i} \rho_j C_{j}(\alpha_i; \hat{x}_\ell) \right)}
$$

(36)

$$
< \frac{\delta \rho_i \Gamma}{2\pi} \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_i) \right]
$$

(37)

where (34) follows from $\frac{\partial \bar{x}(\alpha_i; \hat{x}_\ell)}{\partial \alpha_i} > 0$ and $0 > \Gamma > \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \bar{x}(\alpha_i; \hat{x}_\ell)}$; (35) from applying the implicit function theorem to $\bar{x}(\alpha_i; \hat{x}_\ell)$, which is possible for almost all $\alpha_i \in [0, 1]$; (36) because $\Gamma[u_M(\hat{x}_\ell) - u_M(\hat{y}_i)] < 0$, $\bar{x}_\ell > \bar{x}(\alpha_i; \hat{x}_\ell) > 0$, and $\delta \sum_{j \in N^i} \rho_j C_{j}(\alpha_i; \hat{x}_\ell) \in (0, 1)$; and (37) from using $\hat{y}_i = \frac{\hat{x}_g + \hat{x}_\ell}{2}$ and simplifying.

By Lemma A.4, $\frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha_i} = \frac{\rho_i}{2} (\hat{x}_g - \hat{x}_\ell)^2$. By (34), $\frac{\partial u_g^P(\alpha_i; \hat{x}_\ell)}{\partial \alpha_i} < \frac{\partial v_2(\alpha; \hat{x}_\ell)}{\partial \alpha_i} + \Gamma \frac{\partial \bar{x}(\alpha_i; \hat{x}_\ell)}{\partial \alpha_i}$
for almost all $\alpha_\ell \in [0, 1]$. Thus, (37) implies that \( \frac{\partial U^E_g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0 \) if
\[
\frac{\partial t}{2} (\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho t \Gamma}{2 \tau_\ell} \left[ \frac{1}{4} (\hat{x}_g - \hat{x}_\ell) (3 \hat{x}_\ell + \hat{x}_g) \right] < 0,
\]
which holds for $\hat{x}_\ell > \hat{x}_g \left( \frac{4 \tau_\ell + \delta \Gamma}{4 \tau_\ell - 3 \delta \Gamma} \right)$. Define $x' = \max \left\{ \bar{x}, \hat{x}_g \left( \frac{4 \tau_\ell + \delta \Gamma}{4 \tau_\ell - 3 \delta \Gamma} \right) \right\}$. Note $x' < \hat{x}_g$ because (i) $\bar{x} < \hat{x}_g$ and (ii) $\delta \Gamma < 0$ implies $\frac{4 \tau_\ell + \delta \Gamma}{4 \tau_\ell - 3 \delta \Gamma} < 1$. Thus, $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\frac{\partial U^E_g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ for almost all $\alpha_\ell \in [0, 1]$. Continuity implies $U^E_g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$ for such $\hat{x}_\ell$.

\[\Box\]

**Proposition 3** Assume $\hat{x}_g \in (0, \bar{x}_\ell)$ and either Assumption A.1 or A.2. There are cutpoints satisfying $-\hat{x}_g < x'_{\text{del}} < x'_{\text{dur}} < x''_{\text{del}} < x''_{\text{dur}} < \hat{x}_g$ such that:

(i) $\alpha^{*\text{del}}_\ell > 0$ if $\hat{x}_\ell \in (\chi(\hat{x}_g), x'_{\text{del}}) \cup (\hat{x}_g, \chi(\hat{x}_g))$ and $\alpha^{*\text{del}}_\ell = 0$ if $\hat{x}_\ell \in (x''_{\text{del}}, \hat{x}_g)$; and

(ii) $\alpha^{*\text{dur}}_\ell > 0$ if $\hat{x}_\ell \in (\chi(\hat{x}_g), x'_{\text{dur}}) \cup (\hat{x}_g, \chi(\hat{x}_g))$ and $\alpha^{*\text{dur}}_\ell = 0$ if $\hat{x}_\ell \in (x''_{\text{dur}}, \hat{x}_g)$.

**Proof.** Throughout, I typically spare notation using $\alpha^{*\text{del}}_\ell = \alpha_\ell$ and when necessary refer to $\alpha^{*\text{del}}_\ell$ explicitly.

**Case 1.** Consider $\hat{x}_\ell \in [0, \hat{x}_g)$. By Lemma A.2, there exists $\bar{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. By Lemma A.6, there exists $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $U^E_g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$. Let $x''_{\text{dur}} = \max\{\bar{x}, x'\}$ and consider $\hat{x}_\ell \in (x''_{\text{dur}}, \hat{x}_g)$. Then $z_\ell = \hat{x}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ and $y''_\ell = y' \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. Thus, $g'$s ex-ante expected utility from durable access equals $U^E_g(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$, so $g$ strictly prefers $\alpha^{*\text{dur}}_\ell = 0$. For delayed access, $g'$s ex-ante expected utility equals $U^E_g(\alpha_\ell; \hat{x}_\ell) = \rho u_{\text{del}}(\hat{x}_\ell)$. Because $U^E_g(\alpha_\ell; \hat{x}_\ell) > \rho u_{\text{del}}(\hat{x}_\ell)$ for all $\hat{x}_\ell \in (\chi(\hat{x}_g), \chi(\hat{x}_g))$ and $g'$s ex-ante payoff is continuous, there exists $x''_{\text{del}} < x''_{\text{dur}}$ such that $g$ strictly prefers $\alpha^{*\text{del}}_\ell = 0$ if $\hat{x}_\ell \in (x''_{\text{del}}, \hat{x}_g)$.

**Case 2.** Consider $\hat{x}_\ell \in (\hat{x}_g, \chi(\hat{x}_g))$. It suffices to show that $g'$s ex ante expected utility strictly increases as $\alpha^{*\text{dur}}_\ell$ increases from zero. A similar argument shows the result for $\alpha^{*\text{del}}_\ell$. There are two cases.

- If $\hat{x}_\ell < \bar{x}_\ell$, then $g'$s ex ante expected payoff equals $U^E_g(\alpha_\ell; \hat{x}_\ell)$ for sufficiently small $\alpha_\ell$. By Lemma A.4, $\frac{\partial U^E_g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$. To complete this case, I argue that $U^E_g(\alpha_\ell; \hat{x}_\ell)$ increases for sufficiently small $\alpha_\ell$. Under the maintained assumptions,
Proof. From (9) and (10), we have \( \hat{x}_g \in (\hat{\overline{x}}(0; \hat{x}_e), \overline{x}(0; \hat{x}_e)) \) and \( \hat{y}_e \in (\hat{x}_g, \overline{x}(0; \hat{x}_e)) \). Thus, \( \overline{x}(\alpha \ell; \hat{x}_e) \) strictly decreases for sufficiently small \( \alpha \ell \). Therefore \( u_g(-\overline{x}(\alpha \ell; \hat{x}_e)) \) and \( u_g(\overline{x}(\alpha \ell; \hat{x}_e)) \) are strictly increasing for such \( \alpha \ell \). Lemma A.3 implies \( m^j_\alpha(\alpha \ell; \hat{x}_e) \) weakly decreases in \( \alpha \ell \) for all \( j \in N^L_g \backslash \{ \ell \} \). Thus, \( v^g(\alpha \ell; \hat{x}_e) \) strictly increases over sufficiently small \( \alpha \ell \).

- If \( \hat{x}_e > \overline{x}_e \), then \( \overline{x}(0; \hat{x}_e) = \overline{x}_e \). Thus, \( g \)'s ex ante expected payoff from \( \alpha \ell = 0 \) is

\[
\rho_\ell \left( \alpha \ell \left[ u_g(\hat{y}_e) + u_e(\hat{y}_e) - u_e(\overline{x}_e) \right] + (1 - \alpha \ell) u_g(\overline{x}_e) \right) + \sum_{j \neq \ell} \rho_j \left[ \alpha_j \overline{E}_j^{LB}(0; \hat{x}_e) + (1 - \alpha_j) \overline{E}^{LB}_j(0; \hat{x}_e) \right] u_g(-\overline{x}_e)
\]

\[
+ \alpha_\ell \overline{C}_\ell(0; \hat{x}_e) u_g(\hat{y}_e) + (1 - \alpha_\ell) C_\ell(0; \hat{x}_e) u_g(\hat{x}_e)
\]

\[
- \ell^j_\alpha \alpha_j \overline{m}_g^j(0; \hat{x}_e). \tag{38}
\]

Arguments similar to Case 1 show that (38) strictly increases in \( \alpha \ell \) at \( \alpha \ell = 0 \).

Case 3. Consider \( \hat{x}_e < 0 \). For \( \hat{x}_e \in (\chi(\hat{x}_g), -\hat{x}_g) \), arguments analogous to Case 2 show that \( g \)'s ex-ante expected payoff strictly increases at \( \alpha \ell^{\text{dur}} = 0 \) and \( \alpha \ell^{\text{del}} = 0 \). Because \( g \)'s ex-ante expected payoff from durable access weakly dominates that of delayed access, and both are continuous in \( \hat{x}_e \), there exist \( x^{\text{dur}}_\ell > x^{\text{del}}_\ell > -\hat{x}_g \) such that \( \hat{x}_e \in (x(\hat{x}_g), x^{\text{del}}_\ell) \) implies \( \alpha \ell^{\text{del}} > 0 \) and \( \hat{x}_e \in (x(\hat{x}_g), x^{\text{dur}}_\ell) \) implies \( \alpha \ell^{\text{dur}} > 0 \).

Proposition 4. The interest group never prioritizes \( \alpha \ell^{\text{del}} \).

Proof. From (9) and (10), we have

\[
WTA_{\text{del}} = \left( \rho_R \frac{\partial u_g(\overline{x}_e^\alpha)}{\partial \overline{x}_e^\alpha} - \rho_L \frac{\partial u_g(-\overline{x}_e^\alpha)}{\partial \overline{x}_e^\alpha} \right) \frac{\partial \overline{x}_e^\alpha}{\partial \alpha} \bigg|_{\alpha = 0} + \rho_\ell \frac{\partial u_g(z_0^\alpha) \partial z_0^\alpha}{\partial \alpha} \bigg|_{\alpha = 0}, \tag{39}
\]

\[
WTA_{\text{dur}} = \rho_\ell \left( u_g(y_0^\alpha) + u_e(y_0^\alpha) - u_\ell(\overline{z}_0^\alpha) - u_g(\overline{z}_0^\alpha) \right)
\]

\[
+ \left( \rho_R \frac{\partial u_g(\overline{z}_0^\alpha)}{\partial \overline{z}_0^\alpha} - \rho_L \frac{\partial u_g(-\overline{z}_0^\alpha)}{\partial \overline{z}_0^\alpha} \right) \frac{\partial \overline{z}_0^\alpha}{\partial \alpha} \bigg|_{\alpha = 0} + \rho_\ell \frac{\partial u_g(z_0^\alpha) \partial z_0^\alpha}{\partial \alpha} \bigg|_{\alpha = 0}. \tag{40}
\]

Thus, \( WTA_{\text{dur}} \geq WTA_{\text{del}} \) follows from \( u_g(y_0^\alpha) - u_g(z_0^\alpha) + u_\ell(y_0^\alpha) - u_\ell(z_0^\alpha) \geq 0 \). This observation carries over to the extended model.
Proposition 5. The interest group prioritizes $\alpha_{\ell}^{\text{dur}}$ iff $\text{WTA}_{\text{del}} > 0$.

Proof. From (8), we have $\text{WTA}_{\text{temp}} = \rho_{\ell}\left(u_{g}(y_{0}^{\ell}) + u_{\ell}(y_{0}^{\ell}) - u_{\ell}(z_{0}^{\ell}) - u_{g}(z_{0}^{\ell})\right)$. Thus, (39) and (40) imply $\text{WTA}_{\text{dur}} = \text{WTA}_{\text{temp}} + \text{WTA}_{\text{del}}$. Therefore $\text{WTA}_{\text{dur}} > \text{WTA}_{\text{temp}}$ iff $\text{WTA}_{\text{del}} > 0$. The result carries over to the extended model. \qed

Proposition 6. If $\hat{x}_{g} \in (0, \overline{x}_{\ell})$, then there exist cutpoints satisfying $-\overline{x} < x' < x'' < \overline{x}$ such that $g$ prioritizes:

(i) $\alpha_{\ell}^{\text{temp}}$ if $\hat{x}_{\ell} \in (x'', \hat{x}_{g})$ and only if $\hat{x}_{\ell} \in (x', \hat{x}_{g})$; and

(ii) $\alpha_{\ell}^{\text{dur}}$ if $\hat{x}_{\ell} \in (x(\hat{x}_{g}), x) \cup (\hat{x}_{g}, \chi(\hat{x}_{g}))$ and only if $\hat{x}_{\ell} \in (x(\hat{x}_{g}), x'') \cup (\hat{x}_{g}, \chi(\hat{x}_{g}))$.

Proof. Part (i) follows immediately from Propositions 3 and 5. For Part (ii), arguments in Case 3 in the proof of Proposition 3 implies existence of $x'$ and $x''$ satisfying $-\overline{x} < x' < x'' < \overline{x}$ such that $\text{WTA}_{\text{dur}} > 0$ if $\hat{x}_{\ell} \in (x(\hat{x}_{g}), x') \cup (\hat{x}_{g}, \chi(\hat{x}_{g}))$ and only if $\hat{x}_{\ell} \in (x(\hat{x}_{g}), x'') \cup (\hat{x}_{g}, \chi(\hat{x}_{g}))$. Proposition 5 then implies the result. \qed

We now prove the extended version of Proposition 7.

Proposition 7. Assume $\hat{x}_{g} \geq \overline{x}_{\ell}$.

(i) If $\sum_{i \in N_{L}} \rho_{i} [(1 - \alpha_{i}) I\{\hat{x}_{i} \leq -\overline{x}\} + \alpha_{i} I\{\hat{y}_{i} \leq -\overline{x}\}]$ is sufficiently small, then there exists $x' < 0$ such that $g$ prioritizes $\alpha_{\ell}^{\text{dur}}$ if $\hat{x}_{\ell} \in (x', \overline{x})$.

(ii) If $\sum_{i \in N_{L}} \rho_{i} [(1 - \alpha_{i}) I\{\hat{x}_{i} \geq \overline{x}\} + \alpha_{i} I\{\hat{y}_{i} \geq \overline{x}\}]$ is sufficiently small, then there exists $x'' \geq -\overline{x}$ such that $g$ prioritizes $\alpha_{\ell}^{\text{dur}}$ if $\hat{x}_{\ell} \in (\chi(\hat{x}_{g}), x'')$.

Proof. Let $\alpha_{\ell}^{\text{dur}} = \alpha_{\ell}$.

I prove (i), as (ii) is analogous. Consider $\hat{x}_{\ell} \in [0, \overline{x}_{\ell})$ and assume $\sum_{i \in N_{L}} \rho_{i} [(1 - \alpha_{i}) I\{\hat{x}_{i} \leq -\overline{x}\} + \alpha_{i} I\{\hat{y}_{i} \leq -\overline{x}\}] = 0$. I show that $g$’s ex-ante expected payoff strictly increases at $\alpha_{\ell} = 0$.

We have $\hat{x}_{\ell} \in [0, \overline{x}(0; \hat{x}_{\ell}))$ and $\hat{y}_{\ell} > \hat{x}_{\ell}$. Therefore $0 \leq z_{\ell}(0; \hat{x}_{\ell}) = \hat{x}_{\ell} < y_{g}^{\ell}(0; \hat{x}_{\ell}) \leq \hat{y}_{\ell}$. Furthermore, $-\overline{x}(0; \hat{x}_{\ell})$ is not proposed with positive probability because $\sum_{i \in N_{L}} \rho_{i} [(1 - \alpha_{i}) I\{\hat{x}_{i} \leq -\overline{x}\} + \alpha_{i} I\{\hat{y}_{i} \leq -\overline{x}\}] = 0$. Thus, $g$’s ex-ante expected payoff from $\alpha_{\ell} = 0$ is

$$\rho_{\ell} \left( \alpha_{\ell} \left[ u_{g}(y_{g}^{\ell}(0; \hat{x}_{\ell})) + u_{\ell}(y_{g}^{\ell}(0; \hat{x}_{\ell})) - u_{\ell}(\hat{x}_{\ell}) \right] + (1 - \alpha_{\ell}) u_{g}(\hat{x}_{\ell}) \right) + \sum_{j \neq \ell} \rho_{j} \left[ \alpha_{j} \tilde{E}_{j}^{UB}(0; \hat{x}_{\ell}) + (1 - \alpha_{j}) E_{j}^{UB}(0; \hat{x}_{\ell}) \right] u_{g}(\overline{x}(0; \hat{x}_{\ell})).$$
\[
+ \alpha_j \left( C_j(0; \hat{x}_t) u_g(\hat{y}_j) - I^j g \left( 0; \hat{x}_t \right) \right) + (1 - \alpha_j) C_j(0; \hat{x}_t) u_g(\hat{x}_j). \tag{41}
\]

Three steps show (41) strictly increases at \( \alpha_\ell = 0 \).

- First, \( 0 \leq \hat{x}_\ell \leq y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \) implies \( y^g_\ell(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Therefore \( u_g(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly increases and \( u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) weakly decreases. Because \( u \) is quadratic and \( \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell = \frac{x_g + \hat{x}_\ell}{2} \leq \hat{x}_g \), it follows that \( u_g(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) increases weakly faster than \( u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) \) decreases. Therefore \( u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) \) weakly increases in \( \alpha_\ell \). Furthermore, \( \hat{x}_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell \) also implies \( u_g(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) + u_\ell(y^g_\ell(\alpha_\ell; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0 \). It follows that \( \alpha_\ell \left[ u_g(y^g_\ell(0; \hat{x}_\ell)) + u_\ell(y^g_\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \) weakly increases at \( \alpha_\ell = 0 \).

- Second, \( \bar{v}(0; \hat{x}_\ell) \) strictly increases in \( \alpha_\ell \) because \( 0 \leq z_\ell < y^g_\ell(0; \hat{x}_\ell) \leq \bar{v}(0; \hat{x}_\ell) \). Since \( \bar{v}(0; \hat{x}_\ell) \leq \hat{x}_g \), it follows that \( u_g(\bar{v}(0; \hat{x}_\ell)) \) increases at \( \alpha_\ell = 0 \).

- Third, \( m^g_\ell(0; \hat{x}_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N^L_g \) by Lemma A.3. But it strictly increases only for \( j \in N^L_g \) such that \( \hat{y}_j > \bar{v}(0; \hat{x}_\ell) \). Thus, \( g \)'s lobbying surplus weakly increases in \( \alpha_\ell \) for all \( j \in N^L_g \).

The desired result follows because \( g \)'s ex-ante expected payoff is continuous in \( \sum_{i \in N^L} \rho_i [(1 - \alpha_i) \mathbb{1}\{\hat{x}_i \leq -\bar{v}\} + \alpha_i \mathbb{1}\{\hat{y}_i \leq -\bar{v}\}] \). \( \square \)

**Willingness to Pay for Access**

The following applies to the model in the main text. Define \( \theta = (\hat{x}, \rho, \alpha) \). Let \( U^E_g(\theta) \) be \( g \)'s ex-ante expected utility and let \( \bar{v}_\alpha = \bar{v}(\alpha; \hat{x}_\ell) \). Define \( \frac{\partial \bar{v}_\alpha}{\partial \alpha} = \left. \frac{\partial \bar{v}_\alpha}{\partial \alpha} \right|_{\alpha = 0} \), \( \frac{\partial \bar{v}_\alpha}{\partial \hat{x}_\ell} = \left. \frac{\partial \bar{v}_\alpha}{\partial \hat{x}_\ell} \right|_{\alpha = 0} \), and \( \frac{\partial^2 \bar{v}_\alpha}{\partial \alpha \partial \hat{x}_\ell} = \left. \frac{\partial^2 \bar{v}_\alpha}{\partial \alpha \partial \hat{x}_\ell} \right|_{\alpha = 0} \).

To state the result, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, replace \( \ell \) with two legislators, \( \ell_1 \) and \( \ell_2 \), and replace \( g \) with two groups, \( g_1 \) and \( g_2 \). To isolate differences in proposal power, assume \( \hat{x}_{g_1} = \hat{x}_{g_2} \) and \( \hat{x}_{\ell_1} \neq \hat{x}_{\ell_2} \). These modifications do not qualitatively change the equilibrium characterization. Two identical groups avoid complications arising if one group has access to two legislators, where access to one legislator can affect offers to the other.
Proposition A.1. Consider the modified baseline model with: \(\ell_1\) and \(\ell_2\) such that \(\hat{x}_{\ell_1} = \hat{x}_{\ell_2}\), and \(g_1\) and \(g_2\) satisfying \(\hat{x}_{g_1} = \hat{x}_{g_2}\). For all \(\alpha \in [0, 1]\), \(\rho_{\ell_2} > \rho_{\ell_1}\) implies \(\frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2 = \alpha} \geq \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1 = \alpha}\).

Proof. It suffices to show that \(\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1 = \alpha} \geq 0\) implies \(\frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2 = \alpha} \geq \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1 = \alpha}\) for \(\alpha \in [0, 1]\).

Because \(\hat{x}_{\ell_1} = \hat{x}_{\ell_2}\) and \(\hat{x}_{g_1} = \hat{x}_{g_2}\), we have \(y_{g_1} = y_{g_2}\) and \(z_{\ell_1} = z_{\ell_2}\). Thus, \(m_{g_1} = m_{g_2}\). Denote \(y = y_{g_1}\), \(z = z_{\ell_1}\), and \(m = m_{g_1}\). Assume \(\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1 = \alpha} \geq 0\). There are five cases.

- **Case 1:** Suppose \(z = \bar{x}_\alpha\) and \(y = \bar{y}\). Then,

\[
\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_1}|_{\alpha_1 = \alpha} = \rho_{\ell_1} \left( u_{g_1}(\bar{y}) + u_{\ell_1}(\bar{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) - \frac{\partial \bar{x}_\alpha}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
= \rho_{\ell_1} \left( u_{g_1}(\bar{y}) + u_{\ell_1}(\bar{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)
+ \frac{\delta[u_{M}(\bar{y}) - u_{M}(\hat{x}_\ell)]}{\partial u_{M}(\bar{x}_\alpha)} \left( \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\leq \rho_{\ell_2} \left( u_{g_1}(\bar{y}) + u_{\ell_1}(\bar{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)
+ \frac{\delta[u_{M}(\bar{y}) - u_{M}(\hat{x}_\ell)]}{\partial u_{M}(\bar{x}_\alpha)} \left( \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
= \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_2}|_{\alpha_2 = \alpha},
\]

where (42) follows from \(\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1}[u_{M}(\bar{y}) - u_{M}(\hat{x}_\ell)]}{\delta u_{M}(\bar{x}_\alpha)(1 - \delta(\rho_L + \rho_R))}\); (43) because (i) \(\rho_{\ell_2} > \rho_{\ell_1}\) and (ii) \(\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_2}|_{\alpha_2 = \alpha} \geq 0\) implies the bracketed expression in (42) is positive; and (44) because \(\hat{x}_{\ell_1} = \hat{x}_{\ell_2}\), \(\hat{x}_{g_1} = \hat{x}_{g_2}\), and \(\frac{\partial \bar{x}_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_1}[u_{M}(\bar{y}) - u_{M}(\hat{x}_\ell)]}{\delta u_{M}(\bar{x}_\alpha)(1 - \delta(\rho_L + \rho_R))}\).

- **Case 2:** Suppose \(z = \bar{x}_\alpha\) and \(y = \bar{y}\). In this case, \(\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1}[u_{M}(\bar{y}) - u_{M}(\bar{x}_\alpha)]}{\delta u_{M}(\bar{x}_\alpha)(1 - \delta(\rho_L + \rho_R)(1 - \delta(\rho_{\ell_1} + \rho_{\ell_2}))}\).
and \( \frac{\partial \sigma}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(y) - u_M(x_\alpha)]}{\partial u_M(\partial x_\alpha)} \). Arguments analogous to Case 1 show

\[
\frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}.
\]

The argument for \( z = x_\alpha \) and \( y = \hat{y} \) is symmetric.

**Case 3:** Suppose \( z = x_\ell \) and \( y = x_\alpha \). In this case, \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(x_\alpha) - u_M(\hat{x}_\ell)]}{\partial u_M(\partial x_\alpha)} \(1-\delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2}))\) and \( \frac{\partial \sigma}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(x_\alpha) - u_M(\hat{x}_\ell)]}{\partial u_M(\partial x_\alpha)} \(1-\delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2}))\). Arguments analogous to Case 1 show

\[
\frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}.
\]

The argument for \( z = x_\ell \) and \( y = \tilde{x}_\alpha \) is symmetric.

**Case 4:** Suppose \( z = \tilde{x}_\alpha \) and \( y = -x_\alpha \). In this case, \( \frac{\partial \sigma}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(-x_\alpha) - u_M(x_\alpha)]}{\partial u_M(\partial x_\alpha)} \(1-\delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2}))\) and \( \frac{\partial \sigma}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(-x_\alpha) - u_M(x_\alpha)]}{\partial u_M(\partial x_\alpha)} \(1-\delta(\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2}))\). Arguments analogous to Case 1 show

\[
\frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} \geq \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha}.
\]

The argument for \( z = -x_\alpha \) and \( y = \tilde{x}_\alpha \) is symmetric.

**Case 5:** Suppose \( z = \tilde{x}_\alpha \) and \( y = \tilde{x}_\alpha \). Then,

\[
\frac{\partial U^E_{g_2}(\theta)}{\partial \alpha_2} \bigg|_{\alpha_2 = \alpha} = \frac{\partial U^E_{g_1}(\theta)}{\partial \alpha_1} \bigg|_{\alpha_1 = \alpha} = 0.
\]

The argument for \( z = -x_\alpha \) and \( y = -\tilde{x}_\alpha \) is symmetric.

\( \square \)
Appendix B  Equivalence of Outcome Distribution

A stationary strategy profile $\sigma = (\lambda, \pi, \varphi, \nu)$ is a stationary legislative lobbying equilibrium if it satisfies four conditions. First, for all $g \in N^G$ and $\ell \in N^L_g$, $\lambda^g_\ell$ places probability one on

$$\arg \max_{(y,m)} \nu(y)u(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)] = m$$

subject to

$$\nu(y)u(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)] + m \geq \int_X [\nu(x)u(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)]] \pi_\ell(dx).$$

(45)

Second, for all $\ell \in N^L$ and $(y, m) \in W$,

$$\nu(y)u(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)] + m > \int_X [\nu(x)u(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)]] \pi_\ell(dx).$$

(46)

implies $\varphi_\ell(y, m) = 1$ and the opposite strict inequality implies $\varphi_\ell(y, m) = 0$. Third, for all $\ell \in N^L$,

$$\pi_\ell \left( \arg \max_{x \in X} \nu(x)u(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)] \right) = 1.$$  

(47)

Finally, for all $i \in N^V$ and $x \in X$, $u_i(x) > (1 - \delta)u_i(q) + \delta V_i(\sigma)$ implies $\nu_i(x) = 1$ and the opposite strict inequality implies $\nu_i(x) = 0$.\footnote{Thus, voting strategies are stage-undominated (Baron and Kalai, 1993; Banks and Duggan, 2006a).}

Lemma B.1 shows surplus lobby payments never happen in equilibrium.

Lemma B.1. In every stationary legislative lobbying equilibrium, for all $\ell \in N^L$ every $(y, m) \in \text{supp}(\lambda^g_\ell)$ satisfies

$$\nu(y)u(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)] + m = \int_X [\nu(x)u(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell(\sigma)]] \pi_\ell(dx).$$

(48)
The proof of Lemma B.1 is straightforward and omitted.

From (11), recall \( \xi(\alpha; \sigma) = (1 - \alpha) + \alpha \int_W [1 - \varphi(y, m)] \lambda_g^\ell(dw) \). Define

\[
\hat{\chi}(X') = \sum_{\ell \in N_L} \rho_\ell \left( \xi(\alpha; \sigma) \int_{X'} \varphi(y, m) \lambda_g^\ell(dw) + \alpha \int_{W} \varphi(y, m) \lambda_g^\ell(dw) \right), \tag{49}
\]

the probability some \( x \in X' \subseteq X \) is passed in a given period under \( \sigma \). Next, define

\[
\hat{\tilde{\chi}} = \sum_{\ell \in N_L} \rho_\ell \left( \xi(\alpha; \sigma) \int_{X'} [1 - \varphi(y, m)] \lambda_g^\ell(dw) + \alpha \int_{W} \varphi(y, m) [1 - \varphi(y)] \lambda_g^\ell(dw) \right), \tag{50}
\]

the probability of a failed proposal in a given period under \( \sigma \).

Following Banks and Duggan (2006a), each player’s continuation value can be expressed as a function of a common lottery over policy, denoted \( \chi^\sigma \). Using (49) and (50), define \( \chi^\sigma \) so that for all measurable \( X' \subseteq X \): (i) if \( q \notin X' \), then \( \chi^\sigma(X') = \frac{\hat{\chi}(X')}{1 - \delta \hat{\tilde{\chi}}} \), and (ii) if \( q \in X' \), then \( \chi^\sigma(X') = \frac{\hat{\chi}(X') + (1 - \delta) \hat{\tilde{\chi}}}{1 - \delta \hat{\tilde{\chi}}} \).

Set \( V_{\text{den}}(\sigma) = 1 - \delta \hat{\tilde{\chi}} \) and define

\[
V_{i,\text{num}}(\sigma) = \sum_{\ell \in N_L} \rho_\ell \left( \xi(\alpha; \sigma) \int_{X} \left[ \varphi(y, m) u_i(x) + [1 - \varphi(y)] (1 - \delta) u_i(q) \right] \lambda_g^\ell(dw) 
\right)
\]

\[
+ \alpha \int_{W} \varphi(y, m) \left[ \varphi(y, m) u_i(x) + [1 - \varphi(y)] (1 - \delta) u_i(q) \right] \lambda_g^\ell(dw) \right) \tag{51}
\]

For each \( i \in N \), \( i \)'s continuation value defined in (12) satisfies \( V_i(\sigma) = \frac{V_{i,\text{num}}(\sigma)}{V_{\text{den}}(\sigma)} \). Then we can express \( V_i(\sigma) \) as a lottery over policy, \( V_i(\sigma) = \int_X u_i(x) \chi^\sigma(dx) \).

The policy lottery \( \chi^\sigma \) is common to all players, but committee members may receive payment and interest groups may make payments. Define

\[
\hat{m}_\ell(\sigma) = \rho_\ell \alpha \int_W m \varphi(y, m) \lambda_g^\ell(dw), \tag{51}
\]

which is \( \ell \)'s expected lobby payment in each period until passage. For \( \ell \in N_L \), rearranging (13) yields

\[
\tilde{V}_\ell(\sigma) = \frac{V_{\ell,\text{num}}(\sigma) + \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}
\]

50
\[
\int_X u_\ell(x) \chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}.
\] (52)

Similarly, for \( g \in N^G \) rearranging (14) yields
\[
\hat{V}_g(\sigma) = \frac{V_{\text{num}}^g(\sigma) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}
= \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N^L_g} \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \tag{53}
\]

Finally, define
\[
\tilde{U}_\ell(\sigma) = \int_X \left[ \overline{\nu}_\sigma(x) u_\ell(x) + \left( 1 - \overline{\nu}_\sigma(x) \right) \left( (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right) \right] \pi_\ell(dx), \tag{54}
\]
which is \( \ell \)'s expected dynamic payoff under \( \sigma \) conditional on being recognized as the proposer and rejecting \( g_\ell \)'s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium \( \sigma \) such that \( \chi^\sigma \) is degenerate on \( q \).

**Proof.** Let \( \sigma \) denote an equilibrium. To show a contradiction, assume \( \chi^\sigma(q) = 1 \). Thus, \( V_M(\sigma) = u_M(q) \), which implies \( u_M(q) \geq (1 - \delta) u_M(q) + \delta V_M(\sigma) \) and therefore \( q \in A(\sigma) \). Without loss of generality, assume \( q > 0 \).

By assumption, there exists \( \ell \in N^L \) such that \( \hat{x}_\ell < q \) and at least one of \( \hat{x}_{g_\ell} \leq q \) or \( \alpha_\ell < 1 \) holds. If \( \alpha_\ell < 0 \), then it is straightforward to show that \( \ell \) must have a profitable deviation, a contradiction.

For the other case, suppose \( \hat{x}_\ell < q, \hat{x}_{g_\ell} \leq q \), and \( \alpha_\ell = 1 \). Note that \( u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) is \( g_\ell \)'s expected dynamic payoff from any offer \((y,m)\) such that \( \overline{\nu}_\sigma(y) = 1 \), \( \varphi_\ell(y,m) = 1 \), and \( \ell \) is indifferent between accepting and rejecting. We have \( \hat{y}_\ell = \arg \max_{y \in X} u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) and \( \hat{y}_\ell < q \). Strict concavity and continuity imply existence of \( \varepsilon > 0 \) and \( y^\varepsilon < q \) such that \( \overline{\nu}_\sigma(y^\varepsilon) = 1 \), \( \varphi_\ell(y^\varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1 \), and
\[
u_{g_\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \tilde{U}_\ell(\sigma) - \varepsilon > u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) \tag{55}
\)
\[
\geq u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) - \delta \left( \sum_{j \in N^L_g} \frac{\hat{m}_j(\sigma)}{V_{\text{den}}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \right), \tag{56}
\]
where (56) follows from $\sum_{j \in N^L_y} \frac{m_j(\sigma)}{V^{\text{den}}(\sigma)} \geq \frac{m_q(\sigma)}{V^{\text{den}}(\sigma)}$. The RHS of (55) is weakly greater than $g_\ell$’s expected payoff from lobbying $\ell$ to $q$ if $\nu_\sigma(q) = 1$; and (56) is weakly greater than $g_\ell$’s expected payoff from lobbying $\ell$ to any $y'$ such that $\nu_\sigma(y') = 0$. Thus, $g_\ell$ must have a profitable deviation, a contradiction. \hfill \Box

**Lemma B.3.** Let $\sigma$ denote a stationary legislative lobbying equilibrium. For all $\ell \in N^L$ there exists $(y, m) \in X \times R_+$ such that $\nu_\sigma(y) = 1$ and $g_\ell$ strictly prefers $(y, m)$ to any $(y', m')$ such that $\nu_\sigma(y') = 0$.

*Proof.* Fix an equilibrium $\sigma$. Let $\chi^q$ denote a probability distribution degenerate on $q$. Define the continuation distribution following rejection under $\sigma$ as $\chi = (1 - \delta)\chi^q + \delta\chi^\sigma$, which is non-degenerate because $\delta \in (0, 1)$ and $\chi^\sigma(q) < 1$ by Lemma B.2.

For every player $k \in N$, the expected dynamic policy payoff from a rejected policy proposal satisfies

$$(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).$$

Let $x^\sigma$ denote the mean of $\chi$. Since $u$ is strictly concave and $\chi$ is non-degenerate, Jensen’s Inequality implies

$$u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma). \quad (57)$$

Consider $\ell \in N^L$. First, assume $\phi_\ell(y, m) = 1$ whenever $\ell$ is indifferent. The condition for $g_\ell$ to strictly prefer $(y, m)$ such that $\nu_\sigma(y) = 1$, rather than $(y', m')$ such that $\nu_\sigma(y') = 0$, is

$$u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\sigma).$$

Equivalently,

$$u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma). \quad (58)$$

Notice that

$$\hat{V}_{g_\ell}(\sigma) + \tilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{e' \in N^L_y} \frac{\tilde{m}_{e'}(\sigma)}{V^{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\tilde{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)}. \quad (59)$$
\[ V_g(\sigma) \leq V_{\hat{g}}(\sigma) - \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\hat{m}_M(\sigma)}{V_{\text{den}}(\sigma)} \]

(60)

\[ = V_{\hat{g}}(\sigma) + V_\ell(\sigma), \]

(61)

where (59) follows from substituting for \( \tilde{V}_\ell(\sigma) \) and \( \tilde{V}_g(\sigma) \) using (52) and (53); and (60) from \( \sum_{\ell' \in N_\ell^L} \frac{\hat{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \)

By (57), \( \nu_\sigma(x^\sigma) = 1 \) follows because \( u_M(x^\sigma) > (1 - \delta)u_M(q) + \delta V_M(\sigma). \) Furthermore, (57) implies \( u_{g_\ell}(x^\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma) \) and \( u_\ell(x^\sigma) > (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \). Thus, (61) implies that (58) holds because

\[ u_{g_\ell}(x^\sigma) + u_\ell(x^\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \]

\[ \geq (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma), . \]

Next, assume \( \varphi_\ell(x^\sigma, m) < 1 \) for \( m \) such that \( \ell \) is indifferent between accepting \((x^\sigma, m)\) and rejecting. For sufficiently small \( \epsilon > 0, \varphi_\ell(x^\sigma, m + \epsilon) = 1 \) and the preceding argument implies \( g_\ell \) strictly prefers \((x^\sigma, m + \epsilon)\) over any \((y', m')\) such that \( \nu_\sigma(y') = 0. \)

**Lemma B.4.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

**Proof.** Let \( \sigma \) be an equilibrium. By Duggan (2014), \( M \) is decisive. Quadratic utility and \( \hat{x}_M = 0 \neq q \) together imply \( A(\sigma) = \{x \in X|u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)\} \) is a closed, non-empty interval symmetric about 0. Let \( A(\sigma) = [-\overline{x}(\sigma), \overline{x}(\sigma)]. \) Then \( x \in (\overline{x}(\sigma), -\overline{x}(\sigma)) \) implies \( \nu_\sigma(x) = 1. \)

Fix \( \ell \in N^L. \) By Lemma B.2, \( \chi^\sigma(q) < 1. \) Lemma B.3 implies existence of \((y, m) \in W \) such that \( \nu_\sigma(y) = 1 \) and \( g_\ell \) strictly prefers \((y, m)\) over all \((y', m')\) with \( \nu_\sigma(y') = 0. \) Thus, \( y \in A(\sigma) \) for all \((y, m) \in \text{supp}(\lambda_{g_\ell}). \) Without loss of generality, assume \( \nu_\sigma(-\overline{x}(\sigma)) < 1. \) It suffices to check two cases.

- **Case 1:** If \( \hat{x}_\ell \leq -\overline{x}(\sigma) \) and \( u_\ell(-\overline{x}(\sigma)) > (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma), \) then \( x \in A(\sigma) \) for all \( x \in \text{supp}(\pi_\ell). \) Because \( u_\ell \) is strictly concave and continuous, and \( \nu_\sigma(-\overline{x}(\sigma)) < 1, \) there exists \( \epsilon > 0 \) such that \( \ell \) has a profitable deviation to \(-\overline{x}(\sigma) + \epsilon, \) a contradiction.

- **Case 2:** Assume \( \hat{y}_\ell \leq -\overline{x}(\sigma). \) Continuity, Lemma B.3, and \( \nu_\sigma(-\overline{x}(\sigma)) < 1 \) imply existence of \( \epsilon, \epsilon' > 0 \) such that \( g_\ell \) has a profitable deviation to \((y', m') = (-\overline{x}(\sigma) + \epsilon, \tilde{U}_\ell(\sigma) - u_\ell(-\overline{x}(\sigma) + \epsilon) + \epsilon'), \) a contradiction.

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It follows that either \( \sigma \) must involve deferential voting, or \( \sigma \) is equivalent in outcome distribution to an equilibrium with deferential voting.

\( \square \)

**Lemma B.5.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

\begin{proof}
Let \( \sigma \) denote an equilibrium. By Lemma B.4, we can assume \( \pi_{\sigma}(x) = 1 \) iff \( x \in A(\sigma) \). Fix \( \ell \in N^L \) and define \( y_{\sigma}^* = \arg\max_{y \in A(\sigma)} y \in \pi_{\sigma}(x) \) uniquely defined, and \( m_{\sigma}^* = \sum_{y \in A(\sigma)} m_{\sigma}(y) \).

By Lemma B.2, \( \chi_{\sigma}(q) < 1 \). For sufficiently small \( \varepsilon > 0 \), Lemma B.3 implies \( g \) strictly prefers \( (y_{\sigma}^*, m_{\sigma}^* + \varepsilon) \) over every \( (y', m') \) such that \( y' \notin A(\sigma) \). Thus, if \( \pi_{\ell} \) is not degenerate on \( y_{\sigma}^* \) and \( \varphi_{\ell}(y_{\sigma}^*, m_{\sigma}^*) < 1 \), then there exists \( \varepsilon > 0 \) such that \( g_{\ell} \) has a profitable deviation to \( (y_{\sigma}^*, m_{\sigma}^* + \varepsilon) \), a contradiction. Thus, \( \sigma \) must satisfy either (i) \( \pi_{\ell}(y_{\sigma}^*) = 1 \), or (ii) \( \chi_{\sigma}(y_{\sigma}^*, m_{\sigma}^*) = 1 \) and \( \varphi_{\ell}(y_{\sigma}^*, m_{\sigma}^*) = 1 \), as desired.

A strategy profile \( \sigma \) is no-delay if \( \pi_{\sigma}(x) = 1 \) for all \( x \in \text{supp}(\pi_{\ell}) \) and \( \pi_{\sigma}(y) = 1 \) for all \( (y, m) \in \text{supp}(\lambda_{\sigma}^\ell) \).

**Lemma B.6.** Every stationary legislative lobbying equilibrium is no-delay.

\begin{proof}
Fix an equilibrium \( \sigma \). By Lemma B.2, \( \chi_{\sigma}(q) < 1 \). Thus, Lemma B.3 implies \( g \) strictly prefers some \( (y, m) \in W \) such that \( \pi_{\sigma}(y) = 1 \). Lemma B.4 implies we can assume \( \pi_{\sigma}(x) = 1 \) iff \( x \in A(\sigma) \). Lemma B.5 implies we can assume all \( \ell \in N^L \) use deferential acceptance strategies.

For each \( \ell \in N^L \), the preceding observations and Lemma B.1 imply \( \lambda_{\sigma}^\ell \) puts probability one on \( (y^*, m^*) \) such that \( y^* = \arg\max_{y \in A(\sigma)} u_{\sigma}(y) + u_{\ell}(y) - u_{\ell}(z; \sigma) \), which is unique. Lemmas B.4 and B.5 imply we can assume \( \pi_{\sigma}(y^*) = 1 \) and \( \varphi_{\ell}(y^*, m^*) = 1 \).

It remains to verify that \( z_{\ell} \notin A(\sigma) \) cannot be optimal for any \( \ell \in N^L \). To show a contradiction, assume proposing \( z_{\ell} \notin A(\sigma) \) is optimal for some \( \ell \in N^L \). Let \( z^* = \arg\max_{x \in A(\sigma)} u_{\ell}(x) \). There are two steps. Step 1 establishes useful properties of \( \ell \)'s preferences over lotteries. Step 2 shows a contradiction.

**Step 1:** Recall the continuation lottery induced by \( \sigma \), denoted \( \chi = (1-\delta)\chi^q + \delta\chi^\sigma \) with mean \( x^\sigma \). Jensen’s inequality implies \( u_i(x^\sigma) > \int_X u_{\ell}(x) \chi(dx) = (1-\delta)u_i(q) + \delta V_{\ell}(\sigma) \) for all \( i \in N \), so \( x^\sigma \in \text{int}A(\sigma) \).
Next, let $\chi^{z^*}$ denote the policy lottery nearly equivalent to $\chi$, but shifting probability $\frac{\delta \rho q_{\ell}}{\text{den}(\sigma)}$ from $y^*$ to $z^*$. Let $x^{z^*}$ denote the mean of $\chi^{z^*}$. For all $i \in N$, Jensen’s inequality implies

$$u_i(x^{z^*}) > \int_X u_i(x) \chi^{z^*}(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho q_{\ell} \alpha \varepsilon u_i(y^*)}{\text{den}(\sigma)} + \frac{\delta \rho q_{\ell} \alpha \varepsilon u_i(z^*)}{\text{den}(\sigma)}.$$ 

Moreover, $x^{z^*}$ is located weakly between $x^\sigma$ and $z^*$, implying $x^{z^*} \in A(\sigma)$.

**Step 2:** Since $z_{\ell} \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_{\ell}(q) + \delta \bar{V}_{\ell}(\sigma) - u_{\ell}(y^*)$$

$$= (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) + \frac{\delta \bar{m}_{\ell}(\sigma)}{\text{den}(\sigma)} - u_{\ell}(y^*).$$

Using (51), $\bar{m}_{\ell}(\sigma)$ is expressed recursively as

$$\bar{m}_{\ell}(\sigma) = \rho q_{\ell} \alpha_{\ell} \left( (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) + \frac{\delta \bar{m}_{\ell}(\sigma)}{\text{den}(\sigma)} - u_{\ell}(y^*) \right)$$

$$= \frac{\rho q_{\ell} \alpha_{\ell} \text{den}(\sigma)}{\text{den}(\sigma)} - \delta \rho q_{\ell} \alpha_{\ell} \left( (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) - u_{\ell}(y^*) \right).$$

Because $z_{\ell} \notin A(\sigma)$ is optimal,

$$u_{\ell}(z^*) \leq (1 - \delta)u_{\ell}(q) + \delta \bar{V}(\sigma)$$

$$= (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) + \frac{\delta \rho q_{\ell} \alpha_{\ell} [(1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) - u_{\ell}(y^*)]}{\text{den}(\sigma) - \delta \rho q_{\ell} \alpha_{\ell}},$$

where (65) follows from the definition of $\bar{V}_{\ell}(\sigma)$ and using (63) to substitute for $\bar{m}_{\ell}(\sigma)$. Next, we have $V^{\text{den}}(\sigma) - \delta \rho q_{\ell} \alpha_{\ell} \geq 1 - \delta \sum_{j \in N_{\ell}} \rho_j (1 - \alpha_j) - \delta \rho q_{\ell} \alpha_{\ell} > 0$, where the first inequality follows because Lemma B.3 implies all lobby offers are accepted and passed under $\sigma$, so $V^{\text{den}}(\sigma) \geq 1 - \delta \sum_{j \in N_{\ell}} \rho_j (1 - \alpha_j)$; and the second inequality follows from $\delta [\rho q_{\ell} \alpha_{\ell} + \sum_{j \in N_{\ell}} \rho_j (1 - \alpha_{\ell})] < 1$. Rearranging and simplifying (65),

$$0 \leq V^{\text{den}}(\sigma) \left[ (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) - \delta \rho q_{\ell} \alpha_{\ell} u_{\ell}(y^*) - u_{\ell}(z^*) \right] \left( V^{\text{den}}(\sigma) - \delta \rho q_{\ell} \alpha_{\ell} \right)$$

$$\propto (1 - \delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) - \frac{\delta \rho q_{\ell} \alpha_{\ell} [u_{\ell}(y^*) - u_{\ell}(z^*)]}{V^{\text{den}}(\sigma)} - u_{\ell}(z^*).$$

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\[ = \int_X u_\ell(x) \chi^*(dx) - u_\ell(z^*), \]
a contradiction because \( u_\ell(z^*) \geq u_\ell(x^*) > \int_X u_\ell(x) \chi^*(dx). \]

\[ \square \]

**Lemma B.7.** Every stationary legislative lobbying equilibrium is such that \( \lambda_g \) is degenerate for all \( g \in N^G \) and \( \pi_\ell \) is degenerate for all \( \ell \in N^L \).

**Proof.** Let \( \sigma \) denote an equilibrium. By Duggan (2014), \( A_M(\sigma) = A(\sigma) \), which is nonempty, compact and convex.

First, consider \( g \in N^g \) and \( \ell \in N^L_g \). Recall \( \tilde{U}_\ell(\sigma) \) from (54). Lemmas B.1 and B.6 imply \( \lambda^\ell_g \) puts probability one on the unique \((y^*, m^*)\) satisfying 
\[ y^* = \arg \max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma), \]
and \( m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*) \).

Second, consider \( \ell \in N^L \). Lemma B.6 implies \( \pi_\ell \) puts probability one on \( x^* = \arg \max_{x \in A(\sigma)} u_\ell(x) \), which is unique. \( \square \)

**Proposition 1.2** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

**Proof.** Follows from Lemmas B.4 - B.7. \( \square \)
Appendix C  Partitioning Moderates & Extremists

Consider $\ell \in N^L$. First, I define a function $\zeta^\ell$ that relates to $M$’s equilibrium voting decision. Then, Lemmas C.3 - C.6 characterize $\zeta^\ell$. Finally, Lemma 3 delivers a partitional characterization on $\hat{x}_g$ that facilitates Proposition 3.

Preliminaries to define $\zeta^\ell$. Recall $\bar{\pi}(0) = \bar{\pi}(\hat{x}_g)$ for $\hat{x}_g = 0$. Let $\hat{D}^{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \bar{\pi}(0), j \neq \ell\}$ and $\hat{D}^{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \bar{\pi}(0), j \neq \ell\}$. Next, set $D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y}\}$ and $D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x}\}$. Define $D^{\ell}$ as the unique elements of $D^{\ell,y} \cup D^{\ell,x} \cup \{\bar{\pi}(0)\}$. Let $K^{\ell} + 1 = |D^{\ell}|$. Denote the $k$-th element of $D^{\ell}$ as $d_k^{\ell}$. Index elements $k = 0, \ldots, K^{\ell}$ of $D^{\ell}$ in ascending order so that $d_0^{\ell} = \bar{\pi}(0)$ and $k' > k$ implies $d_{k'}^{\ell} > d_k^{\ell}$.

For each $k$ and $j \neq \ell$, let $C_j^k = \mathbb{I}\{\hat{x}_j \in [-d_k^{\ell}, d_k^{\ell}]\}$ and $\tilde{C}_j^k = \mathbb{I}\{\hat{y}_j \in [-d_k^{\ell}, d_k^{\ell}]\}$. Define

$$I_j^k = (1 - \alpha_j)C_j^k u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^k u_M(\hat{y}_j)$$

and

$$O_j^k = (1 - \alpha_j)(1 - C_j^k) + \alpha_j(1 - \tilde{C}_j^k),$$

suppressing dependence on $\ell$. Let

$$\hat{x}_k^{\ell} = \left(\frac{1}{\delta \rho_\ell} \left[ (1 - \delta)u_M(g) + \delta \sum_{j \neq \ell} \rho_j I_j^k u_M(d_k^{\ell}) \left(1 - \delta \sum_{j \neq \ell} O_j^k \right) \right] \right)^\frac{1}{2}. \tag{66}$$

Because $d_0^{\ell} = \bar{\pi}(0)$, rearranging (66) yields $\hat{x}_0^{\ell} = 0$.

Lemma C.1. For all $\ell \in N^L$ and each $k = 0, \ldots, K^{\ell}$, we have

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^{\ell}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) = \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^{\ell}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k).$$

Proof. Consider $\ell \in N^L$ and fix $k < K^{\ell}$. Then,

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^{\ell}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1})$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^{\ell}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) + \delta u_M(d_{k+1}^{\ell}) \sum_{j \neq \ell} \rho_j O_j^k - \delta u_M(d_{k+1}^{\ell}) \sum_{j \neq \ell} \rho_j O_j^k$$

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$$= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta u_M(d^\ell_{k+1}) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k)$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \tag{67}$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k), \tag{68}$$

where (67) follows because $u_M(d^\ell_{k+1}) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})$ by construction. \qed

Lemma C.2. For all $\ell \in N^\ell$, $\check{x}_k^\ell$ strictly increases in $k$. \newline

\textit{Proof.} Consider $\ell \in N^\ell$ and fix $k < K^\ell$. Lemma C.1 and $0 > u_M(d^\ell_k) > u_M(d^\ell_{k+1})$ together imply

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d^\ell_k) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k)$$ \tag{69}

Thus, $\check{x}_k^\ell < \check{x}_{k+1}^\ell$ follows from (66). \qed

\textit{Definition of $\zeta^\ell$.} For $k = 0, \ldots, K^\ell$, define $\overline{x}_k^\ell : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$\overline{x}_k^\ell(x) = \left( \frac{-(1 - \delta) u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \right)^{\frac{1}{2}} \tag{70}$$

and $\zeta_k^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$\zeta_k^\ell(x) = u_M(x) - \left( (1 - \delta) u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right).$$

By construction, $\overline{x}_k^\ell(\check{x}_k^\ell) = d_k^\ell$ for all $k$. Adopt the convention $d_{K^\ell+1}^\ell = \infty$. Define the piecewise function $\zeta^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$\zeta^\ell(x) = \zeta_k^\ell(x) \text{ if } x \in [d_k^\ell, d_{k+1}^\ell).$$

Lemma C.3. For all $\ell \in N^\ell$, $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. \newline

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Proof. Consider \( \ell \in N^L \). First, we have

\[
\zeta^\ell(0) = \zeta^\ell_0(0) = u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho \varepsilon u_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\bar{x}_0(0)) \sum_{j \neq \ell} \rho_j O_j^0 \right)
\]

\[
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right)
\]

(71)

where (71) follows from \( u_M(0) = 0 \) and \( \bar{x}_0^\ell(0) = \bar{x}_0 \).

Next, I show \( \zeta^\ell(q) \leq 0 \). Let \( k' \) denote the largest \( k \) such that \( \bar{x}_k^\ell \leq q \).

- **Step 1:** Because \( \bar{x}_k^\ell(\bar{x}_k^\ell) = d_k^\ell \), we have

\[
u_M(d_k^\ell) = \frac{(1 - \delta)u_M(q) + \delta \rho \varepsilon u_M(\bar{x}_k^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}
\]

(72)

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho \varepsilon u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}
\]

(73)

\[
\geq \frac{(1 - \delta)u_M(q) + \delta \rho \varepsilon u_M(q) + \delta u_M(d_k^\ell) \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}]}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}
\]

(74)

\[
u_M(d_k^\ell) = \frac{(1 - \delta)u_M(q) + \delta \rho \varepsilon u_M(q) + \delta u_M(d_k^\ell)(1 - \rho \varepsilon - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}
\]

(75)

where (72) follows from rearranging (70); (73) from \( \bar{x}_k^\ell \leq q \); (74) because for all \( j \) the construction of \( I_j^{k'} \) implies \( I_j^{k'} \geq u_M(d_k^\ell)[(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] \); and (75) because \( \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho \varepsilon - \sum_{j \neq \ell} \rho_j O_j^{k'} \) by construction.

Rearranging and simplifying (75) yields \( \nu_M(d_k^\ell) \geq \frac{(1 - \delta + \rho \varepsilon)u_M(q)}{1 - \delta + \rho \varepsilon} = u_M(q) \). Thus,

\[
\sum_{j \neq \ell} \rho_j I_j^{k'} = \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} u_M(\bar{\hat{x}}_j) + \alpha_j \tilde{C}_j^{k'} u_M(\bar{\hat{y}}_j) \right]
\]

(76)

\[
\geq u_M(d_k^\ell) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'} \right]
\]

(77)

\[
u_M(d_k^\ell)(1 - \rho \varepsilon - \sum_{j \neq \ell} \rho_j O_j^{k'})
\]

(78)
Lemma C.4. For all \( \ell \in N^L \), \( \zeta^\ell \) is continuous.

Proof. Consider \( \ell \in N^L \) and fix \( k \). Because \( \pi^\ell_k(x) \) is continuous, \( \zeta^\ell \) is continuous over \((\bar{x}^\ell_k, \bar{x}^\ell_{k+1})\). It suffices to show \( \zeta^\ell_k(\bar{x}^\ell_{k+1}) = \zeta^\ell_{k+1}(\bar{x}^\ell_{k+1}) \).

First, I establish \( d^\ell_{k+1} = \pi^\ell_k(\bar{x}^\ell_{k+1}) \). Rearranging (66) for \( k + 1 \) yields

\[ 0 = u_M(d^\ell_{k+1}) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O^{k+1}_j \right) - (1 - \delta)u_M(q) - \delta \rho_\ell u_M(\bar{x}^\ell_{k+1}) - \delta \sum_{j \neq \ell} \rho_j I^{k+1}_j \]

where (76) follows from the definition of \( I^k_j \); (77) from \( u_M(\bar{x}_j) \geq u_M(d^\ell_k) \) if \( C^\ell_j = 1 \) and \( u_M(\bar{y}_j) \geq u_M(d^\ell_k) \) if \( C^\ell_j = 1 \); (78) because \( \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C^\ell_j + \alpha_j \bar{C}^\ell_j] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^\ell_j \) by construction; and (79) from \( u_M(d^\ell_k) \geq u_M(q) \).

- **Step 2:** We have
  
  \[
  u_M(\pi^\ell_k(q)) = \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^\ell_j}{1 - \delta \sum_{j \neq \ell} \rho_j O^\ell_j} 
  \]

  \[
  \geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^\ell_j)}{1 - \delta \sum_{j \neq \ell} \rho_j O^\ell_j} \]  

  \[
  = u_M(q), \quad (80)
  \]

  where (80) follows from Step 1 and (81) from simplifying.

- **Step 3:** To see \( \zeta^\ell(q) \leq 0 \), note

  \[
  \zeta^\ell(q) = u_M(q) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^\ell_j + \delta u_M(\pi^\ell_k(q)) \sum_{j \neq \ell} \rho_j O^\ell_j \right) 
  \]

  \[
  \leq u_M(q) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^\ell_j) + \delta u_M(q) \sum_{j \neq \ell} \rho_j O^\ell_j \right) \]  

  \[
  = 0, \quad (82)
  \]

  where (82) follows from Steps 1 and 2.

\[\]
where (84) follows from Lemma C.1. Thus, \( u_M(d^\ell_{k+1}) = \frac{(1-\delta)u_M(q) + \delta \rho_x u_M(\hat{x}^\ell_{k+1}) + \delta \sum_{j \neq \ell} \rho_j I^k_j}{1-\delta \sum_{j \neq \ell} \rho_j O^k_j} \), so \( d^\ell_{k+1} = \pi^\ell_k(\hat{x}^\ell_{k+1}) \). Then,

\[
\zeta^\ell_k(\hat{x}^\ell_{k+1}) = u_M(\hat{x}^\ell_{k+1}) - \left( (1-\delta)u_M(q) + \delta \rho_x u_M(\hat{x}^\ell_{k+1}) + \delta \sum_{j \neq \ell} \rho_j I^k_j + \delta u_M(\pi^\ell_k(\hat{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O^k_j \right)
\]

\[
= u_M(\hat{x}^\ell_{k+1}) - \left( (1-\delta)u_M(q) + \delta \rho_x u_M(\hat{x}^\ell_{k+1}) + \delta \sum_{j \neq \ell} \rho_j I^k_{j+1} + \delta u_M(\pi^\ell_k(\hat{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O^k_{j+1} \right)
\]

\[
= \zeta^\ell_{k+1}(\hat{x}^\ell_{k+1}),
\]

where (85) follows from Lemma C.1 because \( d^\ell_{k+1} = \pi^\ell_k(\hat{x}^\ell_{k+1}) \).

\[\square\]

**Lemma C.5.** For all \( \ell \in N^L \), \( \zeta^\ell \) is strictly decreasing.

**Proof.** Consider \( \ell \in N^L \) and fix \( k \). The proof shows that the derivative of \( \zeta^\ell \) is strictly negative at every \( x \in (\hat{x}^\ell_k, \hat{x}^\ell_{k+1}) \). Continuity then implies that \( \zeta^\ell \) is strictly decreasing.

Consider \( x \in (\hat{x}^\ell_k, \hat{x}^\ell_{k+1}) \). Then

\[
\zeta^\ell(x) = u_M(x) - \left( (1-\delta)u_M(q) + \delta \rho_x u_M(x) + \delta \sum_{j \neq \ell} \rho_j I^k_j + \delta u_M(\pi^\ell_k(x)) \sum_{j \neq \ell} \rho_j O^k_j \right)
\]

and

\[
\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x\delta \rho_\ell + \frac{2x\delta \rho_\ell (\delta \sum_{j \neq \ell} \rho_j O^k_j)}{1-\delta \sum_{j \neq \ell} \rho_j O^k_j}
\]

\[
\propto \delta \rho_\ell + \delta \sum_{j \neq \ell} \rho_j O^k_j - 1
\]

\[
< 0,
\]

where (87) follows from\( \frac{\partial u_M(\pi^\ell_k(x))}{\partial \pi^\ell_k(x)} \frac{\partial \pi^\ell_k(x)}{\partial x} = -\frac{2x\delta \rho_\ell}{1-\delta \sum_{j \neq \ell} \rho_j O^k_j} \); and (89) because \( \delta \in (0,1) \) and \( \rho_\ell + \sum_{j \neq \ell} \rho_j O^k_j \leq 1 \).

\[\square\]

**Lemma C.6.** For all \( \ell \in N^L \), there is a unique \( \pi_\ell \in (0,q] \) such that \( \zeta^\ell(x) > 0 \) for all \( x \in [0, \pi_\ell) \), \( \zeta^\ell(\pi_\ell) = 0 \), and \( \zeta^\ell(x) < 0 \) for all \( x > \pi_\ell \).
Proof. Consider \( \ell \in N^L \). Lemma C.3 implies \( \zeta(0) > 0 \) and \( \zeta(q) \leq 0 \). By Lemma C.5, \( \zeta \) is strictly decreasing. Thus, there is a unique \( \overline{x}_\ell \in (0, q] \) such that \( \zeta(x) > 0 \) for all \( x \in [0, \overline{x}_\ell) \) and \( \zeta(x) < 0 \) for all \( x > \overline{x}_\ell \). Lemma C.4 implies \( \zeta(\overline{x}_\ell) = 0 \). \( \qed \)

**Lemma 3.** For all \( \ell \in N^L \), \( \hat{x}_\ell \in (-\overline{x}_\ell, \overline{x}_\ell) \) implies \( \hat{x}_\ell \in \text{int} A(\hat{x}_\ell) \). Otherwise, \( A(\hat{x}_\ell) = [-\overline{x}_\ell, \overline{x}_\ell] \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_\ell = \hat{x}_g \).

**Part 1.** First, suppose \( \hat{x}_g \in (-\overline{x}_\ell, \overline{x}_\ell) \) and assume \( \hat{x}_g \geq 0 \) without loss of generality. I show \( \hat{x}_g \in \text{int} A(\hat{x}_g) \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k^\ell \leq \hat{x}_g \). Define the strategy profile \( \sigma' \) such that it puts probability \( \rho_k \) on \( \hat{x}_g \) and for each \( j \neq \ell \) it (i) puts probability \( (1 - \alpha_j)\rho_j \) on: \( \hat{x}_j \) if \( \hat{x}_j \in [-d_k^\ell, d_k^\ell] \), \( \overline{x}_\ell(\hat{x}_g) \) if \( \hat{x}_j > d_k^\ell \), or \( -\overline{x}_\ell(\hat{x}_g) \) if \( \hat{x}_j < -d_k^\ell \); and (ii) puts probability \( \alpha_j\rho_j \) on: \( \hat{y}_j \) if \( \hat{y}_j \in [-d_k^\ell, d_k^\ell) \), \( \overline{x}_k(\hat{x}_g) \) if \( \hat{y}_j > d_k^\ell \), or \( -\overline{x}_k(\hat{x}_g) \) if \( \hat{y}_j < -d_k^\ell \). By construction, \( \overline{x}(\sigma') = \overline{x}(\hat{x}_g) \). Furthermore, proposal strategies are optimal given \( A(\sigma') = [-\overline{x}(\sigma'), \overline{x}(\sigma')] \).

I now check optimality for \( M \). Because \( \hat{x}_g \in [\hat{x}_k^\ell, \hat{x}_k^{\ell+1}) \), we have \( \overline{x}(\sigma') = \overline{x}_k(\hat{x}_g) \in [d_k^\ell, d_k^{\ell+1}) \). Thus, \( M \) optimally accepts all offers by \( j \neq \ell \). Next, I verify \( \hat{x}_g \in \text{int} A(\sigma') \).

By Lemma C.6, \( \hat{x}_g \in (-\overline{x}_\ell, \overline{x}_\ell) \) implies \( \zeta(\hat{x}_g) > 0 \), which is equivalent to \( u_M(\hat{x}_g) > (1 - \delta)u_M(q) + \delta \sum_{j \neq g} \rho_j I_j^p \). Under \( \sigma' \), this is equivalent to \( \hat{x}_g \in \text{int} A(\sigma') \).

Thus, \( \sigma' \) is equivalent to the equilibrium \( \sigma(\hat{x}_g) \) and \( \hat{x}_g \in \text{int} A(\hat{x}_g) \), as desired.

**Part 2.** Assume \( \hat{x}_g \notin (-\overline{x}_\ell, \overline{x}_\ell) \) and suppose \( \hat{x}_g \geq 0 \) without loss of generality. I verify \( A(\hat{x}_g) = [-\overline{x}_\ell, \overline{x}_\ell] \) in two steps. Step 1 shows \( \overline{x}(\hat{x}_g) \geq \overline{x}_\ell \). Step 2 shows \( \overline{x}(\hat{x}_g) \leq \overline{x}_\ell \).

**Step 1.** Suppose \( \overline{x}(\hat{x}_g) < \overline{x}_\ell \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k^\ell \leq \overline{x}(\hat{x}_g) \). Because \( \hat{x}_g > \overline{x}_\ell > \overline{x}(\hat{x}_g) \), it follows that \( \sigma(\hat{x}_g) \) puts probability \( \rho_\ell \) on \( \overline{x}(\hat{x}_g) \). Thus, \( u_M(\overline{x}(\hat{x}_g)) = (1 - \delta)u_M(q) + \delta \sum_{j \neq g} \rho_j I_j^p \) and rearranging yields \( \zeta(\overline{x}(\hat{x}_g)) = 0 \). Lemma C.6 implies \( \overline{x}(\hat{x}_g) = \overline{x}_\ell \), a contradiction.

**Step 2.** Suppose \( \overline{x}(\hat{x}_g) > \overline{x}_\ell \). If \( \hat{x}_g \geq \overline{x}(\hat{x}_g) \), then the argument from Step 1 shows a contradiction. Assume \( \hat{x}_g < \overline{x}(\hat{x}_g) \). Let \( k' \) be the largest \( k \) such that \( \hat{x}_k^\ell(\hat{x}_g) \leq \overline{x}(\hat{x}_g) \). Then \( \sigma(\hat{x}_g) \) puts probability \( \rho_\ell \) on \( \hat{x}_g \). Next, \( M \) optimally accepts \( \hat{x}_g \) under \( \sigma(\hat{x}_g) \) iff \( u_M(\hat{x}_g) \geq (1 - \delta)u_M(q) + \delta \sum_{j \neq g} \rho_j I_j^p \). Rearranging, this condition is equivalent to \( \zeta(\hat{x}_g) \geq 0 \). By Lemma C.6, this requires \( \hat{x}_g \leq \overline{x}_\ell \), a contradiction. \( \qed \)