Access and Lobbying in Legislatures*

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Abstract

Which legislators and interest groups form relationships to facilitate lobbying? I study a model of legislative policymaking in which interest groups can get access to particular legislators. Access provides lobbying opportunities. Equilibrium spillover effects encourage groups to access certain legislators but discourage access to others. Under broad conditions, groups forgo access to a range of more centrist legislators. In contrast, groups are especially keen to access more extreme legislators. Given access, lobbying expenditures increase with several measures of legislature polarization. The results have implications for two prominent paths to access: campaign contributions and revolving door hiring. For example, forgoing access is consistent with the empirical regularity that many interest groups either do not contribute or contribute well below legal limits.

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Special interests and lobbying are unpopular. A widespread, cynical view is that wealthy interest groups frequently lobby key politicians for favorable policy. Often implicit in concerns about this view are the relative preferences of groups and the politicians they influence. Specifically, observers typically worry that ideologically extreme groups lobby centrist politicians away from majority interests.

To lobby effectively, however, interest groups need access that provides working relationships with politicians (Hansen, 1991; Ainsworth, 1993). Which interest groups and legislators form relationships yielding access? Widely perceived benefits of lobbying suggest that groups always crave access. Do they? And at a finer level, how does the value of access vary across different pairs of groups and legislators? Understanding which legislators are targeted for access by various interest groups can shed light on who lobbies whom, and the resulting welfare implications.

To develop our understanding of these questions, I study a game-theoretic model where access provides interest groups with opportunities to influence legislative policy. The model has three key features: lobbying requires access, lobbying influences policy proposals, and legislators bargain over policy. I now expand on each.

First, to study which connections form, I distinguish access and lobbying. Interest groups choose whether to access particular legislators before policymaking and, if they do so, have chances to lobby those legislators when they control the agenda. The model thus reflects the standard view of access as “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). In practice, groups increase access to particular legislators through several channels, such as campaign contributions (Kalla and Broockman, 2015; Fouirnaies and Hall, 2017) or hiring connected lobbyists who are trusted and more likely to get meetings (Blanes i Vidal, Draca and Fons-Rosen, 2012; Bertrand, Bombardini and Trebbi, 2014).

Second, lobbying allows groups to influence policy proposals before they reach the floor. In the model, groups offer resources in exchange for policy. In practice, interest groups spend substantial effort drafting legislation (Schlozman and Tierney, 1986), and frequently present legislators with model bills (Levy and Razin, 2013; Kroeger, 2016). To get these provisions into bills, interest groups can help legislators in various ways. For example, legislators may gain an inside track on future employment opportunities

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1 Also see, e.g., Milbrath (1976); Hall and Wayman (1990); Hansen (1991); Grossman and Helpman (2002); Hall and Deardorff (2006); Gordon, Hafer and Landa (2007) and Powell (2014).

2 Also see, e.g., Barber (2016) and Grimmer and Powell (2016) for recent evidence that many groups use contributions to get access. For revolving-door hiring, also see Cain and Drutman (2014); Kang and You (2015) and McCrain (2018).
(Diermeier, Keane and Merlo, 2005), or be freed to pursue other tasks such as constituent service and fundraising (Hall and Deardorff, 2006). Groups may also provide valuable political intelligence, or write speeches to promote policies to the legislator’s constituents and co-partisans (Schlozman and Tierney, 1983, 1986; Wright, 1996).

Third, I unpack the legislative black box by modeling a canonical spatial legislative bargaining environment where players can revisit failed proposals, forward-looking legislators anticipate outside influence, agenda power can change hands unpredictably, and passage requires majority approval. Although expanding the scope of application for legislative bargaining models has an independent theoretical interest, I model a rich legislative environment to disentangle access-seeking incentives from lobbying.

There are three primary contributions.

First, a primary insight is that access has indirect effects on equilibrium policymaking under broad conditions. The model thus provides a microfoundation for access-driven spillover effects. These spillovers arise because all legislators account for which connections form and anticipate potential lobbying facilitated by those connections. Their direction and magnitude depends on the relative extremism of groups and targeted legislators, which affects whether access increases or decreases legislature-wide policy extremism. This dependence creates qualitative differences in how various groups value connections to different legislators. I highlight when these spillovers encourage or discourage access, compared to a setting where they are absent.

Second, the main results characterize which pairs of legislators and interest groups are likely to form connections. Naïve intuition suggests groups always want more access. I show that this intuition can fail due to access-driven spillovers. Specifically, a broad set of groups optimally forgo access to a range of more centrist legislators. This behavior does not require costly access, as negative spillovers alone can dominate the benefits of access. On the other hand, groups are keen to access more extreme legislators because these connections have positive spillovers. I also study how spillovers affect the prices that groups are willing to pay for access. The results have empirical implications for campaign contributions and revolving-door hiring, two primary avenues to greater access. For example, forgoing access aligns with the counterintuitive empirical regularity that many groups contribute low amounts (e.g., Tullock, 1972; Ansolabehere, de Figueiredo and Snyder Jr., 2003).

Third, given connections, I characterize how lobbying expenditures vary with access

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3The legislative setting follows Banks and Duggan (2006a) and Cho and Duggan (2003), which integrate key features of Baron and Ferejohn (1989) and Romer and Rosenthal (1978).
and legislative features. Expenditures increase with several measures of polarization, as well as more extreme status quo. They can increase or decrease with access, increasing if the group is more extreme than the targeted legislator and decreasing otherwise. These results have an original logic driven by the legislative interaction.

To begin the analysis, I first study a baseline model with exogenous access. In equilibrium, groups pull policy in their favored direction whenever they can lobby the proposer. But they may be constrained by satisfying a legislative majority. The set of passable policies depends on legislator expectations about future proposals that would follow rejection today. Crucially, these expectations depend in part on access. The baseline analysis also yields clear comparative statics on lobbying expenditures with respect to access and several legislative conditions including ideological polarization, the distribution of agenda power, and status quo policy.

I then endogenize access, which presents groups with an important tradeoff due to access-driven spillovers. On the one hand, access provides more opportunities to lobby during policymaking. On the other hand, access can also have a negative indirect effect on policies proposed when the group is unable to lobby. The logic is as follows. Forward-looking legislators anticipate future lobbying behavior that would follow rejected proposals. Thus, a group’s access to some legislator affects every legislator’s expectation about policymaking. More precisely, access affects each legislator’s reservation value, which is generated endogenously by equilibrium expectations about future policymaking. This effect can change which policies pass. In turn, legislators constrained by majority approval may propose different policy. From a group’s perspective, this indirect effect of access can be good or bad. Furthermore, its magnitude depends on various legislative conditions.

This tradeoff leads a broad set of interest groups to optimally forgo access to a range of legislators, even if it is free. Specifically, groups that are not too extreme forgo access to a range of more centrist legislators. In these cases, access-driven spillovers increase policy extremism and this indirect effect outweighs the group’s gain from more lobbying opportunities. These groups face a time inconsistency problem. When given the opportunity, they always want to lobby. Ex ante, however, they forgo access because it polarizes expected policies too much, relative to the expected gains from lobbying.

On the other hand, groups always want access to a range of more extreme legislators. Access to these legislators increases lobbying opportunities and also favorably
constrains extreme legislators. The analysis thus suggests that centrist and moderate interest groups have strong incentives to access a broad spectrum of legislators.

To illustrate the logic of forgoing access, consider the following stylized example. A regional energy interest group anticipates national legislation regulating emissions. It prefers moderately tighter regulations, perhaps to capitalize on recent investments in clean technology. The group’s local congressman wants to tighten existing regulations more than the group. Greater access makes the group more likely to lobby the congressman increase. Thus, moderate legislators are less optimistic about the eventual regulatory outcome. They know that if the congressman drafts policy, then the group is more likely to lobby. If so, the resulting policy will be more extreme than if the congressman had acted alone. Consequently, rejecting proposals is less attractive for these other legislators, who are now willing to approve a wider range of policies. The group’s access thus indirectly allows extreme pro-energy legislators to pass weaker emissions regulations if they draft policy. Such policies would reduce the group’s benefits from its recent technological investments. I show that this threat of greater extremism can worsen the group’s expectations about policymaking so that it prefers to forgo access.

**Related Literature**


Lobbying has been modeled in many ways. I focus on lobbying to influence policy proposals. Specifically, I study *lobbying as exchange* in the spirit of Grossman and Helpman (1994), where groups provide resources to shape policy proposals.\(^5\) The specific technology used here is similar to Bils, Duggan and Judd (2019), which studies lobbying in a model of repeated elections. There, officeholder ideology exogenously determines access and groups can always lobby their affiliated officeholders. In contrast, I study whether groups want access and allow them to choose.\(^6\)

\(^4\)See Eraslan and McLennan (2013) for a thorough discussion of sequential bargaining models.

\(^5\)See Grossman and Helpman (2002) for an extensive overview of this setting. A similar lobbying technology is used in Martimort and Semenov (2008) and an extension in Acemoglu, Egorov and Sonin (2013).

\(^6\)Another notable difference is that I allow partial access, which does not guarantee lobbying op-
Scholars have also studied less cynical forms of lobbying, such as groups providing useful information (Austen-Smith, 1995; Prat, 2002) or services to politicians and voters (Hall and Deardorff, 2006). Studying these perspectives is also worthwhile. I focus on a cynical form of influence because it aligns with a common outlook and underlies public concern about special interests. I aim to strengthen positive theory under this perspective to develop empirical implications and welfare consequences.

Other work endogenizing agenda access in the sequential bargaining framework allows interest groups to buy agenda control (Yildirim, 2007, 2010; Ali, 2015). Most papers in this vein analyze distributive policies. An exception is Levy and Razin (2013), who study a dynamic setting with an endogenous status quo in a one-dimensional policy space. In each period, a continuum of groups compete in an all-pay auction for temporary agenda control. They characterize conditions for policies to moderate over time. They do not address which connections form, as they do not model politicians and instead implicitly treat them as homogeneous. Furthermore, they do not study persistent access, as groups instead vie for temporary access throughout policymaking. In contrast, I study access that is targeted, persistent, and acquired before bargaining. There are several other differences. Here, bargaining ends when a proposal passes and I abstract from head-to-head competition for access.

I also contribute to a literature that fixes access in various legislative institutions and studies lobbying that influences policy proposals. In Helpman and Persson (2001) and Baron (2006), groups can lobby particular legislators to influence proposals when those legislators control the agenda, as in this paper. Unlike this paper, groups can lobby to influence votes. Baron (2006) allows bargaining to continue after rejected proposals, like this paper, but Helpman and Persson (2001) does not. Furthermore, neither analyzes the one-dimensional policy space studied here.

Scholars have also studied access acquisition in static, informational lobbying environments (Austen-Smith, 1995; Lohmann, 1995; Cotton, 2012, 2016). Closest to this paper is Schnakenberg (2017), where groups can buy access in a legislature. There, groups try to influence a legislative vote over an exogenous policy proposal in a static setting. Access allows groups to provide information. As in this paper, influencing

7 Also see, e.g., Bennedsen and Feldmann (2002); Jackson and Tan (2013); Schnakenberg (2015, 2017); Alonso and Câmara (2016); Awad (2019).

8 A large literature studies lobbying to buy votes in legislatures, mostly studying distributive policies or public goods (Snyder Jr., 1991; Groseclose and Snyder, 1996; Banks, 2000; Dal Bó, 2007; Dekel, Jackson and Wolinsky, 2009). Unlike this paper, proposals are typically exogenous.
a legislature is quite different from influencing a solitary policymaker. In contrast, I study a complete information setting where lobbying affects endogenous policy proposals and policymaking continues after failed proposals. Moreover, groups sometimes optimally forgo free access in this paper, which never happens in Schnakenberg (2017).

A key result of this paper is that groups may forgo access to more centrist legislators. The logic connects to moderation results in spatial models of dynamic bargaining with endogenous status quo (Baron, 1996; Zápal, 2014; Buisseret and Bernhardt, 2017). There, legislators prefer proposing more centrist policies to constrain future proposers. They do not exploit the full power of their temporary agenda control, instead acting to constrain the scale of policy changes by future proposers who may have substantially different preferences. I study a different setting in which policymaking ends once a proposal passes. Yet, incentives to forgo access arise from the same desire to constrain potential future proposers who are ideologically distant.

Model of Legislative Bargaining with Lobbying

I first present and analyze the legislative environment with access fixed exogenously, having implicitly arisen from previous efforts to create connections. This grasp of legislative behavior sets the stage to subsequently study access acquisition and then, given access, how lobbying expenditures vary with legislative features.

The main results can be illustrated in a streamlined setting with four legislators and one interest group. The legislature has a left partisan $L$, a moderate $M$, a right partisan $R$, and a generic legislator $\ell$. The interest group is denoted $g$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Each legislator $i$ has associated ideal point $\hat{x}_i \in X$ and $g$’s ideal point is $\hat{x}_g \in X$. Throughout, I normalize $\hat{x}_M = 0$ and assume $\hat{x}_L < 0 < \hat{x}_R$. Moreover, I maintain $\min\{|\hat{x}_L|, \hat{x}_R\} > |q|$ to model staunchly ideological partisans. Although not crucial, this assumption clarifies key tradeoffs.

Legislative bargaining occurs over an infinite horizon, with periods discrete and indexed by $t \in \{1, 2, \ldots\}$. Let $\rho_i > 0$ denote the probability legislator $i$ is chosen to propose in any period $t$. Then $\rho = (\rho_\ell, \rho_L, \rho_M, \rho_R)$ denotes the distribution of recognition probabilities, which sum to one.

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9Although Forand (2014) is cast as a model of elections, it can be interpreted as a spatial bargaining model with an endogenous status quo and endogenous proposers.

10Appendix A presents a model allowing for more groups and legislators.
The interest group, $g$, has opportunities to influence $\ell$’s policy proposals. Specifically, $g$’s access to $\ell$ is $\alpha \in [0,1]$, which is the probability that $g$ can lobby $\ell$, conditional on $\ell$ being recognized to propose.\textsuperscript{11} Group $g$ does not have access to legislators other than $\ell$.\textsuperscript{12} Although, $g$’s access is exogenously endowed for now, later on I let $g$ choose $\alpha$ to study when groups seek access.

In each period $t$, bargaining proceeds as follows. If no policy has passed before $t$, then each legislator $i$ is recognized as the period-$t$ proposer with probability $\rho_i$. The period-$t$ proposer, $i_t$, is publicly observed. If $i_t \neq \ell$, then $g$ is not active and $i_t$ proposes any policy $x_t \in X$. If $i_t = \ell$, then $g$ can lobby $\ell$ with probability $\alpha$. If $g$ lobbies, then $g$ offers $\ell$ a binding contract $(y_t, m_t)$ consisting of a policy $y_t \in X$ and a transfer $m_t \geq 0$. After observing $g$’s offer, $\ell$ decides to accept or reject. If $\ell$ accepts, then she is committed to propose $x_t = y_t$ and $m_t$ transfers from $g$ to $\ell$. If $\ell$ rejects, then she can propose any $x_t \in X$ and $g$ keeps $m_t$. With probability $1 - \alpha$, $g$ cannot lobby in $t$. Then, $\ell$ simply proposes any $x_t \in X$ and $g$ does not make an offer.

All legislators observe the period-$t$ proposal, $x_t$. Next, the moderate legislator, $M$, chooses to accept or reject $x_t$. If $M$ accepts, then the proposal passes and bargaining ends with $x_t$ enacted in $t$ and all subsequent periods. If $M$ rejects, then the status quo $q \in \mathbb{R}$ is enacted in $t$ and bargaining proceeds to $t + 1$. This setup distills the essence of a larger legislature where all legislators vote and $M$ is a decisive median legislator.\textsuperscript{13}

If $\ell$ accepts $g$’s offer $(y_t, m_t)$ and $x_t$ is the enacted policy in $t$,\textsuperscript{14} then $g$’s stage payoff is $u_g(x_t) - m_t$ and $\ell$’s stage payoff is $u_\ell(x_t) + m_t$. All players have quadratic policy utility and discount streams of stage utility by the common discount factor $\delta \in (0, 1)$.

See Appendix A for formal expressions of dynamic payoffs. Figure 1 illustrates the within-period interaction and accumulation of payoffs for a period in which $\ell$ proposes and $g$ can lobby. For a period in which $\ell$ does not propose or $g$ cannot lobby, the

\textsuperscript{11}The analysis is similar if access is binary, or if access is modeled as $\ell$’s marginal value of money. Also, $\alpha$ can equivalently be viewed as the proportion of legislators in a homogeneous bloc whom the group can lobby.

\textsuperscript{12}I relax this assumption in the appendices, but it reflects the idea that groups are unable to access some legislators due to exogenous factors. For example, regional groups may not be able to access legislators absent geographic ties (Wright, 1989). Alternatively, voters in some districts may be strongly opposed to the group’s mission or tactics (Stratmann, 1992). Finally, the group simply may not be able to afford access to many different legislators.

\textsuperscript{13}Under the maintained assumptions, the median legislator’s decision corresponds to the outcome of majority voting over policy lotteries (Banks and Duggan, 2006b; Duggan, 2014). I abstract from vote buying, to isolate considerations related to lobbying over policy details in committee. The median ideology is a robust statistic in large legislatures, and meaningful vote buying likely requires coordinating deals with several legislators.

\textsuperscript{14}If $y$ does not pass in period $t$, then $x_t = q$. 

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within-period interaction is analogous to Figure 1 following $\ell$ rejecting $g$’s offer.

Figure 1: A period in which the interest group can lobby

Equilibrium Policies and Lobbying Activity

I study stationary legislative lobbying equilibria. Stationarity implies that $g$’s offers are independent of previous play; $\ell$ accepts or rejects based only on the terms of the current offer, and $\ell$’s proposals in lieu of acceptance are independent of the preceding history; legislators other than $\ell$ propose independent of preceding play; and $M$’s voting decision depends only on the current proposal. In Appendix B, I show it is without loss of generality to consider pure strategies that are no-delay.

Informally, a stationary legislative lobbying equilibrium requires four conditions. First, $g$’s policy offer is socially acceptable and $g$ cannot profitably deviate to any other offer. Second, legislator $\ell$ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal. Third, conditional on not receiving a payment from $g$, each legislator proposes socially acceptable policy and cannot profitably deviate to a different proposal. Finally, $M$ uses stage-undominated voting

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15See Appendix A for a formal definition.
16In Appendix B, I define stationary mixed strategy legislative lobbying equilibrium and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium with deferential voting and deferential acceptance.
strategies (Baron and Kalai, 1993), passing a policy if and only if she weakly prefers it relative to rejecting and extending bargaining.\footnote{This equilibrium concept is less restrictive than it may appear. First, although players use straightforward behavioral rules, no player can profitably deviate to any other strategy. Second, \( g \) must make an offer in each period that \( \ell \) proposes and \( g \) can lobby, but this requirement is innocuous because \( g \) can effectively forgo lobbying by offering \( \ell \)'s default proposal and no payment. Finally, \( \ell \) accepts \( g \)'s offer when indifferent, but this restriction is without loss of generality. See Appendix B for details.}

Proposition 1 shows that a stationary legislative lobbying equilibrium exists and, moreover, that there is a unique equilibrium outcome distribution.\footnote{Uniqueness here parallels Cho and Duggan (2003), where lobbying is absent.} Subsequent analysis endogenizing access does not require additional equilibrium selection. In light of Proposition 1, I drop qualifiers and simply use equilibrium throughout.

**Proposition 1.** A stationary legislative lobbying equilibrium exists. Moreover, all stationary legislative lobbying equilibria have the same outcome distribution.

Equilibrium behavior has a sharp characterization that naturally builds upon models without lobbying (Cho and Duggan, 2003; Banks and Duggan, 2006a). As is standard in the legislative bargaining literature, equilibria can be characterized by \( M \)'s acceptance set. This set is denoted \( A^* \) and corresponds to the policies \( M \) accepts in equilibrium. The boundaries of \( A^* \) are the two policies \( M \) is indifferent between approving and rejecting. Formally, the upper bound of \( A^* \), denoted \( \overline{x^*} \), is the positive solution to

\[
u_M(x) = (1 - \delta)u_M(q) + \delta V^*_M,\]

where \( V^*_M \) denotes \( M \)'s equilibrium continuation value.\footnote{See Appendix A for explicit expressions of continuation values.} Thus, \( A^* = [\overline{x^*}, \underline{x^*}] \).

Figure 2 illustrates \( A^* \) and equilibrium proposals for a hypothetical legislature.

In equilibrium, \( M \) proposes \( \hat{x}_M = 0 \) if recognized, legislator \( L \) proposes \( -\overline{x^*} \) and \( R \) proposes \( \overline{x^*} \). The partisans, \( L \) and \( R \), are thus constrained by \( M \)'s voting power because their respective ideal policies will not pass. Each compromises by proposing the closest passable policy. If legislator \( \ell \) is recognized and does not accept a lobby offer, either because \( g \) cannot lobby or because \( \ell \) rejects \( g \)'s offer, then \( \ell \) proposes \( z_\ell = \arg \max_{x \in A^*} u_\ell(x) \).

Next, \( g \) always makes an offer that \( \ell \) accepts. More precisely, \( g \)'s equilibrium offer \((y_g, m_g)\) makes \( \ell \) indifferent between accepting and rejecting.\footnote{To see why, note that it is always feasible for \( g \) to offer \( \ell \)'s independent proposal, \( z_\ell \), with zero}
offer always passes and is skewed away from $\hat{x}_\ell$ towards $\hat{x}_g$. Because $u_\ell(z_\ell)$ does not depend on $g$’s offer, $g$’s equilibrium policy offer is

$$y_g(A^*) = \arg\max_{y \in A^*} u_g(y) + u_\ell(y),$$  \hspace{1cm} (2)$$

which uniquely maximizes the joint surplus of $g$ and $\ell$, subject to the constraint that it passes.\footnote{Uniqueness follows because $u_g + u_\ell$ is strictly concave, and $A^*$ is compact, convex, and nonempty.} Define $g$’s unconstrained policy offer as $\hat{y}_g = y_g(X)$. Because $u_g$ and $u_\ell$ are quadratic, $\hat{y}_g = \hat{x}_g + \hat{x}_\ell^2$. If $\hat{y}_g \in A^*$, then $y_g(A^*) = \hat{y}_g$. Otherwise, strict concavity implies that $y_g(A^*)$ equals the boundary of $A^*$ closest to $\hat{y}_g$.

To ease discussion, I say that $\ell$ and $g$ are aligned if their ideal points are on the same side of 0, e.g. $\max\{\hat{x}_\ell, \hat{x}_g\} \leq 0$. Otherwise, they are opposed. Next, I say $\ell$ is centrist if $\hat{x}_\ell \in \text{int}A^*$ and extremist otherwise, and analogously for $g$.\footnote{Note that this ideological partition depends on an equilibrium object, $A^*$. In the next section analyzing endogenous access, I define an ideological partition that depends only on primitives.} An immediate observation is that non-trivial lobbying does not occur if $g$ and $\ell$ are aligned extremists because $g$ cannot profitably improve upon $\ell$’s independent policy proposal.

Although the model is complicated by lobbying, it can be reinterpreted as a one-dimensional bargaining environment where $\ell$ has $(1 - \alpha)\rho_\ell$ recognition probability and there is an additional legislator with ideal point $\hat{y}_g$ who has $\alpha\rho_\ell$ recognition probability. After expanding the legislature to add this additional proposer representing the effect of $g$’s lobbying, legislators propose bills closest to their ideal point among those that pass. Uniqueness follows from applying Cho and Duggan (2003) to this fictitious enlarged legislature.

In general, the characterization implies that $M$’s equilibrium continuation value from rejecting a proposal is

$$V_M^* = \rho_M u_M(0) + \alpha \rho_\ell u_M(y_g) + (1 - \alpha) \rho_\ell u_M(z_\ell) + \rho_L u_M(-\bar{x}^*) + \rho_R u_M(\bar{x}^*).$$  \hspace{1cm} (3)$$

Rearranging (1) and using (3) to substitute for $V_M^*$ yields the upper bound of $A^*$,

$$\bar{x}^* = \left(\frac{(1 - \delta)u_M(q) + \delta \left(\alpha \rho_\ell u_M(y_g) + (1 - \alpha) \rho_\ell u_M(z_\ell)\right)}{1 - \delta(\rho_L + \rho_R)}\right)^\frac{1}{2}. $$  \hspace{1cm} (4)$$

payment. Because $z_\ell \in A^*$, $g$ weakly prefers to make passable offers that $\ell$ accepts. Of course, $g$ is strictly worse off giving $\ell$ a surplus transfer. Thus, $m_g = u_\ell(z_\ell) - u_\ell(y_g).$
Figure 2: Equilibrium characterization

Figure 2 illustrates equilibrium policy proposals. Arrows point from legislator ideal points to proposals. The bold interval is the acceptance set, \( A^* \). If legislator \( \ell \) is recognized, then she proposes the acceptable policy closest to \( \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \) with probability \( \alpha \) and otherwise proposes the acceptable policy closest to \( \hat{x}_\ell \).

Inspection of (4) shows that \( g \)'s access, \( \alpha \), can affect the boundaries of \( A^* \) and, consequently, proposals by the partisans \( L \) and \( R \). This spillover effect plays a key role in the analysis and requires dynamic concerns, i.e. \( \delta > 0 \).

Endogenous Access

To study where connections form, I now allow the group, \( g \), to choose \( \alpha \), its access to legislator \( \ell \). To isolate key tradeoffs of durable access, I focus on a one-time choice of perfectly persistent access. I discuss other possibilities later.

I allow \( g \) to choose \( \alpha \) freely, abstracting from the particular mapping for access. In practice, the cost of acquiring access depends on idiosyncratic factors such as the connections of the group's lobbyists (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kang and You, 2015), constituent interests within the legislator’s district (Stratmann, 1992), or the number of voters affiliated with the group (Bombardini and Trebbi, 2011).\(^{23}\) The following results are driven purely by policy considerations and hold for standard cost functions.

Proposition 1 and the equilibrium characterization imply that \( g \)'s expected payoff

\(^{23}\)For example, La Raja and Schaffner (2015) emphasize that contributions do not translate into influence the same way for different pairs of interest groups and legislators.
from α access equals
\[ \rho_M u_g(0) + \rho_R u_g(\bar{x}^*) + \rho_L u_g(-\bar{x}^*) + \rho_d \left( \alpha [u_g(y_g) + u_\ell(y_g) - u_\ell(z_\ell)] + (1 - \alpha) u_g(z_\ell) \right), \] (5)

where g’s utility from equilibrium lobbying is \( u_g(y_g) - m_g = u_g(y_g) + u_\ell(y_g) - u_\ell(z_\ell). \)

Access affects (5) in two ways. First, it determines the probability g receives the lobbying surplus \( u_g(y_g) - m_g \), which is weakly positive in equilibrium.\(^{24}\) This direct effect increases (5). Second, \( \bar{x}^* \) implicitly depends on α, as shown in (4). Access can therefore indirectly affect the equilibrium proposals of L and R. Specifically, α affects \( \rho_R u_g(\bar{x}^*) + \rho_L u_g(-\bar{x}^*) \) in (5) by shifting \( \bar{x}^* \). These access-driven spillovers can be good or bad for g, depending on how \( \bar{x}^* \) and \(-\bar{x}^*\) shift relative to \( \hat{x}_g \). If both shift towards \( \hat{x}_g \), then the spillovers benefit g, and vice versa if both shift away from \( \hat{x}_g \). If one shifts towards \( \hat{x}_g \) and the other shifts away, then the relative magnitude of \( \rho_L \) and \( \rho_R \) affect whether g benefits.

Partitioning Interest Groups: Ideologues and Non-ideologues

The subsequent analysis of endogenous access fixes \( \hat{x}_g \) and studies whether g wants access as \( \hat{x}_\ell \) varies. There are two qualitatively distinct cases, differing in whether g is centrist for \( \alpha = 0.\)\(^{25}\) Lemma 1 partitions these cases and Definition 1 labels each.

**Lemma 1.** Suppose the interest group, g, does not have access. Let
\[ \bar{x} = \left( \frac{(1 - \delta) u_M(q)}{1 - \delta(\rho_L + \rho_R + \rho_\ell)} \right)^{1/2}. \] (6)

If \( \hat{x}_g \notin (-\bar{x}, \bar{x}) \), then \( \hat{x}_g \notin \text{int}A^* \). Otherwise, \( \hat{x}_g \notin \text{int}A^* \) only if \( \ell \) is sufficiently more centrist than g.

**Definition 1.** The interest group, g, is a non-ideologue if \( \hat{x}_g \in (-\bar{x}, \bar{x}) \). Otherwise, g is an ideologue.

Crucially, α and \( \hat{x}_\ell \) do not affect whether g is an ideologue, as seen in (6). In contrast, either can affect whether g is extremist or centrist. For example, suppose g is

\(^{24}\)Formally, the marginal effect of α on g’s lobbying surplus in (5) equals \( u_g(y_g) + u_\ell(y_g) - u_\ell(z_\ell) - u_g(z_\ell) \), which is strictly positive if \( y_g \neq z_\ell \) and zero otherwise.

\(^{25}\)Recall g is extremist if \( \hat{x}_g \notin \text{int}A^* \) and centrist otherwise, and similarly for \( \ell \).
a non-ideologue. Then, \( g \) can be extremist if \( \alpha \) is low and \( \ell \) is sufficiently centrist, but \( g \) is centrist if \( \ell \) is sufficiently extreme or \( \alpha \) is sufficiently large. If \( g \) is an ideologue, however, then it is always extremist.

Who do Non-ideologue Groups want to Access?

Proposition 2 is the first main result. It provides a general characterization of whether a non-ideologue group wants access to an aligned legislator. The characterization takes the form of a partition on legislator \( \ell \)'s ideology. It reveals that the relative extremism of \( g \) and \( \ell \) produces qualitative differences in access seeking.

**Proposition 2.** Let the interest group, \( g \), be a non-ideologue aligned with legislator \( \ell \).

1. If \( \ell \) is more centrist than \( g \), but not too centrist, then \( g \) forgoes access.
2. If \( \ell \) is more extreme than \( g \), but not too extreme, then \( g \) acquires access.
3. If \( \ell \) is sufficiently more extreme than \( g \), then \( g \) forgoes costly access.

Figure 3 illustrates Proposition 2 for a right-leaning group. I focus on that case to explain the result.

Figure 3: Who do non-ideologue interest groups want to access?

![Figure 3: Who do non-ideologue interest groups want to access?](image)

Figure 3 illustrates Proposition 2 for a right-leaning interest group, \( g \). If \( \hat{x}_\ell \in (x', \hat{x}_g) \), then \( g \) forgoes access, i.e. \( \alpha = 0 \). If \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( \alpha > 0 \). If \( \hat{x}_\ell \geq x'' \), then \( g \) is indifferent. If \( \hat{x}_\ell \in [0, x'] \), then \( g \)'s preference is ambiguous.

First, \( g \) forgoes access to a range of more centrist legislators. More access increases the probability that \( \ell \) proposes \( y \), at the expense of \( z_\ell \). In this case, \( M \) prefers \( z_\ell \) to \( y \), so \( M \)'s continuation value from rejecting policies decreases with \( \alpha \). Thus, \( M \) passes a wider range of policy proposals. The partisans, \( L \) and \( R \), therefore propose more

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26Recall that \( g \) and \( \ell \) are aligned if \( \hat{x}_\ell \) and \( \hat{x}_g \) are on the same side of 0.
extreme policies. Proposition 2 simply shows that if \( \ell \) is not too far from \( g \), then \( g \)'s marginal benefit from being more likely to lobby \( \ell \) is always dominated by the marginal cost of endogenous spillovers generating more extreme proposals by \( L \) and \( R \). Figure 4 illustrates this logic.

Proposition 2 does not preclude \( g \) wanting access if \( \ell \) is sufficiently centrist, e.g., \( \hat{x}_\ell \in [0, x') \) in Figure 3. In this case, \( g \) receives a larger benefit from lobbying and may want access.

Figure 4: Forgoing access to a more centrist legislator

![Diagram](image1.png)

Figure 4 illustrates why a non-ideologue group, \( g \), forgoes access (\( \alpha = 0 \)) to legislator \( \ell \) if \( \hat{x}_\ell \in (x', \hat{x}_g) \). Part (a) displays equilibrium behavior for \( \alpha = 0 \). Part (b) illustrates \( \alpha > 0 \). Increasing \( \alpha \) has two immediate effects: (i) lobbying is more likely and (ii) \( M \)'s expectations worsen. Effect (ii) expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If \( \hat{x}_g \) and \( \hat{x}_\ell \) are close, then effect (ii) dominates and \( g \) prefers \( \alpha = 0 \).

If \( \ell \) is relatively more extreme, then \( g \)'s preference over access is qualitatively different. As long as \( \ell \) is not too extreme, \( \hat{x}_\ell \in (\hat{x}_g, x'') \), \( g \) wants access. For such \( \hat{x}_\ell \), increasing \( \alpha \) improves \( M \)'s continuation value and shrinks the acceptance set. The indirect effect of access thus forces partisans to propose more centrist policy. Therefore \( g \) strictly benefits from greater access because (i) lobbying opportunities increase and (ii) partisan legislators propose more favorable policies. Figure 5 depicts the logic.

Finally, if \( \ell \) is aligned with \( g \) and sufficiently extreme, then \( g \) cannot profitably lobby to change \( \ell \)'s proposal. Thus, \( g \) is indifferent over access and will not pay for it.

**Who do Ideologue Groups want to Access?**

Next, I study ideologue interest groups. Their preferences over access are ambiguous in general. Specifically, if \( \alpha \) changes the acceptance set, then either \( L \) or \( R \)'s equilibrium
Figure 5: Seeking access to a more extreme legislator

Figure 5 illustrates why a non-ideologue group, $g$, prefers strictly positive access, $\alpha > 0$, if $\hat{x}_\ell \in (\hat{x}_g, x''')$. Part (a) displays equilibrium behavior if $\alpha = 0$. Part (b) illustrates $\alpha > 0$. Access has two immediate effects: (i) $g$’s probability of lobbying increases and (ii) $M$’s expectations improve. Effect (ii) causes the acceptance set to shrink, as shown in (b). Partisans propose more centrist policy. Both effects improve $g$’s expected payoff.

Proposal becomes worse for $g$ and the other partisan’s proposal becomes more favorable. Thus, drawing clear conclusions about $g$’s preference over access requires conditions on the balance of partisan proposal power.

Accordingly, I study a substantively motivated restriction on relative partisan proposal power that also permits a sharp characterization of preferences over access. In U.S. legislatures, the majority party typically exercises substantial control over committee assignments and committee leadership positions (Cox and McCubbins, 2005, 2007). To reflect this observation, I restrict proposal power to one side of the moderate legislator.\footnote{This assumption is stronger than necessary, but eases discussion. Proposition 3 is not knife-edge and holds as long as partisan proposal power is sufficiently imbalanced.} Although majority parties carefully allocate agenda control in the US, the model also aligns with empirical work suggesting that individual legislators possess some freedom from their party and can be influenced by interest groups (Fourirnaies, 2017).

**Definition 2.** The legislature exhibits *minority-party agenda exclusion* if agenda setting power is restricted to legislators on one side of 0. That is, either $\hat{x}_i < 0$ implies $\rho_i = 0$, or vice versa.

I say that $\ell$ is *majority-leaning* if she is aligned with the majority partisan. Proposition 3 shows that a majority-leaning group wants access to majority-leaning non-
ideologue legislators, but is indifferent over access to majority-leaning ideologue legislators. The result focuses on majority-leaning legislators because minority-leaning legislators do not have proposal power and thus it is immediate that $g$ is indifferent. The result suggests $g$ will only pay for access if $\ell$ is majority-leaning and not too extreme.

**Proposition 3.** Assume there is minority-party agenda exclusion and the interest group, $g$, is a majority-leaning ideologue.

1. If legislator $\ell$ is a majority-leaning ideologue, then $g$ forgoes costly access.

2. If $\ell$ is a majority-leaning non-ideologue, then $g$ acquires access.

The first part of Proposition 3, that $g$ forgoes costly access to majority-party ideologues, follows because $g$ cannot profitably change $\ell$’s proposal.

The second part of Proposition 3, that $g$ wants access to majority-leaning non-ideologue legislators, follows because access provides two benefits for $g$ under minority-party agenda exclusion. First, it increases $g$’s chances of enjoying a lobbying surplus. Second, it diminishes $M$’s expectations about future policy and thus expands the acceptance set. Partisans can therefore pass more extreme policy. Because minority-party partisans are unable to propose policy under minority-party agenda exclusion, $g$ benefits from emboldening aligned partisan legislators without risking more extreme proposals by opposing partisans.

**Access and Legislator Welfare**

With a grasp on where access is likely to develop, I discuss legislator welfare.

Whenever group $g$ lobbies legislator $\ell$, it exactly compensates $\ell$ for modifying her proposal. Access thus alters $\ell$’s expected payoff only by indirectly affecting partisan proposals. Depending on the relative extremism of $\ell$ and $g$, more access can be good or bad for $\ell$. For example, increasing $\alpha$ improves $\ell$’s expected payoff if $g$ is slightly more centrist. Here, access acts as a commitment device on $\ell$’s proposals that indirectly constrains partisan proposals. In contrast, $\ell$ is always weakly worse off giving access to a more extreme aligned group because extremism increases. Although not modeled, these observations suggest legislators may price discriminate based on group ideology when selling access.
For the welfare effects of access for other legislators, I focus on $M$. Access that increases the probability of centrist proposals also reduces expected extremism, and vice versa. Thus, $M$’s welfare improves if groups access more extreme legislators and decreases if they access relative centrists. Propositions 2 and 3 therefore have immediate welfare implications for a wide range of aligned group-legislator pairs. In some cases, groups do not acquire access and thus do not effect welfare. Highlighting this possibility, and the conditions producing it, is a key benefit of the formal analysis.

**Persistent vs. Short-term Access**

Thus far, I have studied persistent access, which has direct and indirect effects. The direct effect, opportunities to lobby, always benefits the group. The spillover effect, changing partisan proposals, can be good or bad. Under certain conditions, adverse spillovers dominate, causing groups to forgo access. Overall, the main analysis distinguishes groups that covet access from those less inclined.

Persistence is key for spillovers. Suppose groups can choose access only once and it lasts one period. Then access today does not affect expectations about future policymaking. Thus, the acceptance set and partisan proposals do not change. Immediate, one-shot access provides only a direct benefit, so groups always prefer it over no access.

What if the group can set access freely each period? Then it will choose full access, i.e. $\alpha = 1$, in every period of every stationary equilibrium. Expected payoffs are thus equal to those from persistent full access. Thus, the main analysis implies that some groups would rather commit to forgo access to a range of more centrist legislators. In such cases, however, the group prefers an access contract specifying one period of immediate, one-shot access with no chance for later access. Because the acceptance set is unchanged, this arrangement has a direct benefit without any indirect cost. Thus, one of two possibilities must hold for connections to form in these group-legislator pairs: (i) access decays quickly and commitment is impossible, or (ii) groups can contract against future access.

The preceding discussion focuses on group-legislator pairs for which spillover effects discourage access. But some groups benefit from the spillover effect of persistent access to particular legislators. Groups always want access in these cases, regardless of its

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28In this ordered setting, $M$’s expected payoff captures the spirit of majority welfare (Banks and Duggan, 2006b).

29In a stationary strategy profile, a one-shot access deviation does not change expectations about future policymaking. Thus, the group has a profitable deviation if it is not choosing full access.
persistence or whether they can contract on it. Thus, legislative spillovers increase their desire for persistent access.

**Willingness to Pay for Access**

The preceding analysis considers whether groups want access at all. But how does the value of access vary for access-seeking groups? I make two observations about $g$’s willingness to pay (WTP) for access.$^{30}$ First, and consistent with a large body of empirical work, groups have higher WTP for access to legislators with greater proposal power. Second, under broad conditions I show that groups farther from a given legislator have higher WTP for access.

Proposition 4 shows that groups are willing to pay more for access to who legislators are more likely to propose.

**Proposition 4.** All else equal, an interest group’s willingness to pay for $\alpha$ access weakly increases with the targeted legislator’s proposal power.

The result is intuitive and may appear obvious. Yet, it is not trivial because $\ell$’s proposal power, $\rho_\ell$, can have competing effects. On the one hand, $\rho_\ell$ amplifies the marginal benefit of access by increasing the probability $g$ extracts surplus by lobbying. This increases the value of additional access. On the other hand, it also increases how sensitive the acceptance set is to $\alpha$, which can increase or decrease the value of access. Whenever $g$’s WTP is strictly positive, however, the overall effect is proportional to $\rho_\ell$. Thus, if the group is willing to pay for access, then this desire increases with $\rho_\ell$.

Proposition 4 holds broadly, as it does not depend on the respective ideologies of the legislator and interest group. This theoretical robustness aligns with the robustness of empirical evidence that legislators on important and relevant committees, especially committee chairmen, attract more contributions (Ainsworth, 2002; Grimmer and Powell, 2016; Fouirnaies, 2017; Berry and Fowler, 2018).

Next, I analyze how $g$’s ideology affects its willingness to buy access to a majority-leaning legislator under minority-party exclusion. Proposition 5 fixes $\hat{x}_\ell$ and analyzes $g$’s willingness to pay for increasing $\alpha$ from zero, which I refer to as $g$’s *willingness to acquire access* (WTA). Under broad conditions, $g$’s WTA weakly increases as $\hat{x}_g$ diverges from $\hat{x}_\ell$ in either direction.

$^{30}$See, e.g., Denzau and Munger (1986) and Hall and Deardorff (2006) for previous work on access-seeking campaign contributions studying willingness to pay.
Proposition 5. Suppose there is minority-party agenda exclusion and legislator $\ell$ is majority-leaning. If either (i) the interest group, $g$, is more centrist than $\ell$, or (ii) $g$ is majority-leaning and more extreme than $\ell$, then $g$’s willingness to acquire access weakly increases as $g$ becomes less ideologically similar to $\ell$.

To discuss the logic for Proposition 5, I focus on right-party control, as depicted in Figure 6.

Suppose $g$ is more centrist than $\ell$. Two forces increase $g$’s WTA as $|\hat{x}_g - \hat{x}_\ell|$ increases. First, $g$’s lobbying surplus grows, so it has more to gain from lobbying. Second, $g$’s access forces majority-party partisans to moderate their policy proposals further because $g$’s policy offer gets better for $M$. Thus, $g$ gains more from inducing partisan moderation. These effects increase $g$’s WTA as $\hat{x}_g$ decreases away from $\hat{x}_\ell$.

Figure 6: Willingness to acquire access

![Figure 6: Willingness to acquire access](image)

Figure 6 illustrates Proposition 5 for a majority-centrist legislator, $\ell$, under left-party agenda exclusion. The group’s willingness to acquire access decreases as its ideal point approaches $\hat{x}_\ell$.

Next, suppose $g$ is more extreme than $\ell$ and majority leaning. The logic is best described in two cases. Let $\overline{x}_0$ denote the upper bound of $A^*$ given $\hat{x}_\ell$ and $\alpha = 0$.

First, suppose $\hat{x}_g \geq \overline{x}_0$, as depicted in Figure 6. If $\hat{x}_\ell < \overline{x}_0$, as pictured in Figure 6, then $g$’s WTA decreases as $\hat{x}_g$ shifts towards $\hat{x}_\ell$ for reasons symmetric to those described above: $g$’s lobbying surplus decreases and $g$’s benefit from inciting more extreme partisan proposals also decreases. If $\hat{x}_\ell \geq \overline{x}_0$, then $g$’s lobbying is inconsequential and $g$’s WTA is constant at zero as $\hat{x}_g$ approaches $\hat{x}_\ell$.

Second, suppose $\hat{x}_g \in (\hat{x}_\ell, \overline{x}_0)$. Access has competing effects in this case. By logic similar to Proposition 2, $g$ forgoes access if $\hat{x}_g$ is close to $\hat{x}_\ell$. But $g$’s WTA increases
as \( \hat{x}_g \) shifts away from \( \hat{x}_\ell \). Specifically, whenever \( g \)'s WTA is positive, lobbying surplus grows faster than the loss from inciting worse partisan proposals.

An additional observation is that if \( g \)'s WTA is zero, then \( g \) is not willing to pay for \textit{any} positive amount of access. Proposition 5 thus implies that a majority-leaning group forgoes access if it is slightly more extreme than \( \ell \), mirroring Proposition 2.31

**Lobbying Expenditures**

Having studied access acquisition, I now fix connections and characterize how equilibrium lobbying expenditures vary with several legislative features. A key observation is that expenditures weakly increase as the acceptance set expands. Thus, the characterization of \( \pi^* \) in (4) has direct implications for expenditures.

To state Proposition 6, I first define a general notion of legislature-level polarization. Given the distribution of agenda power, \( \rho \), and access, \( \alpha \), let the moderate legislator’s \textit{unconstrained extremism lottery} be the lottery putting probability \( \alpha \rho_\ell \) on \( |\hat{y}| \), probability \( \rho_\ell(1 - \alpha) \) on \( |\hat{x}_\ell| \), and probability \( \rho_j \) on \( |\hat{x}_j| \) for each legislator \( j \neq \ell \). Thus, the outcomes of an unconstrained extremism lottery are measured in terms of absolute distance between each player’s ideal point and 0. Say that \textit{legislative extremism} under \((\rho', \alpha')\) is higher than \((\rho, \alpha)\) if \( M \)'s unconstrained extremism lottery induced by \((\rho', \alpha')\) first order stochastically dominates the lottery induced by \((\rho, \alpha)\).32 For example, legislative extremism increases if some proposal power transfers away from \( M \) to other legislators.

**Proposition 6.** The interest group’s equilibrium lobbying expenditures weakly increase as either (i) legislative extremism increases, holding constant \( \hat{x}_g \) and \( \hat{x}_\ell \); (ii) the status quo policy becomes more extreme; or (iii) legislator patience decreases.

Expenditures weakly increase as the acceptance set expands and weakly decrease as it shrinks. Each comparative static thus follows from expanding the acceptance set. First, increasing legislative extremism decreases \( M \)'s reservation value because extreme policy proposals become more likely and there is no offsetting increase in the chance of moderate policy proposals. Second, more extreme status quo decrease \( M \)'s reservation

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31See Lemma 7 in Appendix A for more details.
32In this context, the unconstrained extremism lottery \((\rho', \alpha')\) \textit{first order stochastically dominates} another unconstrained extremism lottery \((\rho, \alpha)\) if: (i) for all \( x \in X \), \((\rho', \alpha')\) puts weakly greater probability on \( x' \) such that \( |x'| \geq |x| \) and (ii) for some \( x \in X \), \((\rho', \alpha')\) puts strictly greater probability on \( x' \) such that \( |x'| \geq |x| \).
value. Third, decreasing legislator patience makes $M$ more bothered by enduring the status quo and place less weight on policies that eventually pass.

To see the logic for why changes to the acceptance set drive Proposition 6, recall that $g$’s equilibrium transfer to $\ell$ is $m = u_\ell(z_\ell) - u_\ell(y)$. This transfer increases if either: $g$’s policy offer becomes worse for $\ell$, making $u_\ell(y)$ more negative, or $\ell$ can pass more favorable policy after rejecting $g$’s overtures, increasing $u_\ell(z_\ell)$. Informally, expenditures increase if there is more slack for $g$ to shift $\ell$’s proposal or if $\ell$ has a better outside option.

Figure 7: Increasing lobbying expenditures – extreme group

![Diagram](attachment:figure.png)

Figure 7(a) displays legislator $\ell$’s proposals for a baseline acceptance set where $g$’s offer is constrained and $\ell$’s proposal is unconstrained. Figure 7(b) displays $\ell$’s proposals if the acceptance set expands. Expenditures increase because $g$ can obtain more favorable policy.

If $g$ is extremist and $M$ constrains $g$’s equilibrium policy offer, then greater legislative extremism gives $g$ more slack to lobby $\ell$ to more extreme policy. Consequently, lobbying expenditures increase because $g$’s policy offer is worse for $\ell$. Figure 7 displays this case. If $g$ and $\ell$ are aligned extremists, however, then $z_\ell = y$ and lobbying expenditures are constant for small enough changes in legislative extremism.

Alternatively, if $\ell$ is extremist, then increasing legislative extremism improves $\ell$’s outside option because $z_\ell$ equals the boundary of $A^*$ closest to $\hat{x}_\ell$. This boundary shifts towards $\hat{x}_\ell$ as legislative extremism increases, improving $\ell$’s outside option and forcing $g$ to transfer more to lobby $\ell$ away from $z_\ell$. If $\ell$ is not too extreme, and $g$ is aligned with $\ell$ but not extremist, then $y \neq z_\ell$. Lobbying grows more expensive even though the policy offer does not change. Figure 8 illustrates.

Next, I state three corollaries of Proposition 6 showing how substantively meaning-
ful features of the model affect extremism and, in turn, lobbying expenditures.

First, an important special case of changing legislative extremism is varying $g$’s access, $\alpha$. Greater access causes $M$ to anticipate more frequent lobbying by $g$. Thus, $\alpha$’s effect on the acceptance set depends on $g$ and $\ell$’s relative ideology. If $g$ is more extreme, then increasing $\alpha$ raises legislative extremism and the acceptance set expands. This relationship flips if $g$ is more centrist. Given a group-legislator pair, Proposition 6 yields an immediate corollary on the relationship between access and lobbying expenditures.

**Corollary 6.1.** Suppose the interest group, $g$, is aligned with legislator $\ell$. If $g$ is more extreme than $\ell$, then equilibrium lobbying expenditures weakly increase with access. Otherwise, they weakly decrease with access.

Next, lobbying expenditures increase if $M$ loses proposal power. Substantively, Corollary 6.2 suggests that weakening the agenda setting power of centrist encourages more vigorous lobbying.

**Corollary 6.2.** If proposal power transfers away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.

Finally, lobbying expenditures grow as $\ell$ shifts away from $M$, weakly increasing legislative extremism. Corollary 6.3 suggests that groups will spend more on lobbying in
legislatures that are more polarized in the colloquial sense of having greater ideological spread among legislators.

**Corollary 6.3.** If legislator $\ell$ shifts farther away from the moderate legislator, then equilibrium lobbying expenditures weakly increase.

**Conclusion & Implications**

I analyze a model of legislative policymaking in which access provides interest groups with opportunities to influence policy proposals. Broadly, the analysis unpacks a neglected byproduct of access in legislatures: the prospect of subsequent lobbying can spill over to affect policies proposed by other legislators today. The model provides a tractable framework to explore how access-seeking depends on the larger legislative context.

To study which legislators are targeted for access, the interest group chooses how much access to acquire to particular legislators. In equilibrium, access can have spillover effects by endogenously changing which policies the legislature will pass. In turn, access can indirectly alter policy proposals of other legislators who are constrained by legislative voting. Due to these spillovers, non-extremist groups choose to forego access to a range of more centrist legislators. Policy considerations drive this behavior, as access to these legislators increases policy polarization and counteracts better lobbying prospects. On the other hand, beneficial spillovers make these groups keen to access more extreme legislators.

The analysis has implications for several interest group activities: campaign finance, lobbying expenditures, uncontested lobbying, and the lobbyist labor market.

**Campaign Finance:** In light of evidence that interest groups use campaign contributions to get access (Kalla and Broockman, 2015), there are several implications for contribution patterns. Given interest group ideology, the analysis suggests a potentially multi-modal relationship between contributions and legislator ideology: giving to centrists and some relatively more extreme legislators, but not giving to those who are slightly more centrist or very extreme. Furthermore, a group’s decision to access a nearby legislator can depend critically on their relative extremism. The model also indicates that ideologically centrist groups are especially keen on gaining access to a broad swath of legislators. Empirical evidence aligns with this implication, as contributing groups appear to be overwhelmingly centrist and give broadly (Bonica, 2013;
Barber, 2016).

More broadly, the analysis also speaks to *Tullock’s puzzle*, the empirical regularity that many interest groups do not contribute at all and those that do rarely reach legal limits (Tullock, 1972). Some view Tullock’s Puzzle as evidence that contributions are not valuable, or that donors are unsophisticated (Ansolabehere et al., 2003). Existing work shows that competition can lead to sophisticated interest groups contributing small amounts (Chamon and Kaplan, 2013). I flesh out a new strategic mechanism for Tullock’s Puzzle, legislative considerations. Even in the absence of competition, these considerations can reduce contributions by sophisticated groups precisely because the contributions are valuable for gaining access, which generates adverse spillover effects.

*Lobbying Expenditures*: When evaluating the connection between access and lobbying expenditures, Corollary 6.1 suggests that empirical work should control for relative extremism of groups and legislators. Otherwise, offsetting observations can obscure a meaningful effect. Another implication is that ceteris paribus changes in lobbying expenditures can indicate changes in access levels. Where data on relative ideology exists, one can infer whether access increased or decreased. Finally, for centrist groups targeting extreme legislators, a negative correlation between lobbying expenditures and measures of access can arise because the targeted legislator’s outside option is worse, making the group’s desired policy cheaper. Observing such a correlation thus need not imply that access makes the legislator more responsive to the group’s interests.

*Uncontested Lobbying*: Interest groups frequently lobby unopposed (Baumgartner and Leech, 2001; Dal Bó, 2007). Where are competing interests? Existing explanations include collective action problems, free-riding incentives, and entry costs. Although the main analysis abstracts from competition over access to isolate how legislative considerations shape access-seeking, it can shed light on these competitive voids.

Legislative considerations can discourage competition over access. To illustrate, suppose legislator \( \ell \) is slightly more centrist than a non-ideologue group, \( g \). Say that \( g \) *concedes access* to another group \( g' \) if \( g \) strictly prefers to let \( g' \) acquire access optimally. By Proposition 2, \( g \) forgoes access to \( \ell \). Furthermore, if \( g' \) slightly more centrist than \( \ell \), then it seeks access by Proposition 2 and further constrains \( L \) and \( R \). Under such conditions, \( g \) can strictly prefer to concede access to \( g' \). Why? Although forgoing access avoids increasing expected extremism, \( g \) can be even better off if expected extremism decreases. Thus, if \( g' \) is not too far from \( \ell \), then \( g \) concedes access to \( g' \).

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33Competition over agenda power has been studied in other settings (e.g. Levy and Razin, 2013) and a full analysis is outside the scope of this paper.
**Lobbyist Labor Market:** Another avenue to access is hiring lobbyists with connections to legislators. Thus, the model has implications for revolving-door hiring and points of contact. It suggests that many interest groups will court associates of legislators who are very centrist, or more extreme but not too extreme. A robust prediction is that associates of legislators with greater agenda power will be coveted and enjoy a wage premium, matching empirical evidence (Blanes i Vidal et al., 2012; Bertrand et al., 2014). These implications are especially relevant for groups hiring in-house lobbyists who provide access and faithfully advocate for the group’s interests while lobbying.

Another implication is that, by the logic of conceding access discussed above, groups may prefer to delegate to lobbyists who are more centrist lobbyists and possibly even on the opposite side of the legislator. Substantively, this incentive could manifest as groups using outside lobbyists. To the extent that outsiders have more autonomy than in-house lobbyists, the model suggests a new rationale for why wealthy groups use outside lobbyists in some instances. In a similar vein, this logic could partially explain why some industries lobby through trade associations funded by diverse interests.
Appendix A

Extended Model

I prove the main results in a version of the model that relaxes restrictions on the number of legislators and interest groups. There are three disjoint sets of players: \( n^V \) (finite and odd) voting legislators in \( N^V \); \( n^L \geq 3 \) committee members in \( N^L \); and \( n^G \leq n^L \) interest groups in \( N^G \). Let \( N = N^V \cup N^L \cup N^G \).

Throughout, voting legislators are called voters and denoted by \( i \). To align with the main text, \( M \) denotes the median voter. I denote committee members by \( \ell \) and interest groups by \( g \). Each \( \ell \in N^L \) is associated with only one group, \( g_{\ell} \). Each \( g \in N^G \) can have access to multiple \( \ell \in N^L \) and this set is \( N^L_g \subseteq N^L \). Let \( \alpha_{\ell} \in [0,1] \) denote \( g_{\ell} \)'s access to \( \ell \).

Legislative bargaining occurs over an infinite number of periods \( t \in \{1, 2, \ldots \} \). The policy space \( X \subseteq \mathbb{R} \) is non-empty, compact, and convex. Let \( \rho = (\rho_1, \ldots, \rho_{n^L}) \in \Delta([0,1])^{n^L} \) be the distribution of recognition probability among \( \ell \in N^L \). In each period \( t \), bargaining proceeds as follows. If no policy has passed before \( t \), then \( \ell \) proposes with probability \( \rho_{\ell} > 0 \). All players observe the period-\( t \) proposer, \( \ell_t \). With probability \( 1 - \alpha_{\ell} \), \( g_{\ell_t} \) cannot lobby and \( \ell_t \) freely proposes any \( x_t \in X \). With probability \( \alpha_{\ell} \), \( g_{\ell_t} \) can lobby and offers \( \ell_t \) a binding contract \((y_t, m_t) \in X \times \mathbb{R}_+\). Next, \( \ell_t \) accepts or rejects. Let \( a_t \in \{0, 1\} \) denote \( \ell_t \)'s period-\( t \) acceptance decision, where \( a_t = 1 \) indicates acceptance and \( a_t = 0 \) if either \( \ell_t \) rejects or \( g_{\ell_t} \) is unable to lobby in \( t \). If \( \ell_t \) accepts, then \( \ell_t \) is committed to propose \( x_t = y_t \) in \( t \) and \( g_{\ell_t} \) transfers \( m_t \) to \( \ell_t \). If \( \ell_t \) rejects, then she can propose any \( x_t \in X \) and \( g_{\ell_t} \) keeps \( m_t \). All players observe \( x_t \). There is a simultaneous vote by \( i \in N^V \) using simple majority rule. If \( x_t \) passes, then bargaining ends with \( x_t \) enacted in \( t \) and all subsequent periods. If \( x_t \) fails, then \( q \) is enacted in \( t \) and bargaining proceeds to \( t + 1 \).

Each player \( j \in N \) has quadratic policy utility with ideal point \( \hat{x}_j \in X \). As in the main text, I normalize \( \hat{x}_M = 0 \) and assume \( q \neq 0 \). Additionally, I assume there exists \( \ell \in N^L \) on the same side of \( q \) as \( M \) such that \( \alpha_{\ell} < 1 \) or \( g_{\ell} \) is on the same side of \( q \). For example, assume \( q > 0 \). Then some \( \ell \in N^L \) satisfies \( \hat{x}_\ell < q \) and at least one of the following holds: \( \hat{x}_{g_{\ell}} \leq q \) or \( \alpha_{\ell} < 1 \).

Players discount streams of stage utility by common discount factor \( \delta \in (0,1) \).

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34 An independent legislator is accommodated by \( \alpha_{\ell} = 0 \).

35 Where \( \Delta([0,1])^{n^L} \) denotes the \( n^L \)-dimensional unit simplex.
For convenience, I normalize per-period payoffs by \((1 - \delta)\). Let \(I^\ell_t \in \{0, 1\}\) equal one iff \(\ell\) is the period-\(t\) proposer and \(g_{\ell_t}\) can lobby in \(t\). Given a sequence of offers \((y_1, m_1), (y_2, m_2), \ldots\), a sequence of proposers \(\ell_1, \ell_2, \ldots\) a sequence of acceptance decisions \(a_1, a_2, \ldots\), and a sequence of independent policy proposals \(x_1, x_2, \ldots\) such that bargaining continues until \(t\), the discounted sum of per-period payoffs for \(i \in N^V\) is

\[
(1 - \delta^{t-1})u_i(q) + \delta^{t-1}\left[ (1 - a_t)u_i(x_t) + a_t u_i(y_t) \right];
\]

for \(\ell \in \mathcal{N}^\ell\),

\[
(1 - \delta)\sum_{t'=1}^{t-1} \delta^{t'-1} [u_{\ell_t}(q) + I^\ell_t a_{t'} m_{t'}] + \delta^{t-1}\left[ (1 - a_t)u_{\ell_t}(x_t) + a_t \left( u_{\ell_t}(y_t) + I^\ell_t m_t \right) \right];
\]

and for \(g \in \mathcal{N}^g\),

\[
(1 - \delta)\sum_{t'=1}^{t-1} \delta^{t'-1} \left[ u_{g_t}(q) - a_{t'} m_{t'} \sum_{\ell \in \mathcal{N}^L_g} I^\ell_{t'} \right] + \delta^{t-1}\left[ (1 - a_t)u_{g_t}(x_t) + a_t \left( u_{g_t}(y_t) - m_t \sum_{\ell \in \mathcal{N}^L_g} I^\ell_{t} \right) \right].
\]

Unless noted otherwise, results are proved for this more general setting. The model in the main text is a special case featuring one voter with ideal point \(\hat{x}_M\); four committee members with ideal points \(\hat{x}_L, \hat{x}_M, \hat{x}_\ell,\) and \(\hat{x}_R\); and one group at \(\hat{x}_g\) with access \(\alpha_{\ell} \geq 0\) and \(\alpha_j = 0\) for all \(j \neq \ell\).

**Strategies**

I study a class of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.\(^{36}\)

Let \(\Delta(X)\) be the set of probability measures on \(X\). Let \(W = X \times \mathbb{R}_+\) denote the lobby offer space and \(\Delta(W)\) denote the set of probability measures on \(W\). A stationary mixed strategy for \(g \in \mathcal{N}^G\) is a probability measure \(\lambda_g \in \Delta(W)^{\left|\mathcal{N}^L_g\right|}\) over \(g\)’s offers \((y, m) \in W\) to each \(\ell \in \mathcal{N}^L_g\). A stationary mixed legislative strategy for \(\ell \in \mathcal{N}^L_g\)

\(^{36}\)In Appendix B, I define stationary mixed strategy legislative lobbying equilibria and show that they are all equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium with deferential voting and deferential acceptance.
is a pair \((\pi_\ell, \varphi_\ell)\); where \(\pi_\ell \in \Delta(X)\) specifies a probability measure over \(\ell\)'s independent proposals and \(\varphi_\ell : W \to [0,1]\) is the probability \(\ell\) accepts each \((y, m) \in W\). Finally, voter \(i\)'s stationary mixed strategy \(\nu_i : X \to [0,1]\) specifies the probability \(i\) votes for each \(x \in X\).

Let \(\lambda\) denote a profile of interest group strategies, \((\pi, \varphi)\) a profile of committee member strategies, and \(\nu\) a profile of voter strategies. A stationary strategy profile is \(\sigma = (\lambda, \pi, \varphi, \nu)\). Under \(\sigma\), let \(\nu_\sigma(x)\) represent the probability that \(x\) passes in a given period.

**Continuation Values**

Let \(w = (y, m) \in W\) denote an arbitrary lobby offer. For convenience, define

\[
\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda^{\ell}_y(dw), \tag{7}
\]

which is the probability under \(\sigma\) that \(\ell\) makes an independent policy proposal in any period she is recognized. Given \(\sigma\), \(i \in N^V\) has continuation value

\[
V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)] [(1 - \delta)u_\ell(q) + \delta V_i(\sigma)] \right] \lambda^\ell_y(dw) + \xi_\ell(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)] [(1 - \delta)u_\ell(q) + \delta V_i(\sigma)] \right] \pi_\ell(dx) \right), \tag{8}
\]

the continuation value of \(\ell \in N^L\) is

\[
\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \int_W \varphi_j(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)] [(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda^j_y(dw) + \xi_j(\alpha, \sigma) \int_X \left[ \nu_\sigma(x)u_\ell(x) + [1 - \nu_\sigma(x)] [(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_j(dx) \right),
\]

\[
+ \rho_\ell \left( \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nu_\sigma(y)u_\ell(y) + [1 - \nu_\sigma(y)] [(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m \right] \lambda^\ell_y(dw) \right).
\]
\[ + \xi(\alpha, \sigma) \int_X \left[ \varphi(x)u_\ell(x) + [1 - \varphi(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}(\sigma)] \right] \pi_\ell(dx), \]

(9)

and the continuation value of \( g \in N^G \) is

\[ \hat{V}_g(\sigma) = \sum_{\ell \in N^L_g} \rho(\alpha) \int_W \varphi(y, m) \left[ \varphi(y)u_g(y) + [1 - \varphi(y)][(1 - \delta)u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \lambda^\ell_g(dw) \]

\[ + \xi(\alpha, \sigma) \int_X \left[ \varphi(x)u_g(x) + [1 - \varphi(x)][(1 - \delta)u_g(q) + \delta \tilde{V}_g(\sigma)] \right] \pi_\ell(dx), \]

(10)

**Stationary Legislative Lobbying Equilibrium**

A stationary pure strategy for \( g \in N^G \) is \((y_g, m_g) \in X^{N^L_g} \times \mathbb{R}^{N^L_g} \), where \( y_g \) is \( g \)'s profile of policy offers and \( m_g \) is \( g \)'s profile of monetary offers. A pure stationary strategy for \( \ell \in N^L \) is \((z_\ell, a_\ell)\); where \( z_\ell \in X \) specifies \( \ell \)'s independent proposals, and \( a_\ell : X \times \mathbb{R} \to \{0, 1\} \) equals one iff \( \ell \) accepts \( g_\ell \)'s offer. Finally, for each \( i \in N^V \), \( v_i : X \to \{0, 1\} \) equals one iff \( i \) supports the proposal.

Given \( \sigma \), the set of policies that pass is constant across periods by stationarity and denoted \( A(\sigma) \subset X \). For \( \ell \in N^L \), define

\[ \tilde{U}_\ell(x; \sigma) = \begin{cases} u_\ell(x) & \text{if } x \in A(\sigma) \\ (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) & \text{else}. \end{cases} \]  

(11)

Formally, \( \sigma = (y, m, z, a, v) \) is a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance if it satisfies five conditions. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), \((y_\ell^g, m_\ell^g)\) satisfies

\[ y_\ell^g = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) \]

(12)
and
\[ m_g^\ell = u_\ell(z_\ell) - u_\ell(y_\ell^g). \] (13)

Second, for all \( \ell \in N^L \) and \((y, m) \in W\), \( a_\ell(y, m) = 1 \) iff
\[ \bar{U}_\ell(y; \sigma) + m \geq \bar{U}_\ell(z_\ell; \sigma). \] (14)

Third, for each \( \ell \in N^L \), \( z_\ell \) solves
\[ \max_{x \in A(\sigma)} u_\ell(x). \] (15)

Finally, for each \( i \in N^V \), \( v_i(x) = 1 \) iff
\[ u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma). \] (16)

Appendix B shows that all stationary mixed strategy legislative lobbying equilibria are equivalent in outcome distribution to strategy profiles satisfying (12)-(16).

**Existence**

I now prove part 1 of Proposition 1 from the main text.

**Proposition 1.1.** There exists a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.

**Proof.** There are three parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for \( g \in N^G \) and \( \ell \in N^L \). Part 2 uses the fixed point to construct a strategy profile \( \sigma \). Part 3 verifies \( \sigma \) satisfies (12) - (16).

**Part 1:** Let \((y, z) = (y_1, \ldots, y_{n^L}, z_1, \ldots, z_{n^L}) \in X^{2n^L}\) and for each \( j \in N \) define
\[ r_j(y, z) = \sum_{\ell \in N^L} \rho_\ell \left( \alpha_\ell u_j(y_\ell) + (1 - \alpha_\ell)u_j(z_\ell) \right). \] (17)

Set \( A(r(y, z)) = \{ x \in X \vert u_M(x) \geq (1 - \delta)u_M(q) + \delta r_M(y, z) \} \), which is non-empty, compact, and convex because \( u_M \) is strictly concave, \( q = 0 \), and \( \delta \in (0, 1) \). Moreover,
\(A(r(y, z))\) is continuous in \((y, z)\).

For each \(\ell \in N^L\), define
\[
\tilde{\phi}_\ell(y, z) = \arg \max_{y, z} u_{g_\ell}(y) + u_\ell(y),
\]
which is unique for all \((y, z)\) because the objective function is strictly concave and continuous and \(A(r(y, z))\) is non-empty, compact and convex. Because \(A(r(y, z))\) is continuous, the Theorem of the Maximum implies continuity of \(\tilde{\phi}_\ell(y, z)\). Next, define
\[
\phi_\ell(y, z) = \arg \max_{z \in A(r(y, z))} u_\ell(z),
\]
which is unique for all \((y, z)\) and continuous by the Theorem of the Maximum.

Define the mapping \(\Phi : X^{2n^L} \to X^{2n^L}\) as
\[
\Phi(y, z) = \prod_{\ell \in N^L} \tilde{\phi}_\ell(y, z) \times \prod_{\ell \in N^L} \phi_\ell(y, z),
\]
which is a product of continuous functions and thus continuous in \((y, z)\). By Brouwer’s theorem, a fixed point \((y^*, z^*) = \Phi(y^*, z^*)\) exists because \(\Phi\) is a continuous function mapping a non-empty, compact, and convex set into itself.

**Part 2:** Define a stationary pure strategy profile \(\sigma\) as follows. First, for all \(g \in N^G\) and \(\ell \in N^L_g\), set \(y_g^\ell = y_g^*\) and \(m_g^\ell = u_\ell(z_g^*) - u_\ell(y_g^*)\). Next, for \(\ell \in N^L\), set \(z_\ell = z_\ell^*\) and define
\[
a_\ell(y, m) = \begin{cases} 
1 & \text{if } u_\ell(y) + m \geq u_\ell(z_\ell^*), \text{ for } y \in A(r(y^*, z^*)) \\
1 & \text{if } (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell\alpha_\ell m_\ell^\ell) + m \geq u_\ell(z_\ell^*), \text{ for } y \notin A(r(y^*, z^*)) \\
0 & \text{else.}
\end{cases}
\]

Finally, for each \(i \in N^V\) define \(v_i\) so that \(v_i(x) = 1\) if \(u_i(x) \geq (1 - \delta)u_i(q) + \delta r_\ell(y^*, z^*)\) and \(v_i(x) = 0\) otherwise.

**Part 3:** I check that \(\sigma\) satisfies (12)-(16).

First, I verify (16) to show \(A(\sigma) = A(r(y^*, z^*))\). Note that for each \(g \in N^G\) and all \(\ell \in N^L_g\), \(y_g^\ell \in A(r(y^*, z^*))\) and \(a_\ell(y_g^\ell, m_g^\ell) = 1\). Moreover, \(z_\ell \in A(r(y^*, z^*))\) for all \(\ell \in N^L\). Thus, voter \(i\)'s continuation value under \(\sigma\) is \(V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell[\alpha_\ell u_i(y_i^*) + (1 - \alpha_\ell)u_i(z_i^*)] = r_i(y_i^*, z_i^*)\). Thus, each voter \(i\)'s strategy satisfies (16). Banks and Duggan (2006b) and Duggan (2014) apply, so \(M\) is decisive over lotteries and \(A(\sigma) = A(r(y^*, z^*))\).
To check (12), consider \( g \in N_G \) and \( \ell \in N^L_g \). Focusing on acceptable offers is without loss of generality because \( a_{\ell}(z_{\ell}, 0) = 1 \). Because \( A(\sigma) = A(r(y^*, z^*)) \), (18) implies \( \tilde{\phi}_\ell(y^*, z^*) = \arg\max_{y_{\ell} \in A(\sigma)} u_{g_{\ell}}(y_{\ell}) + u_{\ell}(y_{\ell}) - u_{\ell}(z_{\ell}^*) \). Thus, (12) holds because \( \tilde{\phi}_\ell(y^*, z^*) = y_{\ell}^* = y_{\ell}^g \). Lemma B.6 in Appendix B implies \( y \notin A(\sigma) \) is not a profitable deviation for any \( g \in N_G \).

It is immediate that \( m_{\ell}^g \) satisfies (13).

To check (14), note that \( \ell \)'s expected dynamic payoff from rejecting \( g_{\ell} \)'s offer is \( \tilde{U}_{\ell}(z_{\ell}; \sigma) = u_{\ell}(z_{\ell}^*) \). Thus, \( \ell \) weakly prefers to accept any \((y, m)\) satisfying \( y \in A(r(y^*, z^*)) \) iff \( u_{\ell}(y) + m \geq u_{\ell}(z_{\ell}^*) \). If \( y \notin A(r(y^*, z^*)) \), then \( \ell \) weakly prefers to accept \((y, m)\) iff \((1 - \delta)u_{\ell}(q) + \delta(r_{\ell}(y^*, z^*) + \rho_{\ell}\alpha_{\ell}m_{\ell}^g) + m \geq u_{\ell}(z_{\ell}^*) \). Thus, \( a_{\ell} \) satisfies (14).

To check (15), note that (19) implies \( \phi_{\ell}(y^*, z^*) = \arg\max_{x \in A(\sigma)} u_{\ell}(x) \) because \( A(\sigma) = A(r(y^*, z^*)) \). Thus, (15) holds because \( \phi_{\ell}(y^*, z^*) = z_{\ell}^* = z_{\ell} \) for each \( \ell \in N^L \). By Lemma B.6 in Appendix B, \( x \notin A(\sigma) \) is not a profitable deviation for any \( \ell \in N^L \).

\[ \hat{y}_{\ell} = \arg\max_{y \in X} u_{g_{\ell}}(y) + u_{\ell}(y) = \frac{\hat{x}_{g_{\ell}} + \hat{x}_{\ell}}{2}. \] (21)

Recall \( u_{\ell}(z_{\ell}) \) is \( \ell \)'s expected dynamic payoff in equilibrium, conditional on rejecting \( g_{\ell} \)'s offer. By (12), in equilibrium

\[ y_{g_{\ell}}^\ell = \arg\max_{y \in A(\sigma)} u_{g_{\ell}}(y) + u_{\ell}(y) - u_{\ell}(z_{\ell}) = \arg\max_{y \in A(\sigma)} u_{g_{\ell}}(y) + u_{\ell}(y). \] (22)

If \( \hat{y}_{\ell} \in A(\sigma) \), then \( y_{g_{\ell}}^\ell = \hat{y}_{\ell} \). Otherwise, strict concavity implies \( y_{g_{\ell}}^\ell \) equals the boundary of \( A(\sigma) \) closest to \( \hat{y}_{\ell} \). As this characterization applies to every equilibrium, there is a clear connection to the characterization in Cho and Duggan (2003), where lobbying is absent.

Proposition 1.3 establishes Part 3 of Proposition 1 from the main text.

Equilibrium Analysis

Appendix B shows that every stationary mixed strategy legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential voting and deferential acceptance. The rest of the analysis omits qualifiers, simply referring to equilibria.

Define

\[ \hat{y}_{\ell} = \arg\max_{y \in X} u_{g_{\ell}}(y) + u_{\ell}(y) = \frac{\hat{x}_{g_{\ell}} + \hat{x}_{\ell}}{2}. \] (21)
Proposition 1.3. Every stationary legislative lobbying equilibrium has the same outcome distribution.

Proof. Let $\sigma$ and $\sigma'$ be stationary legislative lobbying equilibria. It suffices to show $(y_g, m_g) = (y'_g, m'_g)$ for all $g \in N^C$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Assume $y_{g\ell} \neq y'_{g\ell}$ or $z_\ell \neq z'_\ell$ for some $\ell \in N^L$. Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, $A(\sigma) = A(\sigma')$. Because $\sigma$ and $\sigma'$ are no-delay by Lemma 2, $\ell$'s expected dynamic payoff from rejecting $g_\ell$’s offer is $u_\ell(z_\ell)$ under both $\sigma$ and $\sigma'$. Lemma B.1 implies $m_\ell^\ell = u_\ell(y_\ell^\ell) - u_\ell(z_\ell)$. Therefore $(y_g, m_g) = (y'_g, m'_g)$ and $z_\ell = z'_\ell$. \hfill \Box

Comparative Statics on Lobbying Expenditures

To facilitate the analysis of endogenous access, it is useful to first prove Proposition 6. Set $\theta = (\hat{x}, \rho, \alpha)$. Let $\mu_\theta$ denote the unconstrained extremism lottery, which puts probability $\rho_\ell \alpha_\ell$ on $|\tilde{y}_\ell|$ and probability $\rho_\ell (1 - \alpha_\ell)$ on $|\hat{x}_\ell|$ for each $\ell \in N^L$. Given $\theta$ and $\theta'$, legislative extremism is greater under $\theta'$ if $\mu_{\theta'}$ first order stochastically dominates $\mu_\theta$.

Lemma 2. The equilibrium acceptance set weakly expands with legislative extremism.

Proof. Consider $\theta$ and $\theta'$, with legislative extremism greater under $\theta'$. By Proposition 1.3, $\theta$ and $\theta'$ each induce a unique equilibrium acceptance set. Let $\overline{x}_\theta$ and $\overline{x}_{\theta'}$ denote the respective upper bounds of these sets. I show $\overline{x}_{\theta'} \geq \overline{x}_\theta$.

For $b \geq 0$, let $C_j^b = \mathbb{I}\{\hat{x}_j \in (-b, b)\}$ and $\tilde{C}_j^b = \mathbb{I}\{\tilde{y}_j \in (-b, b)\}$. Define $C_j'^b$ and $\tilde{C}_j'^b$ analogously for $\hat{x}_j'$ and $\tilde{y}_j'$. For all $b \geq 0$,

$$
(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left((1 - \alpha_j)C_j^b u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^b u_M(\tilde{y}_j)\right) \\
+ \delta u_M(b) \sum_{j \in N^L} \rho_j \left((1 - \alpha_j)(1 - C_j^b) + \alpha_j (1 - \tilde{C}_j^b)\right) \\
\geq (1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j' \left((1 - \alpha'_j)C_j'^b u_M(\hat{x}'_j) + \alpha'_j \tilde{C}_j'^b u_M(\tilde{y}'_j)\right) \\
+ \delta u_M(b) \sum_{j \in N^L} \rho_j' \left((1 - \alpha'_j)(1 - C_j'^b) + \alpha'_j (1 - \tilde{C}_j'^b)\right),
$$

where (24) follows because $\mu_{\theta'}$ FOSD $\mu_\theta$ and $u_M$ is negative quadratic. The equilibrium characterization, and construction of $C_j$ and $\tilde{C}_j$, implies $\overline{x}_\theta$ is the unique $b \geq 0$ such
that \( u_M(b) \) equals (23). Analogously, \( \overline{\pi}_b \) is the unique \( b \geq 0 \) such that \( u_M(b) \) equals (24). Thus, \( \overline{\pi}_b \geq \overline{\pi}_b \).

**Proposition 6.** For all \( \ell \in \mathbb{N}^L \), \( g_\ell \)'s equilibrium lobbying expenditures increase as either (i) legislative extremism increases, fixing \( \hat{x}_\ell \) and \( \hat{x}_g \); (ii) \( |q| \) increases; or (iii) \( \delta \) decreases.

**Proof.** (i) Increase legislative extremism. Let \( \sigma \) denote an equilibrium, suppressing dependence on legislative extremism. By Lemma 2, \( \overline{\pi}(\sigma) \) weakly increases with legislative extremism. There are two cases.

- **Case 1.** Suppose \( \hat{x}_\ell \in A(\sigma) \). Then \( z_\ell = \hat{x}_\ell \). There are two subcases.
  
  First, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y^\ell_{\sigma} = \hat{y}_\ell \). Lemma B.1 and (12) imply \( m^\ell_g = u^\ell_{\sigma}(\hat{x}_\ell) - u^\ell_{\sigma}(\hat{y}_\ell) \). Lemma 2 implies \( z_\ell = \hat{x}_\ell \) and \( y^\ell_g = \hat{y}_\ell \) as legislative extremism increases, so \( m^\ell_g \) is constant.
  
  Second, assume \( \hat{y}_\ell \notin A(\sigma) \). Since \( \hat{x}_\ell \in A(\sigma) \), this requires \( \hat{x}_g \in [\overline{\pi}(\sigma), \overline{\pi}(\sigma)] \). Without loss of generality, assume \( \hat{x}_g > \overline{\pi}(\sigma) \). Thus, \( z_\ell = \hat{x}_\ell \) and \( y^\ell_g = \overline{\pi}(\sigma) \). Lemma B.1 and (12) imply \( m^\ell_g = u^\ell_{\sigma}(\hat{x}_\ell) - u^\ell_{\sigma}(\overline{\pi}(\sigma)) \). By Lemma 2, \( \overline{\pi}(\sigma) \) increases in legislative extremism. Therefore \( m^\ell_g \) increases.

- **Case 2.** Suppose \( \hat{x}_\ell \notin A(\sigma) \). Without loss of generality, assume \( \hat{x}_\ell > z_\ell = \overline{\pi}(\sigma) \). There are three subcases.
  
  First, assume \( \hat{y}_\ell < -\overline{\pi}(\sigma) \). Then \( y^\ell_{\sigma} = -\overline{\pi}(\sigma) \). By Lemma B.1 and (12), \( m^\ell_g = u^\ell(\overline{\pi}(\sigma)) - u^\ell(-\overline{\pi}(\sigma)) \). By Lemma 2, increasing legislative extremism increases \( \overline{\pi}(\sigma) \) and decreases \( -\overline{\pi}(\sigma) \). Thus, \( m^\ell_g \) increases because \( -\overline{\pi}(\sigma) < \overline{\pi}(\sigma) < \hat{x}_\ell \).
  
  Second, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y^\ell_g = \hat{y}_\ell \) and \( y^\ell_g \) is constant as legislative extremism increases. Arguments similar to subcase 2 of Case 1 imply \( m^\ell_g \) increases.
  
  Third, assume \( \hat{y}_\ell \geq \overline{\pi}(\sigma) \), which implies \( y^\ell_g = \overline{\pi}(\sigma) \). By Lemma B.1 and (12), \( m^\ell_g = u^\ell(\overline{\pi}(\sigma)) - u^\ell(\overline{\pi}(\sigma)) = 0 \), which is constant in legislative extremism.

Altogether, \( m^\ell_g \) weakly increases in legislative extremism.

(ii) Increase \( |q| \). First, let \( C_j(\hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \in \text{int}A(\sigma)\} \). Similarly, let \( \tilde{C}_j(\hat{y}_j) = \mathbb{I}\{\hat{y}_j \in \text{int}A(\sigma)\} \).
Then

\[
\pi(\sigma) = \left( \frac{(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left[ C_j(\hat{x}_\ell)(1 - \alpha_j)u_M(\hat{x}_j) + \tilde{C}_j(\hat{y}_j)\alpha_ju_M(\hat{y}_j) \right]}{1 - \delta \sum_{j \in N^L} \rho_j \left[ (1 - C_j(\hat{x}_\ell))(1 - \alpha_j) + (1 - \tilde{C}_j(\hat{y}_j))\alpha_j \right]} \right)^{\frac{1}{\delta}},
\]

(25)

Inspection of (25) shows \(\pi(\sigma)\) strictly increases in \(|q|\) and thus \(A(\sigma)\) expands. Arguments analogous to Part (i) imply \(m^\ell_g\) weakly increases in \(|q|\).

(iii) Decrease \(\delta\). Inspection of (25) shows \(\pi(\sigma)\) strictly decreases as \(\delta\) increases and thus \(A(\sigma)\) shrinks. Arguments analogous to Part (i) imply \(m^\ell_g\) weakly increases as \(\delta\) decreases. \(\square\)

**Endogenous Access**

Fix \(\ell \in N^L\) and recall \(\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}\). For convenience, refer to \(g_\ell\) as \(g\). The results fix \(\hat{x}_g\) and vary \(\hat{x}_\ell\). Let \(\sigma(\alpha_\ell; \hat{x}_\ell)\) denote an equilibrium, given \(\hat{x}_\ell\) and \(\alpha_\ell\). Denote the corresponding social acceptance set as \(A(\alpha_\ell; \hat{x}_\ell)\), with upper bound \(\pi(\alpha_\ell; \hat{x}_\ell)\). Finally, let \(A(\hat{x}_g)\) denote the equilibrium acceptance set if \(\hat{x}_\ell = \hat{x}_g\), suppressing \(\alpha_\ell\) because it is inconsequential.

First, I establish properties used to state analogues of Propositions 2 and 3.

Building upon Lemmas C.1–C.6 in Appendix C, Lemma 1 partitions whether \(\hat{x}_g \in \text{int}A(\hat{x}_g)\) as a function of primitives. See Appendix C for the proof. I state the result here for reference when proving Lemma 3.

**Lemma 1.** For all \(\ell \in N^L\), there exists \(\pi_\ell \in (0, q]\) such that \(\hat{x}_g \in (\pi_\ell, \bar{\pi}_\ell)\) implies \(\hat{x}_g \in \text{int}A(\hat{x}_g)\). Otherwise, \(A(\hat{x}_g) = [-\pi_\ell, \bar{\pi}_\ell]\).

**Lemma 3.** Suppose \(\hat{x}_g \in (0, \bar{\pi}_\ell)\). There exists \(\bar{x} \in [0, \hat{x}_g)\) such that \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). A symmetric result holds if \(\hat{x}_g \in (-\bar{\pi}_\ell, 0)\).

**Proof.** Consider \(\hat{x}_g \in (0, \bar{\pi}_\ell)\). By Lemma 1, \(\hat{x}_\ell = \hat{x}_g\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies \(A(0; \hat{x}_\ell)\) is continuous in \(\hat{x}_\ell\). Thus, there exists \(\bar{x} \in [0, \hat{x}_g)\) such that \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) implies \(\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)\). By Lemma 2, \(\hat{x}_\ell \in (\bar{x}, \hat{x}_g)\) thus implies \(A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell)\) for all \(\alpha_\ell \in [0, 1]\). \(\square\)
For each \( j \in N^L \setminus \{\ell\} \), define \( E_j^{LB}(\alpha; \hat{x}) = \mathbb{I}\{\hat{x} \leq -\bar{\pi}(\alpha; \hat{x})\} \), \( E_j^{UB}(\alpha; \hat{x}) = \mathbb{I}\{\hat{x} \geq \bar{\pi}(\alpha; \hat{x})\} \), and \( C_j(\alpha; \hat{x}) = \mathbb{I}\{\hat{x} \in \text{intA}(\alpha; \hat{x})\} \). Define \( \tilde{E}_j^{LB}(\alpha; \hat{x}) \), \( \tilde{E}_j^{UB}(\alpha; \hat{x}) \), and \( \tilde{C}_j(\alpha; \hat{x}) \) analogously for \( \hat{y}_j \). Let \( I_g \in \{0, 1\} \) indicate whether \( j \in N^L \).

**Assumption A.1.** There exists \( j \in N^L \setminus \{\ell\} \) such that \( \alpha_j < 1 \) and \( \hat{x}_j \notin \text{A}(\sigma(\hat{x}_g)) \).

**Assumption A.2.** There exists \( j \in N^L \setminus \{\ell\} \) such that \( \alpha_j > 0 \) and \( \hat{y}_j \notin \text{A}(\sigma(\hat{x}_g)) \).

Next, define

\[
v_1^g(\alpha; \hat{x}) = \rho_{\ell} \left( \alpha_{\ell} \left[ u_{g}(\hat{y}_{\ell}) + u_{\ell}(\hat{y}_{\ell}) - u_{\ell}(\hat{x}_{\ell}) \right] + (1 - \alpha_{\ell}) u_{g}(\hat{x}_{\ell}) \right)
\]

and

\[
v_2^g(\alpha; \hat{x}) = \sum_{j \neq \ell} \rho_j \left( \alpha_j \tilde{E}_j^{LB}(\alpha; \hat{x}) + (1 - \alpha_j) E_j^{LB}(\alpha; \hat{x}) \right) u_g(-\bar{\pi}(\alpha; \hat{x}))
\]

\[
+ \left( \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}) + (1 - \alpha_j) E_j^{UB}(\alpha; \hat{x}) \right) u_g(\bar{\pi}(\alpha; \hat{x}))
\]

\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha; \hat{x}) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}) u_g(\hat{x}_j)
\]

**Lemma 4.** If \( \hat{x}_\ell \neq \hat{x}_g \), then \( \frac{\partial v_1^g(\alpha; \hat{x})}{\partial \alpha_{\ell}} > 0 \).

**Proof.** Suppose \( \hat{x}_\ell \neq \hat{x}_g \). From (26) and \( \hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2} \), \( \frac{\partial v_1^g(\alpha; \hat{x})}{\partial \alpha_{\ell}} = \frac{\partial}{\partial \alpha_{\ell}} \left( \hat{x}_g - \hat{x}_\ell \right)^2 > 0 \).

**Lemma 5.** Suppose \( 0 \leq \hat{x}_\ell < \hat{x}_g < \bar{\pi}_\ell \) and at least one of Assumption A.1 or A.2 holds. Then \( v_2^g(\alpha; \hat{x}) \) strictly decreases in \( \alpha_{\ell} \). A symmetric result holds for \( \hat{x}_g < 0 \).

**Proof.** Assume \( 0 \leq \hat{x}_\ell < \hat{x}_g < \bar{\pi}_\ell \) and at least one of Assumption A.1 or A.2 holds. It suffices to show that

\[
\left( \alpha_j \tilde{E}_j^{LB}(\alpha; \hat{x}) + (1 - \alpha_j) E_j^{LB}(\alpha; \hat{x}) \right) u_g(-\bar{\pi}(\alpha; \hat{x}))
\]

\[
+ \left( \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}) + (1 - \alpha_j) E_j^{UB}(\alpha; \hat{x}) \right) u_g(\bar{\pi}(\alpha; \hat{x}))
\]

\[
+ \alpha_j \left[ \tilde{C}_j(\alpha; \hat{x}) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha; \hat{x}) \right] + (1 - \alpha_j) C_j(\alpha; \hat{x}) u_g(\hat{x}_j)
\]

decreases in \( \alpha_{\ell} \) for all \( j \in N^L \setminus \{\ell\} \) and strictly decreases for some \( j \).
Thus, both \( x \) for all \( \alpha \) of Assumption A.1 or A.2 holds. I show there exists \( \alpha \) implies \( \hat{g} \in (0, \overline{x}(0; \hat{x}_\ell)) \) by Lemma 1, so \( u_g(-\overline{x}(\alpha; \hat{x}_\ell)) \) and \( u_g(\overline{x}(\alpha; \hat{x}_\ell)) \) both decrease in \( \alpha \). Second, either: \( E^UB_j(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha; C_j(\alpha; \hat{x}_\ell) = 1 \) for all \( \alpha \); or there is a unique \( \overline{x}_\ell^j \in [0, 1] \) such that \( \alpha \in [0, \overline{x}_\ell^j] \) implies \( E^UB_j(\alpha; \hat{x}_\ell) = 1 \), and \( \alpha \in (\overline{x}_\ell^j, 1] \) implies \( C_j(\alpha; \hat{x}_\ell) = 1 \). An analogous observation holds for \( \tilde{E}^UB_j(\alpha; \hat{x}_\ell) \) and \( \tilde{C}_j(\alpha; \hat{x}_\ell) \). Thus, both

\[
E^LB_j(\alpha; \hat{x}_\ell) u_g(-\overline{x}(\alpha; \hat{x}_\ell)) + E^UB_j(\alpha; \hat{x}_\ell) u_g(\overline{x}(\alpha; \hat{x}_\ell)) + C_j(\alpha; \hat{x}_\ell) u_g(\hat{x}_\ell) \tag{29}
\]

and

\[
\tilde{E}^LB_j(\alpha; \hat{x}_\ell) u_g(-\overline{x}(\alpha; \hat{x}_\ell)) + \tilde{E}^UB_j(\alpha; \hat{x}_\ell) u_g(\overline{x}(\alpha; \hat{x}_\ell)) + \tilde{C}_j(\alpha; \hat{x}_\ell) u_g(\hat{y}_j) \tag{30}
\]

decrease in \( \alpha \). Furthermore, at least one of (29) and (30) strictly decreases for some \( j \in N^L \setminus \{\ell\} \) because at least one of Assumptions A.1 or A.2 holds. Proposition 6 implies \( m^j(\alpha; \hat{x}_\ell) \) weakly increases in \( \alpha \) for all \( j \in N^L_g \). Altogether, (28) decreases in \( \alpha \) for all \( j \in N^L \setminus \{\ell\} \) and strictly decreases for some \( j \), as desired. \( \square \)

For \( g \in N^G \), define

\[
U^E_g(\alpha; \hat{x}_\ell) = v^0_g(\alpha; \hat{x}_\ell) + v^2_g(\alpha; \hat{x}_\ell). \tag{31}
\]

**Lemma 6.** Assume \( \hat{x}_g \in (0, \overline{x}) \) and at least one of Assumption A.1 or A.2 holds. There exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U^E_g(\alpha; \hat{x}_\ell) \) strictly decreases in \( \alpha \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_g \in (0, \overline{x}) \) and at least one of Assumption A.1 or A.2 holds. I show \( |\partial_v^g(\alpha; \hat{x}_\ell)| > |\partial_u^g(\alpha; \hat{x}_\ell)| \) for \( \hat{x}_\ell \) sufficiently close to \( \hat{x}_g \).

By Lemma 3, there exists \( \bar{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in \text{int}A(\alpha; \hat{x}_\ell) \) for all \( \alpha \in [0, 1] \). Fix \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) and \( \alpha \in [0, 1] \).

First, I characterize a lower bound on \( |\partial_v^g(\alpha; \hat{x}_\ell)| \). Define

\[
\Gamma = \sum_{j \neq \ell} \rho_j \left[ \alpha_j \tilde{E}^LB_j(\hat{x}_g) + (1 - \alpha_j) E^LB_j(\hat{x}_g) \right] \frac{\partial u_g(-\overline{x}(\bar{x}))}{\partial \overline{x}(\bar{x})} + \left[ \alpha_j \tilde{E}^UB_j(\hat{x}_g) + (1 - \alpha_j) E^UB_j(\hat{x}_g) \right] \frac{\partial u_g(\overline{x}(\bar{x}))}{\partial \overline{x}(\bar{x})}, \tag{32}
\]
Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\overline{\pi}(\hat{x}), \overline{\pi}(\hat{x}))$ implies $\frac{\partial u_g(\pi(\hat{x}))}{\partial \pi(\hat{x})} < 0$ and $\frac{\partial u_g(-\pi(\hat{x}))}{\partial \pi(\hat{x})} < 0$, and (ii) at least one of Assumptions A.1 and A.2 hold.

I claim $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \pi(\alpha; \hat{x}_\ell)} < \Gamma$, where

$$
\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \pi(\alpha; \hat{x}_\ell)} = \sum_{j \neq \ell} \rho_j \left( \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(\alpha; \hat{x}_\ell) \right) \frac{\partial u_g(\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)}
$$

$$
+ \left( \alpha_j \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(\alpha; \hat{x}_\ell) \right) \frac{\partial u_g(\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)}
$$

$$
- J_g^\alpha \alpha_j \frac{\partial m^j_\alpha(\alpha; \hat{x}_\ell)}{\partial \pi(\alpha; \hat{x}_\ell)}.
$$

(33)

Three steps show the claim. First, note $\hat{x}_\ell \in (\hat{x}, \hat{x}_g)$ implies $\overline{\pi}(\hat{x}_g) \geq \overline{\pi}(\alpha; \hat{x}_\ell)$ by Lemma 2. Thus, we have $\tilde{E}_j^{UB}(\hat{x}_\ell) \leq \tilde{E}_j^{UB}(\alpha; \hat{x}_\ell)$, $E_j^{UB}(\hat{x}_\ell) \leq E_j^{UB}(\alpha; \hat{x}_\ell)$, $\tilde{E}_j^{LB}(\alpha; \hat{x}_\ell)$, and $E_j^{LB}(\hat{x}_\ell) \leq E_j^{LB}(\alpha; \hat{x}_\ell)$ for all $j \neq \ell$. Next, $\hat{x}_g < \pi(\hat{x}) < \overline{\pi}(\ell, \alpha)$ implies $\frac{\partial u_g(\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)} < 0$ and symmetrically $\frac{\partial u_g(-\pi(\alpha; \hat{x}_\ell))}{\partial \pi(\alpha; \hat{x}_\ell)} < 0$.

Finally, $\frac{\partial m^j_\alpha(\alpha; \hat{x}_\ell)}{\partial \pi(\alpha; \hat{x}_\ell)} \geq 0$ for all $j \in N_g^L$ by Proposition 6.

For almost all $\ell \alpha \in [0, 1]$, $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial u_2(\alpha; \hat{x}_\ell) \overline{\pi}(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell}$. Define $C_j(\alpha; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_j(\alpha; \hat{x}_\ell)) + \alpha_j(1 - \overline{C}_j(\alpha; \hat{x}_\ell))]$. Then,

$$
\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} < \frac{\partial \overline{\pi}(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell}
$$

(34)

$$
\frac{\delta \rho_\ell \Gamma}{2 \pi(\alpha; \hat{x}_\ell)} \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right]
$$

(35)

$$
< \frac{\delta \rho_\ell \Gamma}{2 \pi(\alpha; \hat{x}_\ell)} \left[ u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right]
$$

(36)

$$
= \frac{\delta \rho_\ell \Gamma}{2 \pi(\alpha; \hat{x}_\ell)} \left[ \frac{1}{4}(\hat{x}_\ell - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_\ell) \right]
$$

(37)

where (34) follows from $\frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$ and $0 > \Gamma > \frac{\partial u_2(\alpha; \hat{x}_\ell) \overline{\pi}(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell}$; (35) from applying the implicit function theorem to $\pi(\alpha; \hat{x}_\ell)$, which is possible for almost all $\alpha \in [0, 1]$; (36) because $\Gamma[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)] < 0$, $\overline{\pi}(\alpha; \hat{x}_\ell) > 0$, and $\delta \sum_{j \in N^L} \rho_j C_j^\ell(\alpha; \hat{x}_\ell) \in (0, 1)$; and (37) from using $\hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_\ell}{2}$ and simplifying.

By Lemma 4, $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\alpha_\ell}{2}(\hat{x}_\ell - \hat{x}_\ell)^2$. By (37), $\frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} < \frac{\partial u_2(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell} + \Gamma \frac{\partial \pi(\alpha; \hat{x}_\ell)}{\partial \alpha_\ell}$

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for almost all $\alpha_\ell \in [0, 1]$. Thus, $\frac{\partial U^E(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ if

$$\frac{\rho_\ell}{2}(\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_\ell \Gamma}{2\bar{\pi}_\ell} \left[\frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g)\right] < 0,$$

which holds for $\hat{x}_\ell > \hat{x}_g \left(\frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta \Gamma}\right)$. Define $x' = \max\left\{\hat{x}, \hat{x}_g \left(\frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta \Gamma}\right)\right\}$. Then $x' < \hat{x}_g$ because (i) $\hat{x} < \hat{x}_g$ and (ii) $\delta \Gamma < 0$ implies $\frac{4\pi_\ell + \delta \Gamma}{4\pi_\ell - 3\delta \Gamma} < 1$. Thus, $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\frac{\partial U^E(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ for almost all $\alpha_\ell \in [0, 1]$. Continuity implies $U^E_g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$ for such $\hat{x}_\ell$.

Next, I prove the analogue of Proposition 2.

**Proposition A.2** Assume $\hat{x}_g \in (-\pi_\ell, \pi_\ell)$ and at least one of Assumptions A.1 and A.2 holds. If $\hat{x}_g > 0$, then there exist $x'$ and $x''$ satisfying $x' < x_g < \pi_\ell < x''$ such that

1. if $\hat{x}_\ell \in (x', \hat{x}_g)$, then $\alpha_\ell = 0$ is uniquely optimal;
2. if $\hat{x}_\ell \in (\hat{x}_g, x'')$, then $\alpha_\ell = 0$ is not optimal; and
3. if $\hat{x}_\ell \geq x''$, then $g$ is indifferent over $\alpha_\ell$.

A symmetric result holds for $\hat{x}_g < 0$.

**Proof.** Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_g \in (0, \pi_\ell)$ and at least one of Assumptions A.1 and A.2 hold.

1. By Lemma 3, there exists $\bar{x} \in [0, \hat{x}_g]$ such that $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. By Lemma 6, there exists $\bar{x} < \hat{x}_g$ such that $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $U^E_g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in $\alpha_\ell$. Consider $\hat{x}_\ell \in (\max\{\bar{x}, \bar{x}'\}, \hat{x}_g)$. Then $z_\ell = \hat{x}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ and $y_\ell^\ell = \hat{y}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. Thus, $g$’s ex ante expected utility equals $U^E_g(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. It follows that $g$ strictly prefers $\alpha_\ell = 0$.

2. Assume $\hat{x}_\ell \in (\hat{x}_g, x'')$, where $x'' = 2\pi_\ell - \hat{x}_g$. It suffices to show $g$’s ex ante expected utility strictly increases at $\alpha_\ell = 0$. There are two cases.

- **Case 1:** If $\hat{x}_\ell < \pi_\ell$, then $g$’s ex ante expected payoff equals $U^E_g(\alpha_\ell; \hat{x}_\ell)$ for sufficiently small $\alpha_\ell$. By Lemma 4, $\frac{\partial U^E(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$. To complete this case, I argue that $v_2^\ell(\alpha_\ell; \hat{x}_\ell)$ increases for sufficiently small $\alpha_\ell$. Under the maintained assumptions, $\hat{x}_g \in (-\pi(0; \hat{x}_\ell), x(0; \hat{x}_\ell))$ and $\hat{y}_\ell \in (\hat{x}_g, \pi(0; \hat{x}_\ell))$. Thus, $\pi(\alpha_\ell; \hat{x}_\ell)$ strictly decreases
for sufficiently small $\alpha$. Therefore $u_g(-\pi(\alpha; \hat{x}_\ell))$ and $u_g(\pi(\alpha; \hat{x}_\ell))$ are strictly increasing for such $\alpha$. Proposition 6 implies $m_g^j(\alpha; \hat{x}_\ell)$ weakly decreases in $\alpha$ for all $j \in N_g^L \setminus \{\ell\}$. Thus, $u_g^g(\alpha; \hat{x}_\ell)$ strictly increases over sufficiently small $\alpha$.

- **Case 2:** If $\hat{x}_\ell > \bar{x}_\ell$, then $\pi(0; \hat{x}_\ell) = \bar{x}_\ell$. Thus, $g$’s ex ante expected payoff from $\alpha = 0$ is

$$
\rho_\ell \left( \alpha \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\bar{x}_\ell) \right] + (1 - \alpha) u_g(\bar{x}_\ell) \right)
$$

$$
+ \sum_{j \neq \ell} \rho_j \left( \left[ \alpha_j \tilde{E}_j^{LB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(0; \hat{x}_\ell) \right] u_g(-\bar{x}_\ell)
\right.
$$

$$
+ \left[ \alpha_j \tilde{E}_j^{UB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(0; \hat{x}_\ell) \right] u_g(\bar{x}_\ell)
$$

$$
+ \alpha_j \tilde{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j)
$$

$$
- I_{g}^j \alpha_j m_g^j(0; \hat{x}_\ell) \right).
$$

(38)

Arguments analogous to Case 1 show (38) strictly increases in $\alpha$ at $\alpha = 0$.

3. Assume $\hat{x}_\ell \geq x''$, where $x''$ is defined as in Case 2 of Part 2. Then $z_\ell = y_\ell = \pi(\alpha; \hat{x}_\ell) = \bar{x}_\ell$ for all $\alpha \in [0, 1]$ and $g$’s ex ante expected payoff is constant. □

I prove the analogue of Proposition 3 from the main text.

**Proposition A.3** Assume $\hat{x}_g \geq \pi_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\pi(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_\ell \geq \pi_\ell$, then $g$ is indifferent over $\alpha$.

2. If $\hat{x}_\ell \in [0, \pi_\ell)$, then $\alpha = 0$ is not optimal.

A symmetric result holds if $\hat{x}_g \leq -\pi_\ell$ and $\max\{\hat{x}_j, \hat{y}_j\} < \pi(0; \hat{x}_\ell)$ for all $j \in N^L$.

**Proof.** Suppose $\hat{x}_g \geq \pi_\ell$ and $\min\{\hat{x}_j, \hat{y}_j\} > -\pi(0; \hat{x}_\ell)$ for all $j \in N^L$.

1. If $\hat{x}_g \geq \pi_\ell$, then apply arguments analogous to Part 3 of Proposition A.2.

2. Assume $\hat{x}_\ell \in [0, \pi_\ell)$. I show $g$’s ex ante expected utility strictly increases at $\alpha = 0$.  

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We have $\hat{x}_\ell \in [0, \bar{x}(0; \hat{x}_\ell))$ and $\hat{y}_\ell > \hat{x}_\ell$. Therefore $0 \leq z_\ell(0; \hat{x}_\ell) = \hat{x}_\ell < y_g(0; \hat{x}_\ell) \leq \hat{y}_\ell$. Furthermore, no $j \in N^L$ proposes $-\bar{x}(0; \hat{x}_\ell)$ because $\min\{\hat{x}_j, \hat{y}_j\} > -\bar{x}(0; \hat{x}_\ell)$. Thus, $g$’s ex ante expected payoff from $\alpha_\ell = 0$ is

$$\rho_\ell\left(\alpha_\ell \left[ u_g(y_g(0; \hat{x}_\ell)) + u_e(y_g(0; \hat{x}_\ell)) - u_e(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell)\right)$$

$$+ \sum_{j \neq \ell} \rho_j \left[ \alpha_j \bar{E}_j(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(0; \hat{x}_\ell) \right] u_g(\bar{x}(0; \hat{x}_\ell))$$

$$+ \alpha_j \left[ \bar{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) - I_j m_g(0; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j) \right).$$

(39)

Three steps show (39) strictly increases at $\alpha_\ell = 0$.

- First, $0 \leq \hat{x}_\ell < y_g(0; \hat{x}_\ell) \leq \hat{y}_\ell$ implies $y_g(0; \hat{x}_\ell)$ weakly increases in $\alpha_\ell$. Therefore $u_g(y_g(\alpha_\ell; \hat{x}_\ell))$ weakly increases and $u_e(y_g(\alpha_\ell; \hat{x}_\ell))$ weakly decreases. Because $u$ is quadratic and $\hat{x}_\ell < y_g(0; \hat{x}_\ell) \leq \hat{y}_\ell = \frac{x_g + \hat{x}_\ell}{2} < \hat{x}_g$, it follows that $u_g(y_g(\alpha_\ell; \hat{x}_\ell))$ increases weakly faster than $u_e(y_g(\alpha_\ell; \hat{x}_\ell))$ decreases. Therefore $u_g(y_g(0; \hat{x}_\ell)) + u_e(y_g(0; \hat{x}_\ell)) - u_e(\hat{x}_\ell)$ weakly increases in $\alpha_\ell$. Furthermore, $\hat{x}_\ell < y_g(0; \hat{x}_\ell) \leq \hat{y}_\ell < \hat{x}_g$ also implies $u_g(y_g(0; \hat{x}_\ell)) + u_e(y_g(0; \hat{x}_\ell)) - u_e(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0$. It follows that

$$\alpha_\ell \left[ u_g(y_g(0; \hat{x}_\ell)) + u_e(y_g(0; \hat{x}_\ell)) - u_e(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell)$$

weakly increases at $\alpha_\ell = 0$.

- Second, $0 \leq z_\ell < y_g(0; \hat{x}_\ell) \leq \bar{x}(0; \hat{x}_\ell)$ implies $\bar{x}(0; \hat{x}_\ell)$ strictly increases in $\alpha_\ell$ by Lemma 2. Since $\bar{x}(0; \hat{x}_\ell) < \hat{x}_g$, it follows that $u_g(\bar{x}(0; \hat{x}_\ell))$ increases at $\alpha_\ell = 0$.

- Third, Proposition 6 implies $m_g(0; \hat{x}_\ell)$ weakly increases in $\alpha_\ell$ for all $j \in N_g^L$. However, $\hat{y}_j > \bar{x}(0; \hat{x}_\ell)$ for all $j \in N_g^L$ such that $m_g(0; \hat{x}_\ell)$ strictly increases in $\alpha_\ell$, which implies $g$’s lobbying surplus weakly increases in $\alpha_\ell$ for any such $j \in N_g^L$.

\[ \square \]

### Willingness to Pay for Access

The following results apply to the model in the main text. Define $\theta = (\hat{x}, \rho, \alpha)$. Let $U_g^E(\theta)$ be $g$’s ex ante expected utility. Additionally, let $\bar{x}_\alpha = \bar{x}(\alpha; \hat{x}_\ell)$ denote the upper bound of $A(\alpha; \hat{x}_\ell)$. Define $\frac{\partial \bar{x}_\alpha}{\partial \alpha} = \frac{\partial \bar{x}_\alpha}{\partial \alpha} |_{\alpha = 0}$, $\frac{\partial \bar{x}_\alpha}{\partial x_\ell} = \frac{\partial \bar{x}_\alpha}{\partial x_\ell} |_{\alpha = 0}$, and $\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial x_\ell} = \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial x_\ell} |_{\alpha = 0}$.
To state Proposition 4, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, consider the baseline model, but replace \( \ell \) with two legislators, \( \ell_1 \) and \( \ell_2 \), and replace \( g \) with two groups, \( g_1 \) and \( g_2 \). To isolate differences in proposal power, assume \( \hat{x}_{g_1} = \hat{x}_{g_2} \) and \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), but \( \rho_{\ell_1} \neq \rho_{\ell_2} \). These modifications do not qualitatively change the equilibrium characterization. I use two identical groups to avoid complications arising if one group has access to two legislators, because the group accounts for how access to one legislator affects its lobby offer to the other.

**Proposition 4.** Consider the modified baseline model with: \( \ell_1 \) and \( \ell_2 \) such that \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), and \( g_1 \) and \( g_2 \) satisfying \( \hat{x}_{g_1} = \hat{x}_{g_2} \). For all \( \alpha \in [0, 1] \), \( \rho_{\ell_2} > \rho_{\ell_1} \) implies

\[
\frac{\partial U^{E}_{g_2}(\theta)}{\partial \alpha_2} |_{\alpha_2 = \alpha} \geq \frac{\partial U^{E}_{g_1}(\theta)}{\partial \alpha_1} |_{\alpha_1 = \alpha}.
\]

**Proof.** Assume \( \rho_{\ell_2} > \rho_{\ell_1} \) and fix \( \alpha \in [0, 1] \). It suffices to show \( \frac{\partial U^{E}_{g_2}(\theta)}{\partial \alpha_2} |_{\alpha_2 = \alpha} \geq \frac{\partial U^{E}_{g_1}(\theta)}{\partial \alpha_1} |_{\alpha_1 = \alpha} \) for \( \alpha \in [0, 1] \).

Because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \) and \( \hat{x}_{g_1} = \hat{x}_{g_2} \), we have \( y_{g_1} = y_{g_2} \) and \( z_{\ell_1} = z_{\ell_2} \). Thus, \( m_{g_1} = m_{g_2} \).

For convenience, let \( y = y_{g_1} \), \( z = z_{\ell_1} \), and \( m = m_{g_1} \). Assume \( \frac{\partial U^{E}_{g_1}(\theta)}{\partial \alpha_1} |_{\alpha_1 = \alpha} \geq 0 \). There are five cases:

- **Case 1:** Suppose \( z = \hat{x}_{\ell} \) and \( y = \hat{y} \). Then,

\[
\frac{\partial U^{E}_{g_1}(\theta)}{\partial \alpha_1} |_{\alpha_1 = \alpha} = \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)
\]

\[
- \frac{\partial u}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\]

\[
= \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)
\]

\[
+ \frac{\delta [u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\]

\[
\leq \rho_{\ell_2} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right)
\]

\[
+ \frac{\delta [u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\partial \alpha_1} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\]

(40)
Without loss of generality, assume than \( \ell \) is majority-leaning and \( \ell \) is majority-leaving and \( \ell \) is more extreme than \( \ell \), then \( \frac{\partial U^E_\ell (\theta)}{\partial \alpha_1} |_{\alpha_2=\alpha} \) weakly increases in \( |\hat{x}_\ell - \hat{x}_\ell| \).

**Proposition 5.** Assume minority-party agenda exclusion and \( \ell \) is majority-leaving.

If either (i) \( g \) is more centrist than \( \ell \), or (ii) \( g \) is majority-leaving and more extreme than \( \ell \), then \( \frac{\partial U^E_\ell (\theta)}{\partial \alpha} |_{\alpha=0} \) weakly increases in \( |\hat{x}_g - \hat{x}_\ell| \).

**Proof.** Without loss of generality, assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \).

First, \( g \)'s ex ante expected utility for \( \alpha \in [0,1] \) is

\[
U^E_g (\theta) = \rho_\ell \left( \alpha [u_g(y) + u_\ell(y) - u_\ell(z_\ell)] + (1 - \alpha) u_g(z_\ell) \right) + \rho_M u_g(0) + \rho_R u_g(\pi_\alpha). \tag{43}
\]
Thus, $g$’s willingness to acquire access to $\ell$ is
\[
\left.\frac{\partial U^E_g(\theta)}{\partial \alpha}\right|_{\alpha=0} = \rho_{\ell} \left( u_g(y) - u_g(z_\ell) + u_\ell(y) - u_\ell(z_\ell) \right) + \rho_R \frac{\partial u_g(\pi_0)}{\partial \pi_0} \frac{\partial \pi_0}{\partial \alpha}.
\] (44)

The cross-partial with respect to $\hat{x}_g$ satisfies
\[
\left.\frac{\partial^2 U^E_g(\theta)}{\partial \alpha \partial \hat{x}_g}\right|_{\alpha=0} = \rho_{\ell} \left( \frac{\partial u_g(y)}{\partial y} + \frac{\partial u_\ell(y)}{\partial y} \right) \frac{\partial y}{\partial \hat{x}_g} + \frac{\partial u_g(z_\ell)}{\partial \hat{x}_g} + \rho_R \left( \frac{\partial^2 u_g(\pi_0)}{\partial \pi_0^2} \frac{\partial \pi_0}{\partial \alpha} + \frac{\partial^2 \pi_0}{\partial \pi_0 \partial \hat{x}_g} \right)
+ \rho_R \left( \frac{\partial^2 u_g(\pi_0)}{\partial \pi_0^2} \frac{\partial \pi_0}{\partial \alpha} + \frac{\partial u_g(\pi_0)}{\partial \pi_0} \frac{\partial \pi_0}{\partial \alpha} \hat{x}_g \right),
\] (45)
\[
= \rho_{\ell} \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(z_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial^2 u_g(\pi_0)}{\partial \pi_0^2} \frac{\partial \pi_0}{\partial \alpha} + \frac{\partial u_g(\pi_0)}{\partial \pi_0} \frac{\partial \pi_0}{\partial \alpha} \hat{x}_g \right),
\] (46)

where (46) follows because either (i) $y = \pi_0$, which implies $\frac{\partial y}{\partial \hat{x}_g} = 0$, or (ii) $y = \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$, which implies $\frac{\partial u_g(y)}{\partial \hat{x}_g} = -\frac{\partial u_g(y)}{\partial y}$.

**Part (i)** Consider $\hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell]$. There are two cases.

**Case 1**: Suppose $\hat{x}_\ell \geq \pi_0$. Then $z_\ell = \pi_0$. Since $\hat{x}_g \geq -\hat{x}_\ell$, we have $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \geq 0$.

There are two subcases.

- First, consider $\hat{x}_g \geq 2\pi_0 - \hat{x}_\ell$. Then $y = z_\ell = \pi_0$. For $\alpha \in [0, 1]$, if $y = z_\ell = \pi_\alpha$, then $\pi_\alpha$ solves

\[
0 = (1 - \delta) u_M(q) + \delta \rho_M u_M(0) - [1 - \delta(\rho_R + \rho_\ell)] u_M(\pi_\alpha).
\] (47)

Applying the implicit function theorem to (47) yields $\frac{\partial \pi_\alpha}{\partial \alpha} = 0$ and thus $\frac{\partial \pi_\alpha}{\partial \alpha} = 0$. Therefore $\left.\frac{\partial U^E_g(\theta)}{\partial \alpha}\right|_{\alpha=0} = 0$ over $\hat{x}_g \in [2\pi_0 - \hat{x}_\ell, \hat{x}_\ell]$.

- Second, consider $\hat{x}_g < 2\pi_0 - \hat{x}_\ell$. Then $y = \hat{y}$. For $\alpha \in [0, 1]$, if $y = \hat{y}$ and $z_\ell = \pi_\alpha$, then $\pi_\alpha$ solves

\[
0 = (1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \alpha \rho_\ell u_M(\hat{y}) \right) - \left( 1 - \delta(\rho_R + (1 - \alpha)\rho_\ell) \right) u_M(\pi_\alpha).
\] (48)
Applying the implicit function theorem to (48) yields
\[
\frac{\partial \tau_\alpha}{\partial \alpha} = \frac{\delta \rho \ell [u_M(\hat{y}) - u_M(\tau_\alpha)]}{(1 - \delta [\rho_R + (1 - \alpha)\rho_\ell]) \frac{\partial u_M(\tau_\alpha)}{\partial \tau_\alpha}}
\]
(49)

\[
\frac{\partial \tau_\alpha}{\partial \hat{x}_g} = \frac{\alpha \delta \rho \ell \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta [\rho_R + (1 - \alpha)\rho_\ell]) \frac{\partial u_M(\tau_\alpha)}{\partial \tau_\alpha}}.
\]
(50)

and
\[
\frac{\partial^2 \tau_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho \ell \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\tau_\alpha)}{\partial \tau_\alpha^2} \frac{\partial \tau_\alpha}{\partial \hat{x}_g} \frac{\partial \tau_\alpha}{\partial \alpha} \right) \left( \frac{\partial u_M(\tau_\alpha)}{\partial \tau_\alpha} \right)^{-1}.
\]
(51)

Inspecting (50) reveals \(\frac{\partial \tau_0}{\partial \hat{x}_g} = 0\). Thus,
\[
\frac{\partial^2 \tau_0}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho \ell \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta [\rho_R + \rho_\ell]) \frac{\partial u_M(\tau_0)}{\partial \tau_0}} > 0,
\]
(52)

which follows because (i) \(\frac{\partial \hat{y}}{\partial \hat{x}_g} > 0\) and (ii) \(0 < \hat{y} < \tau_0\) implies \(0 > \frac{\partial u_M(y)}{\partial y} > \frac{\partial u_M(\tau_0)}{\partial \tau_0}\). Thus,
\[
\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} \bigg|_{\alpha=0} = \rho \ell \left( \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_M(\tau_0)}{\partial \hat{x}_g} \right) + \rho_R \frac{\partial u_M(\tau_0)}{\partial \tau_0} \frac{\partial^2 \tau_0}{\partial \alpha \partial \hat{x}_g}
\]
(53)
\[
< \rho \ell \left( \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_M(\tau_0)}{\partial \hat{x}_g} \right)
\]
(54)
\[
< 0,
\]
(55)

where (54) follows from (52) and \(\frac{\partial u_M(\tau_0)}{\partial \tau_0} < 0\); and (55) because \(\hat{x}_g < \hat{y} < \tau_0\) implies \(\frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} < \frac{\partial u_M(\tau_0)}{\partial \hat{x}_g}\).

**Case 2:** Suppose \(\hat{x}_\ell < \tau_0\). Then \(z_\ell = \hat{x}_\ell\). Furthermore, \(\hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell)\) implies \(y = \hat{y} \geq 0\). For \(\alpha \in [0, 1]\), if \(y = \hat{y}\) and \(z_\ell = \hat{x}_\ell\), then \(\tau_\alpha\) solves
\[
u M(\tau_\alpha) = \frac{(1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \rho \ell [\alpha u_M(\hat{y}) + (1 - \alpha) u_M(\hat{x}_\ell)] \right)}{(1 - \delta \rho_R)}.
\]
(56)
Applying the implicit function theorem yields

\[
\frac{\partial \bar{x}_\alpha}{\partial \alpha} = \frac{\delta \rho \{u_M(\bar{y}) - u_M(\hat{x}_\ell)\}}{(1 - \delta \rho_R) \frac{\partial u_M(\bar{x}_\alpha)}{\partial x_\alpha}},
\]

(57)

and

\[
\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \alpha^2} \frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} \cdot \frac{\partial \bar{x}_\alpha}{\partial \alpha} \right) \cdot \left( \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)^{-1}.
\]

(58)

Inspecting (58) reveals \(\frac{\partial \bar{x}_\alpha}{\partial \hat{x}_g} = 0\), which implies

\[
\frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g}}{(1 - \delta \rho_R) \frac{\partial u_M(\bar{x}_0)}{\partial \bar{x}_0}} > 0.
\]

(59)

Because \(0 \leq \hat{y} < \hat{x}_\ell\), a inequalities analogous to (53)-(55) imply \(\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g}|_{\alpha=0} < 0\).

In both cases, \(g\)'s willingness to acquire access weakly decreases in \(\hat{x}_g - \hat{x}_\ell\).

**Part (ii) Assume \(\hat{x}_g \geq \hat{x}_\ell\).** There are two cases.

**Case 1:** Consider \(\hat{x}_\ell \geq \bar{x}_0\). Then \(y = z_\ell = \bar{x}_0\) at \(\alpha = 0\), implying \(\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g}|_{\alpha=0} = 0\).

**Case 2:** Consider \(\hat{x}_\ell < \bar{x}_0\). Then \(z_\ell = \hat{x}_\ell\). There are three subcases.

- First, assume \(\hat{x}_g \in [\hat{x}_\ell, \bar{x}_0]\). Then \(y = \hat{y}\). I show \(\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g}|_{\alpha=0} \geq 0\) implies \(\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g}|_{\alpha=0} > 0\). Since \(y = \hat{y}\) and \(z_\ell = \hat{x}_\ell\), case 2 of Part (ii) implies \(\frac{\partial \bar{x}_0}{\partial \alpha} = 0\), and \(\frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g}\) is given by (57), \(\frac{\partial \bar{x}_0}{\partial \alpha} = 0\), and \(\frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g}\) is (59). Therefore

\[
\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g}|_{\alpha=0} = \rho e \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \frac{\partial^2 \bar{x}_0}{\partial \alpha \partial \hat{x}_g} \right)
\]

(60)

\[
= \rho e \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial u_g(\bar{x}_0)}{\partial \bar{x}_0} \frac{\delta \rho e \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g}}{(1 - \delta \rho_R) \frac{\partial u_M(\bar{x}_0)}{\partial \bar{x}_0}} \right)
\]

(61)

\[
\geq \rho e \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right)
\]

\[
- \rho_R \left( \frac{\rho e \{u_g(\hat{y}) - u_g(\hat{x}_\ell) + u_e(\hat{y})\}}{\rho_R \frac{\partial \bar{x}_0}{\partial \alpha}} \right) \left( \frac{\delta \rho e \frac{\partial u_M(\bar{y})}{\partial \bar{y}} \frac{\partial \bar{y}}{\partial \hat{x}_g}}{(1 - \delta \rho_R) \frac{\partial u_M(\bar{x}_0)}{\partial \bar{x}_0}} \right)
\]

46
\[= \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) \]
\[+ \rho \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g} \left( \frac{u_g(\hat{y}) - u_g(\hat{x}_\ell) + u(\hat{y})}{u_M(\hat{y}) - u_M(\hat{x}_\ell)} \right) \]
\[= \frac{5\rho}{4}(\hat{x}_g - \hat{x}_\ell) \]
\[> 0, \]  
\[
\text{where (60) follows from using (59) to substitute for } \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g}; \text{ (61) because (i) } \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g} > 0 \text{ and (ii) } \frac{\partial U^E(\theta)}{\partial \alpha} |_{\alpha=0} \geq 0 \text{ and } \frac{\partial \varpi_\alpha}{\partial \alpha} > 0 \text{ together imply } \frac{\partial u_g(\hat{x}_0)}{\partial \varpi_0} \geq -\frac{\rho \rho_\ell}{\rho (\varpi_0)} \frac{\partial u_g(\hat{y}) - u_g(\hat{x}_\ell) + u(\hat{y})}{\partial \varpi_0}; \text{ and (62) from using (57) to substitute for } \frac{\partial \varpi_\alpha}{\partial \alpha} \text{ and simplifying.} \]

- Second, assume \( \hat{x}_g \in [\hat{x}_0, 2\hat{x}_0 - \hat{x}_\ell] \). Then \( y = \hat{y} \). Thus, \( \frac{\partial \varpi_0}{\partial \alpha}, \frac{\partial \varpi_0}{\partial \hat{x}_g}, \) and \( \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g} \) are defined as in subcase 1. Therefore
\[\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} = \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\hat{x}_0)}{\partial \varpi_0} - \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g} \right) \]
\[\geq \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) \]
\[> 0, \]
\[
\text{where (65) follows because (i) } \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g} > 0 \text{ and (ii) } \hat{x}_g \geq \hat{x}_0 \text{ implies } \frac{\partial u_g(\hat{x}_0)}{\partial \varpi_0} \geq 0; \text{ and (66) because } \hat{x}_\ell < \hat{y} < \hat{x}_g \text{ implies } \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} < \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g}. \]

- Third, assume \( \hat{x}_g \geq 2\hat{x}_0 - \hat{x}_\ell \). Then \( y = \hat{x}_0 \). For \( \alpha \in [0, 1] \), \( y = \varpi_\alpha \) and \( z_\ell = \hat{x}_\ell \) imply \( \varpi_\alpha \) solves
\[0 = (1 - \delta)u_M(q) + \delta \left( \rho M u_M(0) + \rho \ell (1 - \alpha) u_M(\hat{x}_\ell) \right) - \left( 1 - \delta \rho_\ell + \alpha \rho_\ell \right) u_M(\varpi_\alpha). \]
\[
\text{Applying the implicit function theorem to (67) yields}
\]
\[\frac{\partial \varpi_\alpha}{\partial \alpha} = \frac{\partial u_g(\hat{x}_0) - u_M(\varpi_\alpha)}{(1 - \delta \rho_\ell + \alpha \rho_\ell) \frac{\partial u_M(\varpi_\alpha)}{\partial \varpi_\alpha} \frac{\partial \varpi_\alpha}{\partial \alpha} \frac{\partial \varpi_\alpha}{\partial \hat{x}_g} = 0, \text{ and } \frac{\partial^2 \varpi_\alpha}{\partial \alpha \partial \hat{x}_g} = 0. \]
\[\text{Thus,}
\]
\[\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha=0} = \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\hat{x}_0)}{\partial \varpi_0} - \frac{\partial^2 \varpi_0}{\partial \alpha \partial \hat{x}_g} \right) \]
\[= \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) > 0. \]
Lemma 7. Assume $\rho_L = 0$ and $\hat{x}_\ell \geq 0$. There exists $x' > \hat{x}_\ell$ such that $\hat{x}_g \in (\hat{x}_\ell, x')$ implies $\alpha = 0$ is optimal. An symmetric result holds if $\rho_R = 0$ and $\hat{x}_\ell \leq 0$.

Proof. Without loss of generality, assume $\rho_L = 0$ and $0 < \hat{x}_\ell$.

If $\hat{x}_\ell \geq \bar{x}_0$, then $\hat{x}_g > \hat{x}_\ell$ implies $g$ is indifferent, so $\alpha = 0$ is optimal. As in the proof of Proposition 5, $\frac{\partial U^E_g(\theta)}{\partial \alpha}|_{\alpha=0} = 0$ for all $\alpha \in [0, 1]$. Setting $x' = \infty$ delivers the result.

Next, suppose $\hat{x}_\ell < \bar{x}_0$. Lemma 5 implies that it suffices to show $\frac{\partial U^E_g(\theta)}{\partial \alpha}|_{\alpha=0} \leq 0$ and $\frac{\partial^2 U^E_g(\theta)}{\partial \alpha^2} < 0$ for $\hat{x}_g$ sufficiently close to $\hat{x}_\ell$.

An argument analogous to Part 1 of Proposition 2 shows existence of $x' > \hat{x}_\ell$ such that $\frac{\partial U^E_g(\theta)}{\partial \alpha}|_{\alpha=0} < 0$ if $\hat{x}_g \in [\hat{x}_\ell, x']$. Also, $x' \in (\hat{x}_\ell, \bar{x}_0)$ because $\rho_L = 0$ implies $\frac{\partial U^E_g(\theta)}{\partial \alpha}|_{\alpha=0} > 0$ for $\hat{x}_g \geq \bar{x}_0$.

Consider $\hat{x}_g \in [\hat{x}_\ell, x']$. I show $\frac{\partial^2 U^E_g(\theta)}{\partial \alpha^2} < 0$. Applying the implicit function theorem to (56) yields

$$\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha^2} = -\frac{\partial^2 u_M(\bar{x}_\alpha)(\frac{\partial \bar{x}_\alpha}{\partial \alpha})^2}{\partial u_M(\bar{x}_\alpha)} < 0 \quad (69)$$

because $\frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \alpha^2} < 0$ and $\hat{x}_g < \bar{x}_\alpha$ implies $\frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} < 0$.

We have $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$, so

$$\frac{\partial^2 U^E_g(\theta)}{\partial \alpha^2} = \rho_R \left( \frac{\partial^2 u_g(\bar{x}_\alpha)}{\partial \bar{x}_\alpha^2} \left( \frac{\partial \bar{x}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_g(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha^2} \right)$$

$$= \rho_R \left( \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha^2} \left( \frac{\partial \bar{x}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha^2} \right) \quad (70)$$

$$< \rho_R \left( \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha^2} \left( \frac{\partial \bar{x}_\alpha}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \frac{\partial^2 \bar{x}_\alpha}{\partial \alpha^2} \right) \quad (71)$$

$$= 0, \quad (72)$$

where (70) follows from $\frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha^2} = \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha^2}$; (71) because $\frac{\partial^2 \bar{x}_\alpha}{\partial \alpha^2} < 0$ and $0 < \hat{x}_g < \bar{x}_\alpha$ implies $\frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} < \frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} < 0$; and (72) from simplifying using (69). Thus, $\frac{\partial U^E_g(\theta)}{\partial \alpha}|_{\alpha=0} \leq 0$ for all $\alpha \in [0, 1]$. Proposition 5 delivers the result.

\[\square\]
Appendix B

A stationary strategy profile \( \sigma = (\lambda, \pi, \varphi, \nu) \) is a stationary legislative lobbying equilibrium if it satisfies four conditions. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), \( \lambda^g_\ell \) places probability one on

\[
\arg\max_{y, m} \mathcal{V}_\sigma(y)u_\ell(y) + [1 - \mathcal{V}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] - m
\]

subject to

\[
\mathcal{V}_\sigma(y)u_\ell(y) + [1 - \mathcal{V}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] + m
\]

\[
\geq \int_X \left[ \mathcal{V}_\sigma(x)u_\ell(x) + [1 - \mathcal{V}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{73}
\]

Second, for all \( \ell \in N^L \) and \((y, m) \in W\),

\[
\mathcal{V}_\sigma(y)u_\ell(y) + [1 - \mathcal{V}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] + m
\]

\[
> \int_X \left[ \mathcal{V}_\sigma(x)u_\ell(x) + [1 - \mathcal{V}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{74}
\]

implies \( \varphi_\ell(y, m) = 1 \) and the opposite strict inequality implies \( \varphi_\ell(y, m) = 0 \). Third, for all \( \ell \in N^L \),

\[
\pi_\ell\left( \arg\max_{x \in X} \mathcal{V}_\sigma(x)u_\ell(x) + [1 - \mathcal{V}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] \right) = 1. \tag{75}
\]

Finally, for all \( i \in N^V \) and \( x \in X \), \( u_i(x) > (1 - \delta)u_i(q) + \delta V_i(\sigma) \) implies \( \nu_i(x) = 1 \) and the opposite strict inequality implies implies \( \nu_i(x) = 0 \).37

Lemma B.1 shows surplus lobby payments never happen in equilibrium.

**Lemma B.1.** In every stationary legislative lobbying equilibrium, for all \( \ell \in N^L \) every \((y, m) \in \text{supp}(\lambda^g_\ell)\) satisfies

\[
\mathcal{V}_\sigma(y)u_\ell(y) + [1 - \mathcal{V}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] + m
\]

\[
= \int_X \left[ \mathcal{V}_\sigma(x)u_\ell(x) + [1 - \mathcal{V}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\mathcal{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{76}
\]

37Thus, voting strategies are stage-undominated (Baron and Kalai, 1993; Banks and Duggan, 2006a).
The proof of Lemma 76 is straightforward and omitted.

From (7), recall

\[ \xi_{\ell}(\alpha; \sigma) = (1 - \alpha_{\ell}) + \alpha_{\ell} \int_{W} [1 - \varphi_{\ell}(y, m)] \lambda_{g}^{\ell}(dw). \]

Define

\[ \hat{\chi}(X') = \sum_{\ell \in N_{L}} \rho_{\ell} \left( \xi_{\ell}(\alpha; \sigma) \int_{X'} \varphi_{\sigma}(x) \pi_{\ell}(dx) + \alpha_{\ell} \int_{W} \varphi_{\ell}(y, m) \varphi_{\sigma}(y) \lambda_{g}^{\ell}(dw) \right), \tag{77} \]

the probability some \( x \in X' \subseteq X \) is passed in a given period under \( \sigma \). Next, define

\[ \tilde{\chi} = \sum_{\ell \in N_{L}} \rho_{\ell} \left( \xi_{\ell}(\alpha; \sigma) \int_{X} [1 - \varphi_{\sigma}(x)] \pi_{\ell}(dx) + \alpha_{\ell} \int_{W} \varphi_{\ell}(y, m) [1 - \varphi_{\sigma}(y)] \lambda_{g}^{\ell}(dw) \right), \tag{78} \]

the probability of a failed proposal in a given period under \( \sigma \).

Following Banks and Duggan (2006a), each player’s continuation value can be expressed as a function of a common lottery over policy, denoted \( \chi_{\sigma} \). Using (77) and (78), define \( \chi_{\sigma} \) so that for all measurable \( X' \subseteq X \):

(i) if \( q \notin X' \), then \( \chi_{\sigma}(X') = \hat{\chi}(X') \frac{1}{1 - \delta \tilde{\chi}} \),

and (ii) if \( q \in X' \), then \( \chi_{\sigma}(X') = \frac{\hat{\chi}(X') + (1 - \delta) \tilde{\chi}}{1 - \delta \tilde{\chi}} \).

Set \( V_{den}(\sigma) = 1 - \delta \tilde{\chi} \) and define

\[ V_{i}^{num}(\sigma) = \sum_{\ell \in N_{L}} \rho_{\ell} \left( \xi_{\ell}(\alpha; \sigma) \int_{X} \varphi_{\sigma}(x) u_{i}(x) + [1 - \varphi_{\sigma}(x)] (1 - \delta) u_{i}(q) \right) \pi_{\ell}(dx) \]

\[ + \alpha_{\ell} \int_{W} \varphi_{\ell}(y, m) \varphi_{\sigma}(y) u_{i}(x) + [1 - \varphi_{\sigma}(y)] (1 - \delta) u_{i}(q) \lambda_{g}^{\ell}(dw) \right). \]

For each \( i \in N_{V} \), \( i \)'s continuation value defined in (8) satisfies \( V_{i}(\sigma) = \frac{V_{i}^{num}(\sigma)}{V_{den}(\sigma)} \). Then we can express \( V_{i}(\sigma) \) as a lottery over policy, \( V_{i}(\sigma) = \int_{X} u_{i}(x) \chi_{\sigma}(dx) \).

The policy lottery \( \chi_{\sigma} \) is common to all players, but committee members may receive payment and interest groups may make payments. Define

\[ \hat{m}_{\ell}(\sigma) = \rho_{\ell} \alpha_{\ell} \int_{W} m \varphi_{\ell}(y, m) \lambda_{g}^{\ell}(dw), \tag{79} \]

which is \( \ell \)'s expected lobby payment in each period until passage. For \( \ell \in N_{L} \), rearranging (9) yields

\[ \tilde{V}_{\ell}(\sigma) = \frac{V_{\ell}^{num}(\sigma) + \hat{m}_{\ell}(\sigma)}{V_{den}(\sigma)} \]
\[
\int_X u_\ell(x) \chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \tag{80}
\]

Similarly, for \( g \in N^G \) rearranging (10) yields

\[
\hat{V}_g(\sigma) = \frac{V_g^{\text{num}}(\sigma) - \sum_{\ell \in N_g^{L}} \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} = \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N_g^{L}} \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}. \tag{81}
\]

Finally, define

\[
\tilde{U}^\ell(\sigma) = \int_X \left[ \nu \sigma(x) u_\ell(x) + (1 - \nu \sigma(x)) \left( (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \right) \right] \pi(\sigma), \tag{82}
\]

which is \( \ell \)'s expected dynamic payoff under \( \sigma \) conditional on being recognized as the proposer and rejecting \( g \)'s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium \( \sigma \) such that \( \chi^\sigma \) is degenerate on \( q \).

**Proof.** Let \( \sigma \) denote an equilibrium. To show a contradiction, assume \( \chi^\sigma(q) = 1 \). Thus, \( V_M(\sigma) = u_M(q) \), which implies \( u_M(q) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma) \) and therefore \( q \in A(\sigma) \). Without loss of generality, assume \( q > 0 \).

By assumption, there exists \( \ell \in N^L \) such that \( \hat{x}_\ell < q \) and at least one of \( \hat{x}_{g_\ell} \leq q \) or \( \alpha_\ell < 1 \) holds. If \( \alpha_\ell < 0 \), then it is straightforward to show that \( \ell \) must have a profitable deviation, a contradiction.

For the other case, suppose \( \hat{x}_\ell < q, \hat{x}_{g_\ell} \leq q \), and \( \alpha_\ell = 1 \). Note that \( u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) is \( g_\ell \)'s expected dynamic payoff from any offer \( (y, m) \) such that \( V_\sigma(y) = 1, \varphi_\ell(y, m) = 1 \), and \( \ell \) is indifferent between accepting and rejecting. We have \( \hat{y}_\ell = \arg \max_{y \in X} u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) \) and \( \hat{y}_\ell < q \). Strict concavity and continuity imply existence of \( \varepsilon > 0 \) and \( y^\varepsilon < q \) such that \( V_\sigma(y^\varepsilon) = 1, \varphi_\ell(y^\varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1 \), and

\[
u_{g_\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \tilde{U}_\ell(\sigma) - \varepsilon > u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) \tag{83}
\]

\[
geq u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) - \delta \left( \sum_{j \in N_g^{L}} \frac{\hat{m}_j(\sigma)}{V_{\text{den}}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \right), \tag{84}
\]
where (84) follows from $\sum_{j \in N_y} \frac{m_j(\sigma)}{\bar{V}_{\text{den}}(\sigma)} \geq \frac{m_\sigma(\sigma)}{\bar{V}_{\text{den}}(\sigma)}$. The RHS of (83) is weakly greater than $g_\ell$’s expected payoff from lobbying $\ell$ to $q$ if $\nu_\sigma(q) = 1$; and (84) is weakly greater than $g_\ell$’s expected payoff from lobbying $\ell$ to any $y'$ such that $\nu_\sigma(y') = 0$. Thus, $g_\ell$ must have a profitable deviation, a contradiction. \hfill \Box

**Lemma B.3.** Let $\sigma$ denote a stationary legislative lobbying equilibrium. For all $\ell \in N^L$ there exists $(y, m) \in X \times \mathbb{R}_+$ such that $\nu_\sigma(y) = 1$ and $g_\ell$ strictly prefers $(y, m)$ to any $(y', m')$ such that $\nu_\sigma(y') = 0$.

**Proof.** Fix an equilibrium $\sigma$. Let $\chi^q$ denote a probability distribution degenerate on $q$. Define the continuation distribution following rejection under $\sigma$ as $\chi = (1 - \delta)\chi^q + \delta \chi^\sigma$, which is non-degenerate because $\delta \in (0, 1)$ and $\chi^\sigma(q) < 1$ by Lemma B.2.

For every player $k \in N$, the expected dynamic policy payoff from a rejected policy proposal satisfies

$$(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).$$

Let $x^\sigma$ denote the mean of $\chi$. Since $u$ is strictly concave and $\chi$ is non-degenerate, Jensen’s Inequality implies

$$u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma). \quad (85)$$

Consider $\ell \in N^L$. First, assume $\phi_\ell(y, m) = 1$ whenever $\ell$ is indifferent. The condition for $g_\ell$ to strictly prefer $(y, m)$ such that $\nu_\sigma(y) = 1$, rather than $(y', m')$ such that $\nu_\sigma(y') = 0$, is

$$u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\sigma).$$

Equivalently,

$$u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma). \quad (86)$$

Notice that

$$\tilde{V}_{g_\ell}(\sigma) + \tilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{\ell' \in N^L_y} \frac{\tilde{m}_{\ell'}(\sigma)}{\bar{V}_{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\tilde{m}_\ell(\sigma)}{\bar{V}_{\text{den}}(\sigma)} \quad (87)$$

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\[
\begin{align*}
\leq V_{\ell}(\sigma) - \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} + V_\ell(\sigma) + \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} & \quad \text{(88)} \\
= V_{\ell}(\sigma) + V_\ell(\sigma), & \quad \text{(89)}
\end{align*}
\]

where (87) follows from substituting for \( \tilde{V}_\ell(\sigma) \) and \( \tilde{V}_g(\sigma) \) using (80) and (81); and (88) from \( \sum_{\ell' \in N^1} \frac{\tilde{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{\tilde{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \).

By (85), \( \nu_{\sigma}(x^\sigma) = 1 \) follows because \( u_M(x^\sigma) > (1-\delta)u_M(q) + \delta V_M(\sigma) \). Furthermore, (85) implies \( u_{\ell}(x^\sigma) > (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) \) and \( u_{\ell}(x^\sigma) > (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) \). Thus, (89) implies that (86) holds because

\[
\begin{align*}
u_{\ell}(x^\sigma) + u_{\ell}(x^\sigma) > (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) + (1-\delta)u_{\ell}(q) + \delta V_{\ell}(\sigma) \\
\geq (1-\delta)u_{\ell}(q) + \tilde{V}_{\ell}(\sigma) + (1-\delta)u_{\ell}(q) + \tilde{V}_{\ell}(\sigma).
\end{align*}
\]

Next, assume \( \varphi_{\ell}(x^\sigma, m) < 1 \) for \( m \) such that \( \ell \) is indifferent between accepting \( (x^\sigma, m) \) and rejecting. For sufficiently small \( \varepsilon > 0 \), \( \varphi_{\ell}(x^\sigma, m + \varepsilon) = 1 \) and the preceding argument implies \( g_{\ell} \) strictly prefers \( (x^\sigma, m + \varepsilon) \) over any \( (y', m') \) such that \( \nu_{\sigma}(y') = 0 \).

**Lemma B.4.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

**Proof.** Let \( \sigma \) be an equilibrium. By Duggan (2014), \( M \) is decisive. Quadratic utility and \( \hat{x}_M = 0 \neq q \) together imply \( A(\sigma) = \{ x \in X | u_M(x) \geq (1-\delta)u_M(q) + \delta V_M(\sigma) \} \) is a closed, non-empty interval symmetric about 0. Let \( A(\sigma) = [-\overline{\sigma}(\sigma), \overline{\sigma}(\sigma)] \). Then \( x \in (-\overline{\sigma}(\sigma), \overline{\sigma}(\sigma)) \) implies \( \nu_{\sigma}(x) = 1 \).

Fix \( \ell \in N^L \). By Lemma B.2, \( \chi^\sigma(q) < 1 \). Lemma B.3 implies existence of \( (y, m) \in W \) such that \( \nu_{\sigma}(y) = 1 \) and \( g_{\ell} \) strictly prefers \( (y, m) \) over all \( (y', m') \) with \( \nu_{\sigma}(y') = 0 \). Thus, \( y \in A(\sigma) \) for all \( (y, m) \in \text{supp}(\lambda_{g_{\ell}}) \). Without loss of generality, assume \( \nu_{\sigma}(-\overline{\sigma}(\sigma)) < 1 \). It suffices to check two cases.

- **Case 1:** If \( \hat{x}_\ell \leq -\overline{\sigma}(\sigma) \) and \( u_{\ell}(-\overline{\sigma}(\sigma)) > (1-\delta)u_{\ell}(q) + \delta \tilde{V}_{\ell}(\sigma) \), then \( x \in A(\sigma) \) for all \( x \in \text{supp}(\pi_\ell) \). Because \( u_{\ell} \) is strictly concave and continuous, and \( \nu_{\sigma}(-\overline{\sigma}(\sigma)) < 1 \), there exists \( \varepsilon > 0 \) such that \( \ell \) has a profitable deviation to \( -\overline{\sigma}(\sigma) + \varepsilon \), a contradiction.

- **Case 2:** Assume \( \hat{y}_{\ell} \leq -\overline{\sigma}(\sigma) \). Continuity, Lemma B.3, and \( \nu_{\sigma}(-\overline{\sigma}(\sigma)) < 1 \) imply existence of \( \varepsilon, \varepsilon' > 0 \) such that \( g_{\ell} \) has a profitable deviation to \( (y', m') = (-\overline{\sigma}(\sigma) + \varepsilon, U_{\ell}(\sigma) - u_{\ell}(-\overline{\sigma}(\sigma) + \varepsilon) + \varepsilon') \), a contradiction.

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It follows that either $\sigma$ must involve deferential voting, or $\sigma$ is equivalent in outcome distribution to an equilibrium with deferential voting. \hfill \square

**Lemma B.5.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

*Proof.* Let $\sigma$ denote an equilibrium. By Lemma B.4, we can assume $\pi_\sigma(x) = 1$ iff $x \in A(\sigma)$. Fix $\ell \in N^L$ and define $y_{g\ell}^* = \arg \max_{y \in A(\sigma)} u_{g\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$, which is uniquely defined, and $m_{g\ell}^* = \tilde{U}_\ell(\sigma) - u_\ell(y_{g\ell}^*)$.

By Lemma B.2, $\chi^\sigma(q) < 1$. For sufficiently small $\varepsilon > 0$, Lemma B.3 implies $g$ strictly prefers $(y_{g\ell}^*, m_{g\ell}^* + \varepsilon)$ over every $(y', m')$ such that $y' \notin A(\sigma)$. Thus, if $\pi_\ell$ is not degenerate on $y_{g\ell}^*$ and $\varphi_\ell(y_{g\ell}^*, m_{g\ell}^*) < 1$, then there exists $\varepsilon > 0$ such that $g_\ell$ has a profitable deviation to $(y_{g\ell}^*, m_{g\ell}^* + \varepsilon)$, a contradiction. Thus, $\sigma$ must satisfy either (i) $\pi_\ell(y_{g\ell}^*) = 1$, or (ii) $\lambda_\ell(y_{g\ell}^*, m_{g\ell}^*) = 1$ and $\varphi_\ell(y_{g\ell}^*, m_{g\ell}^*) = 1$, as desired. \hfill \square

A strategy profile $\sigma$ is no-delay if $\pi_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$ and $\pi_{\sigma}(y) = 1$ for all $(y, m) \in \text{supp}(\lambda_\ell^\ell)$.

**Lemma B.6.** Every stationary legislative lobbying equilibrium is no-delay.

*Proof.* Fix an equilibrium $\sigma$. By Lemma B.2, $\chi^\sigma(q) < 1$. Thus, Lemma B.3 implies $g$ strictly prefers some $(y, m) \in W$ such that $\pi_{\sigma}(y) = 1$. Lemma B.4 implies we can assume $\pi_{\sigma}(x) = 1$ iff $x \in A(\sigma)$. Lemma B.5 implies we can assume all $\ell \in N^L$ use deferential acceptance strategies.

For each $\ell \in N^L$, the preceding observations and Lemma B.1 imply $\lambda_\ell^\ell$ puts probability one on $(y^*, m^*)$ such that $y^* = \arg \max_{y \in A(\sigma)} u_{g\ell}(y) + u_\ell(y) - u_\ell(z_\ell; \sigma)$, which is unique. Lemmas B.4 and B.5 imply we can assume $\pi_{\sigma}(y^*) = 1$ and $\varphi_\ell(y^*, m^*) = 1$.

It remains to verify that $z_\ell \notin A(\sigma)$ cannot be optimal for any $\ell \in N^L$. To show a contradiction, assume proposing $z_\ell \notin A(\sigma)$ is optimal for some $\ell \in N^L$. Let $z^* = \arg \max_{x \in A(\sigma)} u_\ell(x)$. There are two steps. Step 1 establishes useful properties of $\ell$’s preferences over lotteries. Step 2 shows a contradiction.

**Step 1:** Recall the continuation lottery induced by $\sigma$, denoted $\chi = (1 - \delta)\chi^q + \delta\chi^\sigma$ with mean $x^\sigma$. Jensen’s inequality implies $u_i(x^\sigma) > \int_X u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma)$ for all $i \in N$, so $x^\sigma \in \text{int} A(\sigma)$. 

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Next, let $\chi^{z^*}$ denote the policy lottery nearly equivalent to $\chi$, but shifting probability $\frac{\delta \rho_{\ell} \alpha_{\ell}}{V_{\text{den}}(\sigma)}$ from $y^*$ to $z^*$. Let $x^{z^*}$ denote the mean of $\chi^{z^*}$. For all $i \in N$, Jensen’s inequality implies

$$u_i(x^{z^*}) > \int_x u_i(x) \chi^{z^*}(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho_{\ell} \alpha_{\ell} u_i(y^*)}{V_{\text{den}}(\sigma)} + \frac{\delta \rho_{\ell} \alpha_{\ell} u_i(z^*)}{V_{\text{den}}(\sigma)}.$$ 

Moreover, $x^{z^*}$ is located weakly between $x^*$ and $z^*$, implying $x^{z^*} \in A(\sigma)$.

**Step 2:** Since $z_\ell \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_\ell(q) + \delta \hat{V}_\ell(\sigma) - u_\ell(y^*)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} - u_\ell(y^*).$$ \hspace{1cm} (90)

Using (79), $\hat{m}_\ell(\sigma)$ is expressed recursively as

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} - u_\ell(y^*) \right)$$

$$= \frac{\rho_\ell \alpha_\ell V_{\text{den}}(\sigma)}{V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell} \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*) \right).$$ \hspace{1cm} (91)

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \hat{V}(\sigma)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)] V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell}{V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell},$$ \hspace{1cm} (92)

where (93) follows from the definition of $\hat{V}_\ell(\sigma)$ and using (91) to substitute for $\hat{m}_\ell(\sigma)$. Next, we have $V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in N_L} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell > 0$, where the first inequality follows because Lemma B.3 implies all lobby offers are accepted and passed under $\sigma$, so $V_{\text{den}}(\sigma) \geq 1 - \delta \sum_{j \in N_L} \rho_j (1 - \alpha_j)$; and the second inequality follows from $\delta[\rho_\ell \alpha_\ell + \sum_{j \in N_L} \rho_j (1 - \alpha_j)] < 1$. Rearranging and simplifying (93),

$$0 \leq V_{\text{den}}(\sigma) \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*) \right) \left( V_{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \right)$$

$$\propto (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta \rho_\ell \alpha_\ell [u_\ell(y^*) - u_\ell(z^*)]}{V_{\text{den}}(\sigma)} - u_\ell(z^*)$$

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\[ = \int_X u_\ell(x) \chi^*(dx) - u_\ell(z^*), \]
a contradiction because \( u_\ell(z^*) \geq u_\ell(x^*) > \int_X u_\ell(x) \chi^*(dx). \]

Lemma B.7. Every stationary legislative lobbying equilibrium is such that \( \lambda_g \) is degenerate for all \( g \in N^G \) and \( \pi_\ell \) is degenerate for all \( \ell \in N^L \).

Proof. Let \( \sigma \) denote an equilibrium. By Duggan (2014), \( A_M(\sigma) = A(\sigma) \), which is nonempty, compact and convex.

First, consider \( g \in N^g \) and \( \ell \in N^L_g \). Recall \( \tilde{U}_\ell(\sigma) \) from (82). Lemmas B.1 and B.6 imply \( \lambda^g_\ell \) puts probability one on the unique \((y^*, m^*)\) satisfying \( y^* = \arg \max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma), \) and \( m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*) \).

Second, consider \( \ell \in N^L \). Lemma B.6 implies \( \pi_\ell \) puts probability one on \( x^* = \arg \max_{x \in A(\sigma)} u_\ell(x) \), which is unique.

Proposition 1.2 corresponds to Part 2 of Proposition 1 in the text.

Proposition 1.2 Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

Proof. Follows from Lemmas B.4 - B.7.
Appendix C

Consider \( \ell \in N^L \). First, I define a function \( \zeta^\ell \) that relates to \( M \)'s equilibrium voting decision. Then, Lemmas C.3 - C.6 characterize \( \zeta^\ell \). Finally, Lemma 1 delivers a partitional characterization on \( \hat{x}_g \) that facilitates Proposition 2.

Preliminaries to define \( \zeta^\ell \). Recall \( \pi(0) = \pi(\hat{x}_g) \) for \( \hat{x}_g = 0 \). Let \( \hat{D}^{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \pi(0), j \neq \ell \} \) and \( \hat{D}^{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \pi(0), j \neq \ell \} \). Next, set \( D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y} \} \) and \( D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x} \} \). Define \( D^\ell \) as the unique elements of \( D^{\ell,y} \cup D^{\ell,x} \cup \{\pi(0)\} \). Let \( K^\ell + 1 = |D^\ell| \). Denote the \( k \)-th element of \( D^\ell \) as \( d^\ell_k \). Index elements \( k = 0, \ldots, K^\ell \) of \( D^\ell \) in ascending order so that \( d^\ell_0 = \pi(0) \) and \( k' > k \) implies \( d^\ell_{k'} > d^\ell_k \).

For each \( k \) and \( j \neq \ell \), let \( C^\ell_j = \|\hat{x}_j \in [-d^\ell_k, d^\ell_k]\| \) and \( \hat{C}^\ell_j = \|\hat{y}_j \in [-d^\ell_k, d^\ell_k]\| \). Define

\[
I^\ell_j = (1 - \alpha_j)C^\ell_j u_M(\hat{x}_j) + \alpha_j \hat{C}^\ell_j u_M(\hat{y}_j)
\]

and

\[
O^\ell_j = (1 - \alpha_j)(1 - C^\ell_j) + \alpha_j(1 - \hat{C}^\ell_j),
\]

suppressing dependence on \( \ell \). Let

\[
\hat{x}^\ell_k = \left( \frac{1}{\delta \rho^\ell} \left[ (1 - \delta)u_M(g) + \delta \sum_{j \neq \ell} \rho_j I^\ell_j - u_M(d^\ell_k) \left( 1 - \delta \sum_{j \neq \ell} O^\ell_j \right) \right] \right)^{\frac{1}{2}}.
\]  

(94)

Because \( d^\ell_0 = \pi(0) \), rearranging (94) yields \( \hat{x}^\ell_0 = 0 \).

Lemma C.1. For all \( \ell \in N^L \) and each \( k = 0, \ldots, K^\ell \), we have

\[
\delta \sum_{j \neq \ell} \rho_j I^{\ell+1}_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell+1}_j) = \delta \sum_{j \neq \ell} \rho_j I^{\ell}_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell}_j).
\]

Proof. Consider \( \ell \in N^L \) and fix \( k < K^\ell \). Then,

\[
\delta \sum_{j \neq \ell} \rho_j I^{\ell+1}_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell+1}_j)
\]

\[
= \delta \sum_{j \neq \ell} \rho_j I^{\ell+1}_j - u_M(d^\ell_{k+1}) (1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell+1}_j) + \delta u_M(d^\ell_{k+1}) \sum_{j \neq \ell} \rho_j O^{\ell}_j - \delta u_M(d^\ell_{k+1}) \sum_{j \neq \ell} \rho_j O^{\ell}_j
\]

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Lemma C.3. For all $\ell \in N^\ell$, $\zeta$ strictly increases in $k$.\hfill\Box

Proof. Consider $\ell \in N^\ell$ and fix $k < K^\ell$. Lemma C.1 and $0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell)$ together imply
\begin{align*}
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^k - O_j^k)
= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})
= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k),
\end{align*}
where (95) follows because $u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^k - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})$ by construction. \hfill\Box

Lemma C.2. For all $\ell \in N^\ell$, $\hat{x}_k^\ell$ strictly increases in $k$.\hfill\Box

Proof. Consider $\ell \in N^\ell$ and fix $k < K^\ell$. Lemma C.1 and $0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell)$ together imply
\begin{align*}
\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k)
\end{align*}
Thus, $\hat{x}_k^\ell < \hat{x}_{k+1}^\ell$ follows from (94). \hfill\Box

Definition of $\zeta^\ell$. For $k = 0, \ldots, K^\ell$, define $\overline{x}_k^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ as
\begin{align*}
\overline{x}_k^\ell(x) = \left(\frac{- (1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}\right)^{\frac{1}{2}}
\end{align*}
and $\zeta_k^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}$ as
\begin{align*}
\zeta_k^\ell(x) = u_M(x) - \left(1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k\right).
\end{align*}
By construction, $\overline{x}_k^\ell(\hat{x}_k^\ell) = d_k^\ell$ for all $k$. Adopt the convention $d_{K^\ell+1} = \infty$. Define the piecewise function $\zeta^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}$ as
\begin{align*}
\zeta^\ell(x) = \zeta_k^\ell(x) \text{ if } x \in [d_k^\ell, d_{k+1}^\ell).\hfill\Box
\end{align*}

Lemma C.3. For all $\ell \in N^L$, $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$.  

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Proof. Consider $\ell \in N^L$. First, we have

$$
\zeta^{\ell}(0) = \zeta_0^{\ell}(0)
= u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho \epsilon u_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\pi_0(0)) \sum_{j \neq \ell} \rho_j O_j^0 \right)
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right)
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right)
> 0,
$$

where (99) follows from $u_M(0) = 0$ and $\pi_0^\ell(0) = \hat{x}_0$.

Next, I show $\zeta^{\ell}(q) \leq 0$. Let $k'$ denote the largest $k$ such that $\hat{x}_k^\ell \leq q$.

- **Step 1:** Because $\pi^{k'}(\hat{x}_{k'}^\ell) = d_{k'}^\ell$, we have

$$
u_M(d_{k'}^\ell) = \frac{(1 - \delta)u_M(q) + \delta \rho \epsilon u_M(\hat{x}_{k'}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho \epsilon u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho \epsilon u_M(q) + \delta u_M(d_{k'}^\ell)(1 - \rho \epsilon - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} = u_M(q),$$

where (100) follows from rearranging (98); (101) from $\hat{x}_{k'}^\ell \leq q$; (102) because for all $j$ the construction of $I_j^{k'}$ implies $I_j^{k'} \geq u_M(d_{k'}^\ell)((1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'})$; and (103) because $\sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho \epsilon - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction.

Rearranging and simplifying (103) yields $u_M(d_{k'}^\ell) \geq \frac{(1 - \delta + \delta \rho \epsilon u_M(q))}{1 - \delta + \delta \rho \epsilon} = u_M(q)$. Thus,

$$
\sum_{j \neq \ell} \rho_j I_j^{k'} = \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^{k'} u_M(\hat{y}_j) \right] \geq u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'} \right] = u_M(d_{k'}^\ell)(1 - \rho \epsilon - \sum_{j \neq \ell} \rho_j O_j^{k'}).$$

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\[ \geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_{j}^{k'}) \]  

(107)

where (104) follows from the definition of \( I_j^{k'} \); (105) from \( u_M(\hat{x}_j) \geq u_M(d_k^j) \) if \( C_j^{k'} = 1 \) and \( u_M(\hat{y}_j) \geq u_M(d_k^j) \) if \( \tilde{C}_j^{k'} = 1 \); (106) because \( \sum_{j \neq \ell} \rho_j[(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'} \) by construction; and (107) from \( u_M(d_k^j) \geq u_M(q) \).

- **Step 2:** We have

\[
\begin{align*}
    u_M(\pi_{k}^{\ell}(q)) & = \frac{(1 - \delta)u_M(q) + \delta \rho \ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \\
    & \geq \frac{(1 - \delta)u_M(q) + \delta \rho \ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \\
    & = u_M(q),
\end{align*}
\]

(108) (109)

where (108) follows from Step 1 and (109) from simplifying.

- **Step 3:** To see \( \zeta^{\ell}(q) \leq 0 \), note

\[
\begin{align*}
    \zeta^{\ell}(q) & = u_M(q) - \left( (1 - \delta)u_M(q) + \delta \rho \ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'} + \delta u_M(\pi_{k}^{\ell}(q)) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \\
    & \leq u_M(q) - \left( (1 - \delta)u_M(q) + \delta \rho \ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) + \delta u_M(q) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \\
    & = 0,
\end{align*}
\]

(110) (111)

where (110) follows from Steps 1 and 2.

\[ \square \]

**Lemma C.4.** For all \( \ell \in N^L \), \( \zeta^{\ell} \) is continuous.

**Proof.** Consider \( \ell \in N^L \) and fix \( k \). Because \( \pi_{k}^{\ell}(x) \) is continuous, \( \zeta^{\ell} \) is continuous over \((\hat{x}_{k}, \hat{x}_{k+1})\). It suffices to show \( \zeta_k^{\ell}(\hat{x}_{k+1}) = \zeta_{k+1}^{\ell}(\hat{x}_{k+1}) \).

First, I establish \( d_{k+1}^\ell = \pi_{k}^{\ell}(\hat{x}_{k+1}) \). Rearranging (94) for \( k + 1 \) yields

\[
0 = u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) - (1 - \delta)u_M(q) - \delta \rho \ell u_M(\hat{x}_{k+1}) - \delta \sum_{j \neq \ell} \rho_j I_j^{k+1}
\]

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\begin{equation}
= u_M(d_{k+1}^\ell) \left( 1 - \delta \sum_{j \neq \ell} \rho_j O_j^k \right) - (1 - \delta) u_M(q) - \delta \rho_\ell u_M(\ddot{x}_{k+1}^\ell) - \delta \sum_{j \neq \ell} \rho_j I_j^k, \quad (112)
\end{equation}

where (112) follows from Lemma C.1. Thus, \( u_M(d_{k+1}^\ell) = \frac{(1 - \delta) u_M(q) + \delta \rho_\ell u_M(\ddot{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \), so \( d_{k+1}^\ell = \pi_k^\ell(\ddot{x}_{k+1}^\ell) \). Then,

\begin{align*}
\zeta_{k+1}(\ddot{x}_{k+1}^\ell) &= u_M(\ddot{x}_{k+1}^\ell) - \left( (1 - \delta) u_M(q) + \delta \rho_\ell u_M(\ddot{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\pi_k^\ell(\ddot{x}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_j^k \right) \\
&= u_M(\ddot{x}_{k+1}^\ell) - \left( (1 - \delta) u_M(q) + \delta \rho_\ell u_M(\ddot{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_{j+1}^k + \delta u_M(\pi_k^\ell(\ddot{x}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_{j+1}^k \right) \\
&= \zeta_{k+1}(\ddot{x}_{k+1}^\ell), \quad (113)
\end{align*}

where (113) follows from Lemma C.1 because \( d_{k+1}^\ell = \pi_k^\ell(\ddot{x}_{k+1}^\ell) \).

\begin{lemma}
For all \( \ell \in N^L \), \( \zeta^\ell \) is strictly decreasing.
\end{lemma}

\begin{proof}
Consider \( \ell \in N^L \) and fix \( k \). The proof shows that the derivative of \( \zeta^\ell \) is strictly negative at every \( x \in (\ddot{x}_k^\ell, \ddot{x}_{k+1}^\ell) \). Continuity then implies that \( \zeta^\ell \) is strictly decreasing.

Consider \( x \in (\ddot{x}_k^\ell, \ddot{x}_{k+1}^\ell) \). Then

\[ \zeta^\ell(x) = u_M(x) - \left( (1 - \delta) u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\pi_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right) \]

and

\begin{equation}
\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x \delta \rho_\ell + \frac{2x \delta \rho_\ell (\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \quad (115)
\end{equation}

\begin{equation}
\propto \delta \rho_\ell + \delta \sum_{j \neq \ell} \rho_j O_j^k - 1 \quad (116)
\end{equation}

\begin{equation}
< 0, \quad (117)
\end{equation}

where (115) follows from \( \frac{\partial u_M(\pi_k^\ell(x))}{\partial \pi_k^\ell(x)} \frac{\partial \pi_k^\ell(x)}{\partial x} = -\frac{2x \delta \rho_\ell}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \); and (117) because \( \delta \in (0, 1) \) and \( \rho_\ell + \sum_{j \neq \ell} \rho_j O_j^k \leq 1 \).

\begin{lemma}
For all \( \ell \in N^L \), there is a unique \( \pi^\ell \in (0, q] \) such that \( \zeta^\ell(x) > 0 \) for all \( x \in [0, \pi^\ell) \), \( \zeta^\ell(\pi^\ell) = 0 \), and \( \zeta^\ell(x) < 0 \) for all \( x > \pi^\ell \).
\end{lemma}

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Proof. Consider $\ell \in N^L$. Lemma C.3 implies $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. By Lemma C.5, $\zeta^\ell$ is strictly decreasing. Thus, there is a unique $\bar{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}_\ell)$ and $\zeta^\ell(x) < 0$ for all $x > \bar{x}_\ell$. Lemma C.4 implies $\zeta^\ell(\bar{x}_\ell) = 0$. □

Lemma 1. For all $\ell \in N^L$, $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Otherwise, $A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell]$.

Proof. Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_\ell = \hat{x}_g$.

Part 1. First, suppose $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ and assume $\hat{x}_g \geq 0$ without loss of generality. I show $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Let $k'$ be the largest $k$ such that $\hat{x}_g^k \leq \hat{x}_g$. Define the strategy profile $\sigma'$ such that it puts probability $\rho_k$ on $\hat{x}_g$ and for each $j \neq \ell$ it (i) puts probability $(1 - \alpha_j)\rho_j$ on: $\hat{x}_j$ if $\hat{x}_j \in [d_k, d_k']$, $\bar{\pi}_k(\hat{x}_g)$ if $\hat{x}_j > d_k'$, or $-\bar{\pi}_k(\hat{x}_g)$ if $\hat{x}_j < -d_k'$; and (ii) puts probability $\alpha_j\rho_j$ on: $\hat{y}_j$ if $\hat{y}_j \in [-d_k', d_k']$, $\bar{\pi}_k(\hat{x}_g)$ if $\hat{y}_j > d_k'$, or $-\bar{\pi}_k(\hat{x}_g)$ if $\hat{y}_j < -d_k'$. By construction, $\bar{\pi}(\sigma') = \bar{\pi}_k(\hat{x}_g)$. Furthermore, proposal strategies are optimal given $A(\sigma') = [-\pi(\sigma'), \pi(\sigma')]$.

I now check optimality for $M$. Because $\hat{x}_g \in [\hat{x}_g^k, \hat{x}_g^{k+1}]$, we have $\pi(\sigma') = \bar{\pi}_k(\hat{x}_g) \in [d_k', d_k'+1)$. Thus, $M$ optimally accepts all offers by $j \neq \ell$. Next, I verify $\hat{x}_g \in \text{int}A(\sigma')$. By Lemma C.6, $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\zeta(\hat{x}_g) > 0$, which is equivalent to $u_M(\hat{x}_g) > (1-\delta)u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j I_{\hat{x}_g}^{k'}$. Under $\sigma'$, this is equivalent to $\hat{x}_g \in \text{int}A(\sigma')$.

Thus, $\sigma'$ is equivalent to the equilibrium $\sigma(\hat{x}_g)$ and $\hat{x}_g \in \text{int}A(\hat{x}_g)$, as desired.

Part 2. Assume $\hat{x}_g \notin (-\bar{x}_\ell, \bar{x}_\ell)$ and suppose $\hat{x}_g \geq 0$ without loss of generality. I verify $A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell]$ in two steps. Step 1 shows $\bar{\pi}(\hat{x}_g) \geq \bar{x}_\ell$. Step 2 shows $\bar{\pi}(\hat{x}_g) \leq \bar{x}_\ell$.

Step 1. Suppose $\bar{\pi}(\hat{x}_g) < \bar{x}_\ell$. Let $k'$ be the largest $k$ such that $\hat{x}_g^k \leq \bar{\pi}(\hat{x}_g)$. Because $\hat{x}_g > \bar{x}_\ell > \bar{\pi}(\hat{x}_g)$, it follows that $\sigma(\hat{x}_g)$ puts probability $\rho_k$ on $\bar{\pi}(\hat{x}_g)$. Thus, $u_M(\bar{\pi}(\hat{x}_g)) = \frac{(1-\delta)u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j I_{\hat{x}_g}^{k'}}{1-\delta \sum_{j \neq \ell} \rho_j O_{\hat{x}_g}^{k'}}$ and rearranging yields $\zeta(\bar{\pi}(\hat{x}_g)) = 0$. Lemma C.6 implies $\bar{\pi}(\hat{x}_g) = \bar{x}_\ell$, a contradiction.

Step 2. Suppose $\bar{\pi}(\hat{x}_g) > \bar{x}_\ell$. If $\hat{x}_g \geq \bar{x}_\ell$, then the argument from Step 1 shows a contradiction. Assume $\hat{x}_g < \bar{x}_\ell(\hat{x}_g)$. Let $k'$ be the largest $k$ such that $\hat{x}_g^k(\hat{x}_g) \leq \bar{x}_\ell(\hat{x}_g)$. Then $\sigma(\hat{x}_g)$ puts probability $\rho_k$ on $\hat{x}_g$. Next, $M$ optimally accepts $\hat{x}_g$ under $\sigma(\hat{x}_g)$ if $u_M(\hat{x}_g) \geq \frac{(1-\delta)u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j O_{\hat{x}_g}^{k'}}{1-\delta \sum_{j \neq \ell} \rho_j O_{\hat{x}_g}^{k'}}$. Rearranging, this condition is equivalent to $\zeta(\hat{x}_g) \geq 0$. By Lemma C.6, this requires $\hat{x}_g \leq \bar{x}_\ell$, a contradiction. □
References


