Access and Lobbying in Legislatures*

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Abstract

I study a model of legislative policymaking with interest groups. To lobby, groups must have access. Access provides opportunities to lobby particular legislators when they control the agenda. In equilibrium, persistent access creates a tradeoff. It changes legislature-wide expectations, thereby affecting which policies pass today. Thus, access to particular legislators can indirectly affect proposals by other legislators. These endogenous spillovers encourage access to some legislators but discourage access to others. Under broad conditions, groups forgo access to a range of more centrist legislators. In contrast, they are keen to access more extreme legislators. These results have implications for campaign finance and revolving door hiring. I also show that lobbying expenditures increase with several measures of legislature polarization. Expenditures can increase or decrease with access depending on the relative extremism of the group and targeted legislator.

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Fears about interest group influence are widespread. Under the popular cynical view, wealthy groups lobby key politicians to shift policy in their own favor whenever possible. Implicit in the public’s fear are the relative preferences of groups and the politicians they influence, as a chief concern is ideologically extreme groups pulling centrist politicians away from majority interests. To create opportunities for effective lobbying, however, scholars widely believe that groups must acquire access by forming working relationships with politicians.\footnote{See, e.g., Wright (1989, 1990); Hall and Wayman (1990); Hansen (1991); Ainsworth (1993); de Figueiredo and Silverman (2006) and Powell (2014).}

Studying who influences whom, and the corresponding welfare implications, thus requires a clear understanding of the connections groups cultivate with different politicians. To develop this understanding, I ask: which combinations of interest groups and politicians form connections that provide access? Beyond this main question, I study two subsequent questions. First, given connections that form, how do political conditions influence observed levels of lobbying? Second, what are the policy and welfare effects of access and lobbying?

Given the apparent benefits of lobbying, and evidence that access is a necessary precursor, a natural expectation is that groups covet access. Naïve intuition suggests groups must be better off increasing access to any legislator because lobbying opportunities improve. In legislatures, however, this expectation overlooks an important consideration: spillover effects. Because groups cannot feasibly influence every legislator, they must account for how their access to particular legislator(s) spills over and affects behavior by other legislators. In some cases, spillovers may be harmful and discourage access, but in other cases they may be helpful and further encourage access. I provide a microfoundation for access-driven spillover effects by disentangling distinct legislative forces. In this paper, spillovers arise endogenously because legislators anticipate (i) where connections develop and (ii) the policy implications of subsequent lobbying. Moreover, I shed light on how these spillovers depend on specific legislative features.

A key result is that access-driven spillovers produce qualitative differences in how groups value connections to different legislators. Specifically, some groups optimally forgo access to particular legislators. This behavior does not require costly access. Instead, it stems from adverse spillover effects of access. Specifically, some groups forgo access to more centrist legislators because the prospect of lobbying these legislators polarizes policymaking. In general, however, access can have either a moderating or polarizing effect on policymaking. The direction depends on the relative preferences of groups and targeted legislators. Although forgoing free access may be counterintuitive, it aligns with longstanding puzzles about campaign finance in the US.

The model has three key features. First, I model lobbying as an instrument to influence policy proposals before reaching the floor. Second, to study which connections form, I distinguish between access and lobbying. Interest groups choose whether to access particular legislators before policymaking begins and, if they acquire access, they can lobby those legislators when they control the agenda during policymaking. Finally, I unpack the legislative black box by studying an environment where failed proposals can be revisited and forward-looking legislators anticipate outside influence. Legislators are forward-looking, agenda power can change hands unpredictably, and passage requires majority approval. These features generate access-driven spillovers, and thus microfound potential drawbacks from greater access, none of which arises in the static model of, e.g., Romer and Rosenthal (1978).

To introduce lobbying into this legislative setting, I first study a baseline model where interest group access is exogenous. Access gives groups a chance to influence policy proposals by particular legislators. Policymaking proceeds until a proposal passes and interest groups lobby throughout as their access permits. Legislators and interest groups are dynamically sophisticated, accounting for expectations about future play. I model a rich legislative environment to disentangle access-seeking incentives from lobbying. Expanding the scope of application for the canonical bargaining framework is of independent theoretical interest.

The baseline model illustrates how lobbying affects legislative policymaking. I establish equilibrium existence and provide sharp characterization. Equilibrium behavior has clear connections to the setting without lobbying. As expected, interest groups lobby to pull policy in their favored direction whenever they have access to the proposer. Groups may be constrained, however, because successful policy proposals must satisfy a majority. Depending on the respective preferences of groups and their associated legislators, lobbying can increase or decrease policy extremism. Additionally, the characterization yields clear comparative statics about the relationships between lobbying expenditures and various legislative conditions.

Access-driven spillover effects create an important tradeoff for interest groups. On the one hand, access provides groups with more opportunities to lobby for favorable policy during

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2 See Eraslan and McLennan (2013) for a more thorough discussion of models using the Baron and Ferejohn (1989) framework.
3 Banks and Duggan (2006a) generalize this setting to multiple dimensions.
policymaking. If the group can lobby, then it always receives more favorable policy. On the other hand, access can have a negative indirect effect on proposals by legislators the group does not access. Because legislators are forward-looking, they account for anticipated lobbying behavior after rejected proposals. Thus, access can change which policies will pass. In turn, it can indirectly affect policies enacted by proposers constrained by majority voting. From the group’s perspective, this effect can be either good or bad. Moreover, its size depends on legislative conditions, including the group’s amount of access.

To study how groups resolve this tradeoff, I extend the baseline model so that groups choose access before policymaking begins. The sharp characterization of legislative behavior pins down how access affects a group’s welfare. In practice, access must be acquired well ahead of time, before active policymaking.

I show that interest groups optimally forgo access to particular legislators. Whether a group wants access to a legislator depends on their relative extremism. Some groups choose not to access their neighboring, more centrist legislators because access-driven spillovers increase policy extremism. This indirect effect outweighs the group’s gain from more lobbying opportunities. These connections are foregone even if they are free. In these cases, groups face a time inconsistency problem. They always want to lobby when given the opportunity. Ex ante, however, they forgo access if it polarizes the policymaking environment too much, relative to their expected gain from lobbying.

In contrast, groups always want access to nearby, more extreme legislators. In this case, access increases lobby opportunities and also favorably constrains extreme legislators. Substantively, the analysis suggests that centrist and moderate interest groups have especially strong incentives to acquire access to a broad spectrum of legislators.

To illustrate the logic, consider the following stylized example. A regional energy interest group anticipates national legislation regulating emissions. It prefers moderately tighter regulations to capitalize on recent investments in clean technology. The group’s local congressman wants to tighten existing regulations more than the group. If the group gets access, its chances of lobbying the congressman increase. Thus, moderate and pro-environment legislators are less optimistic about the eventual regulatory outcome. These legislators know that if the congressman drafts policy, then the group may be able to lobby. If so, the resulting policy will be more extreme than if the congressman had acted alone. Consequently, these other legislators are willing to approve more extreme policies. The group’s access thus indirectly allows extreme pro-energy legislators to pass weaker emissions regulations if they draft policy. Such policies would reduce the group’s benefits from its recent technological investments. I show that this threat of greater extremism can worsen the group’s expectations about policymaking so that it prefers to forgo access altogether.
The analysis provides implications for welfare and empirical work. First, the consequences of access depend on the relative preferences of groups and targeted politicians. Many fear that groups pull otherwise public-minded politicians away from majority interests. But some groups may moderate otherwise extreme politicians. Studying which pairs of groups and legislators are likely to form connections highlights when society may want to restrict access, encourage it, or do nothing. Second, empirical evidence suggests that campaign contributions and hiring lobbyists with revolving door connections are two ways that groups get valuable access (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kalla and Broockman, 2015). Thus, identifying who groups want to access provides direct implications for both (i) how groups allocate contributions and (ii) which lobbyists they hire.

I contribute to a small literature analyzing lobbying to influence the agenda within legislative institutions. Helpman and Persson (2001) introduce interest groups into a static version of Baron and Ferejohn (1989). They focus on comparing the consequences of lobbying in different legislative institutions. As in this paper, groups can lobby particular legislators if they control the agenda and lobbying influences proposals. Notable differences are that they study distributive policies, bargaining ends immediately, and groups can also lobby to influence votes. Moreover, they do not study access.

Other work incorporating interest groups into the Baron and Ferejohn (1989) framework allows groups to buy agenda control (Yildirim, 2010, 2007; Ali, 2015). Most papers in this vein analyze distributive policies. An exception is Levy and Razin (2013), who study a dynamic setting with an endogenous status quo in a one-dimensional policy space. In each period, a continuum of interest groups compete in an all-pay auction for temporary agenda control. They provide conditions for policies to moderate over time. They do not address which connections form because they do not model politicians, implicitly treating them as homogeneous. Furthermore, they do not study persistent access, as groups vie for temporary access throughout policymaking as needed. In contrast, I study targeted access that is persistent and acquired before policymaking. There are several other differences. Here, bargaining ends when a proposal passes and groups do not compete for access.

Lobbying has been modeled in a variety of ways. I focus on lobbying that influences policy content. Specifically, I study lobbying as exchange in the spirit of Grossman and Helpman (1994), where interest groups provide resources to directly influence the content of policy proposals.\footnote{See Grossman and Helpman (2002) for an extensive overview of this setting, which they apply to campaign contributions.} The lobbying technology in this paper is closely related to that of Bils, Duggan and Judd (2017), which studies lobbying in a model of repeated elections.\footnote{See Martimort and Semenov (2008) and an extension in Acemoglu et al. (2013) for recent studies that use a similar approach to model lobbying.} There,
politician ideology exogenously determines access, and the interest group is always able to
lobby officeholders to which it has access. Here, I study whether interest groups want access.
Also, I study legislative policymaking, where policymaking is collaborative. Finally, I allow
for partial access, where the group is not guaranteed a chance to lobby.

Although I focus on lobbying to influence the agenda, other work studies lobbying to
influence legislative voting on a fixed agenda. First, there is a prominent literature on vote
buying in legislatures (Snyder Jr., 1991; Groseclose and Snyder, 1996; Banks, 2000; Dal Bó,
2007; Dekel et al., 2009). Many of these papers focus on either distributive policies or public
goods. Second, there are several models where groups influence votes by strategically pro-
viding information (Bennedsen and Feldmann, 2002; Jackson and Tan, 2013; Schnakenberg,

Scholars have studied access acquisition in informational lobbying environments (Austen-
Smith, 1995; Cotton, 2012, 2016). Closest to this paper is Schnakenberg (2017), where inter-
gest groups can buy access in a legislature. Groups want to influence a legislative vote over
exogenous policy proposals in a static setting. Access allows groups to provide information.
As in this paper, influencing a legislature is substantially different from influencing an exec-
utive. In contrast, I analyze a complete information setting where lobbying affects endogenous
policy proposals. Additionally, I study an environment where policymaking continues after
rejected proposals. Finally, groups can optimally forgo free access in this paper, which never

Finally, a key result of this paper is that interest groups prefer to forgo access to certain
more centrist legislators. The logic connects to moderation results in spatial models of
dynamic bargaining with an endogenous status quo (Baron, 1996; Zápal, 2014; Forand, 2014;
Buisseret and Bernhardt, 2017). There, legislators prefer to propose more centrist policies
to constrain future proposers in equilibrium. They forgo the full power of their current
agenda control to constrain the scale of policy changes by future proposers who may have
substantially different preferences. I study a different setting, as here policymaking ends
once a proposal passes. But the incentive to forgo access arises from the same desire to
constrain potential proposers who are ideologically distant.

Model of Legislative Bargaining with Lobbying

To study access, it is important to firmly understand the downstream effects through lob-
bying. Thus, I first present and analyze the legislative environment with access fixed exoge-

\footnote{Although Forand (2014) is cast as a model of elections, it can be interpreted as a spatial bargaining
model with an endogenous status quo and endogenous proposers.}
nously, having implicitly arisen from previous interest group efforts to create connections. With a handle on legislative behavior, I then take note of the determinants of lobbying expenditures, which will set the stage to analyze access acquisition.

In the model, legislators bargain to set a common policy. Throughout policymaking, ideological interest groups may receive opportunities to influence policy by providing favors. The logic for the main results can be illustrated in a streamlined setting with four legislators and one interest group. I provide several comments after the baseline model is in place.

There are four legislators: a left-partisan \( L \), a moderate \( M \), a right-partisan \( R \), and a generic legislator \( \ell \). The interest group is denoted \( g \). The policy space is \( X \subseteq \mathbb{R} \). Each legislator \( i \) is associated with ideal point \( \hat{x}_i \in X \). Similarly, \( g \)’s ideal point is \( \hat{x}_g \in X \). Throughout, I normalize \( \hat{x}_M = 0 \). Furthermore, I assume \( \hat{x}_L < 0 < \hat{x}_R \). To reflect that each partisan is staunchly committed in its favored direction, suppose \( -\hat{x}_L, \hat{x}_R > |q| \). This assumption is not crucial, but clarifies key tradeoffs.

Legislative bargaining occurs over an infinite horizon, with periods discrete and indexed by \( t \in \{1, 2, \ldots \} \). Let \( \rho_i \) denote the probability that legislator \( i \) is chosen to propose policy in any period \( t \). Then \( \rho = (\rho_{\ell}, \rho_L, \rho_M, \rho_R) \) denotes the distribution of legislator recognition probabilities, which sum to one. An important feature of the model is that the interest group, \( g \), has opportunities to influence legislator \( \ell \)’s policy proposals. Specifically, \( g \) has access to \( \ell \). The strength of \( g \)’s access is given by \( \alpha \in [0, 1] \). Access determines the probability that \( g \) can lobby \( \ell \), conditional on \( \ell \) begin recognized to propose. This technology reflects the standard view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). To reflect targeted access, \( g \) does not have access to legislators other than \( \ell \) in the baseline model. Additionally, \( g \)’s access is exogenously endowed. Later, to study how interest groups use contributions to buy access, I allow \( g \) to choose \( \alpha \).

In each legislative period \( t \), bargaining proceeds as follows. If no policy has passed before period \( t \), then legislator \( i \) is recognized as the period-\( t \) proposer with probability \( \rho_i > 0 \). The identity of the period-\( t \) proposer, \( i_t \), is publicly observed. If \( i_t \) is some legislator other than \( \ell \), then \( g \) is not active and \( i_t \) proposes any policy \( x_t \in X \). If \( \ell \) is the period-\( t \) proposer, then \( g \) can lobby \( \ell \) with probability \( \alpha \). If \( g \) lobbies, then \( g \) offers \( \ell \) a binding contract \((y_t, m_t)\) that consists of a policy \( y_t \in X \) and a transfer \( m_t \geq 0 \). After observing \( g \)’s offer, \( \ell \) decides whether to accept or reject. If \( \ell \) accepts, then she is committed to propose \( x_t = y_t \) and \( m_t \) transfers from \( g \) to \( \ell \). On the other hand, if \( \ell \) rejects, then she is free to propose any \( x_t \in X \) and \( g \) keeps \( m_t \). With probability \( 1 - \alpha \), the group cannot lobby in period \( t \). In this case, \( \ell \)

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\(^7\)See Appendix A for the full model.

\(^8\)I study a more general setting in the appendix.

\(^9\)Also see, e.g., Milbrath (1976); Hall and Wayman (1990); Hansen (1991); Grossman and Helpman (2002); Hall and Deardorff (2006); Gordon et al. (2007) and Powell (2014).
simply proposes any $x_t \in X$ and $g$ does not make an offer.

In each case, all legislators observe the period-$t$ proposal, $x_t$. Next, the moderate legislator, $M$, chooses whether to accept or reject the proposal. If $M$ accepts, then the proposal passes and bargaining ends with $x_t$ enacted in period $t$ and all subsequent periods. If $M$ rejects, then the status quo $q \in \mathbb{R}$ is enacted in period $t$ and bargaining proceeds to period $t + 1$. This approach aims to capture a more general setting where all legislators vote. Because policy is one-dimensional and $u$ is quadratic, $M$’s decision corresponds to the outcome of majority voting.

If $\ell$ is the period-$t$ proposer, $\ell$ accepts $g$’s offer $(y, m)$, and $x_t$ is the enacted policy in $t$, then $g$’s stage payoff is $u_g(x_t) - m$ and $\ell$’s stage payoff is $u_\ell(x_t) + m$. All players have quadratic policy utility and discount streams of stage utility by the common discount factor $\delta \in (0, 1)$. See Appendix A for explicit expressions of dynamic payoffs. Figure 1 illustrates the within-period interaction and accumulation of payoffs for a period in which $\ell$ proposes and $g$ can lobby. For a period in which $\ell$ does not propose or $g$ cannot lobby, the within-period interaction is analogous to the section of Figure 1 following from $\ell$ rejecting $g$’s offer.

Figure 1: A period in which the interest group can lobby

Model Discussion

I make several comments before proceeding to the analysis.

\footnote{Notice that $x_t = q$ if $y$ is not passed in period $t$.}
First, there are multiple interpretations for access, $\alpha$. One is personal connections possessed by $g$’s lobbyists that affect their chances of meeting with $\ell$ (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Cain and Drutman, 2014; Kang and You, 2015). Another is access gained using campaign contributions that $\ell$ receives from $g$ in a preceding, yet unmodeled, election.\(^{11}\) A third is $\ell$’s value from using policy proposals to appeal to her constituents. This value may affect her propensity to meet with lobbyists.

Second, $g$ has access to only one legislator. I relax this assumption in the appendix, but it reflects the idea that groups are unable to access some legislators due to exogenous factors. For example, regional interest groups may not be able to access legislators absent a geographic connection (Wright, 1989). Alternatively, voters in some districts may be strongly opposed to the group’s mission or tactics (Stratmann, 1992).

Third, I model access as $g$’s probability of being able to lobby when $\ell$ controls the agenda. Other ways of modeling lobbying capacity also produce tradeoffs for $g$. Qualitatively similar results hold if access is binary, or if $g$ chooses $\ell$’s marginal value of money.

Finally, I conclude by interpreting the lobbying technology. In the model, lobby offers are binding contracts that exchange resources for policy. In practice, interest groups spend substantial effort drafting legislation (Schlozman and Tierney, 1986) and frequently present legislators with model bills (Levy and Razin, 2013; Kroeger, 2016). In the model, this corresponds to the policy offer, $y$. In exchange, legislators may gain an inside track on future employment opportunities after they leave office (Diermeier et al., 2005). Moreover, legislators are freed to pursue other tasks such as constituent service and fundraising, in the spirit of Hall and Deardorff (2006). Finally, interest groups may also provide valuable political intelligence or write speeches to help sell policies to the legislator’s constituents and co-partisans (Schlozman and Tierney, 1983, 1986; Hall and Wayman, 1990; Wright, 1996). The group’s transfer, $m$, captures these benefits.

### Equilibrium Policies and Lobbying Activity

I study a selection of the model’s subgame perfect equilibria (SPE), applying standard refinements from the legislative bargaining literature. In particular, I focus on no-delay pure strategy stationary legislative lobbying equilibria.\(^{12}\) These are strategy profiles in which (i) $g$’s offers to $\ell$ are independent of previous play; (ii) $\ell$ accepts or rejects $g$’s offer based only on the terms of the offer, and $\ell$’s policy proposals in lieu of acceptance are independent of

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\(^{11}\)See, e.g., Langbein (1986); Romer and Snyder Jr. (1994); Kalla and Broockman (2015); Barber (2016); Grimmer and Powell (2016) and Fourinaies and Hall (2017) for evidence suggesting that many interest groups use campaign contributions to buy access.

\(^{12}\)See Appendix A for a formal definition of this equilibrium concept.
the preceding history; (iii) legislators other than \( \ell \) propose policy independent of preceding play; and (iv) \( M \)’s voting decision depends only on current proposal.

A strategy profile \( \sigma \) is *no-delay* if (i) it specifies that all legislators propose socially acceptable policy and (ii) the interest group’s policy offer is socially acceptable. Informally, a no-delay pure strategy stationary legislative lobbying equilibrium requires four conditions: (i) \( g \)’s policy offer is socially acceptable and \( g \) cannot profitably deviate to another offer; (ii) legislator \( \ell \) accepts a lobby offer if and only if she weakly prefers the offer over the alternative of making her own proposal; (iii) each legislator proposes socially acceptable policy and cannot profitably deviate to a different proposal conditional on not receiving a payment from \( g \); and (iv) \( M \) supports a policy if and only if she weakly prefers it relative to rejecting and extending bargaining.\(^{13}\)

Before proceeding, there are several noteworthy features of stationary legislative lobbying equilibrium. First, although players use straightforward behavioral rules, no player can profitably deviate to any other strategy. Second, \( g \) must make an offer in each period that \( \ell \) is the proposer and \( g \) can lobby. This requirement is innocuous, however, as \( g \) can effectively forgo lobbying by offering a contract composed of \( \ell \)’s default proposal and no payment. Third, \( \ell \) always accepts \( g \)’s offer when indifferent, but this restriction is without loss of generality. Finally, I focus on no-delay strategy profiles for convenience, as this restriction is inconsequential.\(^{14}\)

Proposition 1 provides three results. First, I show existence of a no-delay pure strategy stationary legislative lobbying equilibrium. Along the way, I obtain a sharp characterization of equilibrium behavior. Next, I show that a large class of more general equilibria are equivalent in a strong sense to the equilibria I study.\(^{15}\) Finally, I prove there is a unique equilibrium outcome distribution, by capitalizing on Cho and Duggan (2003). This property ensures that the extension to endogenous access does not require consequential equilibrium selection.

**Proposition 1.**

1. There exists a no-delay pure strategy stationary legislative lobbying equilibrium.

2. Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay pure strategy stationary legislative lobbying equilibrium.

\(^{13}\)Specifically, \( M \) uses *stage-undominated* voting strategies (Baron and Kalai, 1993).

\(^{14}\)See Appendix B for more details regarding these comments.

\(^{15}\)In Appendix B I define *mixed strategy stationary legislative lobbying equilibrium* and show that every such equilibrium is equivalent in outcome distribution to a no-delay pure strategy stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.
3. Every stationary legislative lobbying equilibrium has the same outcome distribution.

In light of Proposition 1, I refer to no-delay pure strategy stationary legislative lobbying equilibria as *equilibria* throughout the rest of the analysis. As is standard in the legislative bargaining literature, equilibria can be characterized by their *social acceptance set*, which is denoted \( A(\sigma) \) and corresponds to the set of policies that \( M \) votes to accept under the strategy profile \( \sigma \).\(^{16}\)

In an equilibrium \( \sigma \), the boundaries of \( A(\sigma) \) are the two policies that \( M \) is indifferent between approving and rejecting. Formally, the upper bound of \( A(\sigma) \), denoted \( x(\sigma) \), is the positive solution to

\[
u_M(x) = (1 - \delta)u_M(q) + \delta V_M(\sigma),
\]

where \( V_M(\sigma) \) denotes \( M \)'s continuation value under \( \sigma \). The acceptance set is \( A(\sigma) = [-x(\sigma), x(\sigma)] \), facilitating a sharp characterization of proposal strategies. In the hypothetical legislature illustrated in Figure 2, \( M \) proposes \( \hat{x}_M \) if recognized, legislator \( L \) proposes \( -x(\sigma) \), and \( R \) proposes \( x(\sigma) \). The partisans, \( L \) and \( R \), are thus constrained by \( M \)'s voting power because their respective ideal policies will not pass. In equilibrium, each compromises by proposing its favorite passable policy.

If legislator \( \ell \) is recognized to propose and does not accept an offer from group \( g \), either because \( g \) cannot lobby or because \( \ell \) rejects \( g \)'s offer, then \( \ell \) proposes \( z_\ell \), her favorite policy in \( A(\sigma) \). Notably, however, \( \ell \) never rejects \( g \)'s offers in equilibrium because \( g \) always makes an offer \( \ell \) accepts. In particular, \( g \)'s equilibrium lobby payment exactly satisfies \( \ell \)'s acceptance condition given \( g \)'s policy offer. Clearly, \( g \) is strictly worse off giving \( \ell \) a surplus transfer and will not do so in equilibrium. Additionally, \( g \) always offers policy that passes, that is \( y^* \in A(\sigma) \).

Formally, \( g \)'s equilibrium offer \((y_g, m_g)\) consists of the policy

\[
y_g = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - u_\ell(z_\ell)
\]

and transfer \( m_g = u_\ell(z_\ell) - u_\ell(y) \). Because \( u_\ell(z_\ell) \) does not depend on \( g \)'s offer,

\[
y_g = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y),
\]

which is unique and maximizes the joint surplus of \( g \) and \( \ell \), subject to the constraint that \( y_g \) passes.\(^{17}\) It is always feasible for \( g \) to offer \( \ell \)'s independent proposal, \( z_\ell \), with zero payment.

\(^{16}\)See, e.g., Cho and Duggan (2003) and Banks and Duggan (2006a).

\(^{17}\)Uniqueness follows because \( u_g \) and \( u_\ell \) are strictly concave, and \( A(\sigma) \) is compact and nonempty.
By $z_\ell \in A(\sigma)$, $g$ weakly prefers to make successful offers that $\ell$ accepts. For convenience, define $g$’s *unconstrained policy offer* as

$$\hat{y} = \arg \max_{y \in X} u_g(y) + u_\ell(y). \quad (4)$$

Because $u_g$ and $u_\ell$ are quadratic, $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$. Notice that $\hat{y}$ is solely a function of primitives. Furthermore, $\hat{y} \in A(\sigma)$ implies $y_g = \hat{y}$. Otherwise, strict concavity implies $y_g$ equals the boundary of $A(\sigma)$ closest to $\hat{y}$.

Figure 2 illustrates the equilibrium social acceptance set, $A(\sigma)$, for a hypothetical legislature, along with the corresponding equilibrium proposals. In general, $A(\sigma)$ is a closed interval symmetric about $\hat{x}_M$. Additionally, $y_g$ is skewed away from $\hat{x}_\ell$ towards $\hat{x}_g$.

The model, although complicated by lobbying, can be reinterpreted as a one-dimensional spatial bargaining environment with an additional legislator who has positive recognition probability equal to $\alpha \rho_\ell$ and an ideal point, $\hat{y}$, located between $\hat{x}_g$ and $\hat{x}_\ell$. After expanding the legislature to add this additional proposer representing the effect of $g$’s lobbying, legislators propose bills that are closest to their ideal point among those that are acceptable. Uniqueness follows from applying Cho and Duggan (2003) to this fictitious enlarged legislature.

Figure 2: Equilibrium characterization

Figure 2 depicts the characterization of equilibrium policy proposals. Arrows point from legislator ideal points to proposals. The bold interval is the acceptance set, $A(\sigma)$. If legislator $\ell$ is recognized, then she proposes the acceptable policy closest to $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$ with probability $\alpha$ and otherwise proposes the acceptable policy closest to $\hat{x}_\ell$.

The preceding characterization of equilibrium policy proposals implies that $M$’s continuation value from rejecting a proposal in equilibrium is

$$V_M(\sigma) = \rho_M u_M(\hat{x}_M) + \alpha \rho_\ell u_M(y_g) + (1 - \alpha) \rho_\ell u_M(z_\ell) + \rho_L u_M(-\bar{x}(\sigma)) + \rho_R u_M(\bar{x}(\sigma)). \quad (5)$$

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Using (5) and $u_M(\hat{x}_M) = 0$, we can recursively express

$$
\bar{x}(\sigma) = \left( \frac{(1 - \delta)u_M(q) + \delta\left(\alpha \rho_\ell u_M(y_\ell) + (1 - \alpha) \rho_\ell u_M(z_\ell)\right)}{1 - \delta(\rho_L + \rho_R)} \right)^{\frac{1}{2}}.
$$

It is the unique $x \geq \hat{x}_M$ making $M$ indifferent between accepting and rejecting. Inspection of (6) shows that $\alpha$ can affect the boundaries of $A(\sigma)$ and, consequently, the policies proposed by the partisans $L$ and $R$. This indirect effect requires dynamic concerns, $\delta > 0$, and plays a key role in the analysis.

Before proceeding, I define useful terminology for the respective ideologies of $g$ and $\ell$.

**Definition 1.** Legislator $\ell$ is *extremist* if $\hat{x}_\ell \notin \text{int}A(\sigma)$. Analogously, the interest group, $g$, is extremist if $\hat{x}_g \notin \text{int}A(\sigma)$. Otherwise, say that $\ell$ or $g$ is *centrist*.

**Definition 2.** Legislator $\ell$ and the interest group, $g$, are *aligned* if their ideal points are on the same side of $\hat{x}_M$, e.g. $\hat{x}_\ell, \hat{x}_g \leq \hat{x}_M$. Conversely, $\ell$ and $g$ are *opposed* if they are not aligned.

An initial observation regarding equilibrium behavior is that two conditions are necessary for non-trivial lobbying. First, $\alpha_\ell > 0$, as otherwise $g$ never lobbies $\ell$. Second, $g$ and $\ell$ cannot be aligned extremists. In this case, $g$ cannot profitably lobby to improve upon $\ell$’s independent policy proposal.

**Who do Interest Groups Want to Access?**

To study the connections that form, I now allow the group, $g$, to choose $\alpha$, its access to legislator $\ell$. I focus on a one-time choice of perfectly persistent access. This exercise reveals the key tradeoffs of durable access. I discuss other arrangements later. Substantively, this analysis can be interpreted as allowing $g$ to gain access before policymaking by making campaign contributions or hiring connected lobbyists. Proposition 1 ensures that $g$’s choice of $\alpha$ pins down equilibrium play in the legislature.

I abstract from the particular mapping that determines access by allowing the interest group to freely choose $\alpha$. In practice, this mapping almost certainly depends on idiosyncratic factors such as the connections of the interest group’s lobbyists (Blanes i Vidal et al., 2012; Bertrand et al., 2014; Kang and You, 2015), constituent interests within the legislator’s district (Stratmann, 1992), or the number of voters affiliated with the interest group (Bom-
bardini and Trebbi, 2011).\textsuperscript{18} The following results are driven purely by policy considerations and hold even if acquiring access is free.

Propositions 2 and 3 fix $\hat{x}_g$ and study whether $g$ wants strictly positive access, as a function of $\hat{x}_\ell$. Importantly, $\hat{x}_\ell$ and $\alpha$ can affect equilibrium legislative behavior by changing policy proposals and the social acceptance set. Propositions 2 and 3 are distinguished by whether or not $g$ is extremist if $\hat{x}_g = \hat{x}_\ell$.\textsuperscript{19} This distinction is important. It determines whether $g$ is centrist for all $\alpha \in [0, 1]$ if $\hat{x}_g$ and $\hat{x}_\ell$ are sufficiently close. Furthermore, it has a simple partitional characterization. In particular, define

$$\bar{x} = \left( \frac{-\delta u_M(q)}{1 - \delta (\rho_L + \rho_R + \rho_e)} \right)^{\frac{1}{2}},$$

which satisfies $\bar{x} \geq \bar{x}(\sigma) > 0$. Then $g$ is centrist when $\hat{x}_g = \hat{x}_\ell$ if and only if $\hat{x}_g \in (-\bar{x}, \bar{x})$.\textsuperscript{20}

**Definition 3.** The interest group, $g$, is a non-ideologue if $\hat{x}_g \in (-\bar{x}, \bar{x})$. Otherwise, $g$ is an ideologue.

Notably, $g$’s status as an ideologue or non-ideologue does not vary with $\alpha$ or $\hat{x}_\ell$. But $g$’s status as a extremist or centrist can change with these features. For example, suppose $g$ is a non-ideologue. Then $g$ is centrist if $\ell$ is sufficiently extreme or $\alpha$ is sufficiently large. Yet, $g$ is extremist if $\alpha$ is low and $\ell$ is sufficiently centrist. If $g$ is an ideologue, however, then it is always extremist.

**Non-ideologue Interest Groups**

Proposition 2 shows that non-ideologue interest groups can optimally forgo access. Recall that $g$ and $\ell$ are aligned if $\hat{x}_g$ and $\hat{x}_\ell$ are on the same side of $\hat{x}_M$.

**Proposition 2.** Suppose the interest group, $g$, satisfies $\hat{x}_g \in (0, \bar{x})$. There exist $x', x''$ satisfying $0 < x' < \hat{x}_g < \bar{x} < x''$ such that:

(i) if legislator $\ell$ satisfies $\hat{x}_\ell \in (x', \hat{x}_g)$, then $g$ forgoes access;

(ii) if $\hat{x}_\ell \in [\hat{x}_g, x'')$, then $g$ acquires access;

(iii) if $\hat{x}_\ell \geq x''$, then $g$ is indifferent over access.

An analogous result holds if $\hat{x}_g \in (-\bar{x}, 0)$.

\textsuperscript{18}For example, La Raja and Schaffner (2015) emphasize that contributions do not translate into influence in the same fashion for different pairs of interest groups and legislators.

\textsuperscript{19}Recall that $g$ is extremist if $\hat{x}_g \notin \text{int} A(\sigma)$ and centrist otherwise, and similarly for $\ell$.

\textsuperscript{20}See Lemma 2 in Appendix A for a formal statement and proof.
The following discussion explains the logic for Proposition 2 using the case illustrated in Figure 3, where \( g \) is a right-leaning non-ideologue. The symmetric case is analogous.

**Figure 3:** Does a non-ideologue interest group want access?

![Figure 3](image)

Figure 3 illustrates Proposition 2 for a right-leaning interest group, \( g \). If \( \hat{x}_{\ell} \in (x', \hat{x}_g) \), then \( g \) forgoes access, i.e. \( \alpha = 0 \). If \( \hat{x}_{\ell} \in (\hat{x}_g, x'') \), then \( \alpha > 0 \). If \( \hat{x}_{\ell} \geq x'' \), then \( g \) is indifferent. If \( \hat{x}_{\ell} \) is sufficiently centrist, \( \hat{x}_m \in [\hat{x}_m, x'] \), then \( g \)'s preference is ambiguous and depends on parameters.

By Proposition 2, \( g \) may prefer to forgo access if \( \ell \) is more centrist than \( g \). Concretely, \( \alpha \) affects \( g \)'s ex ante expected utility in two ways. First, it changes the probability that \( g \) enjoys the surplus from lobbying \( \ell \). Second, it changes proposals by \( L \) and \( R \). Specifically, if \( g \) is more extreme than \( \ell \), then higher \( \alpha \) increases the probability that \( \ell \) proposes \( y \) at the expense of \( z_{\ell} \). Thus, in this case \( M \)'s continuation value from rejecting policies decreases with \( \alpha \) because \( M \) prefers \( z_{\ell} \) to \( y \). As a result, \( M \) passes more extreme policy proposals. Because \( L \) and \( R \) are always partisan, their legislative proposals are thus more extreme. Proposition 2 simply establishes existence of \( x' < \hat{x}_g \) such that if \( \hat{x}_{\ell} \in (x', \hat{x}_g) \), then \( g \)'s expected cost from enabling \( L \) and \( R \) to pass more extreme policy outweighs \( g \)'s expected benefit from being more likely to lobby \( \ell \). But Proposition 2 leaves open the possibility that \( g \) wants access if \( \ell \) is sufficiently centrist, i.e. \( \hat{x}_{\ell} \in [\hat{x}_M, x'] \). In this case, \( g \) receives a larger benefit from lobbying and it may want access. Figure 4 illustrates this logic.

In light of Proposition 2, \( g \) may not want access to \( \ell \) when they are ideologically close. But this does not imply that \( g \) never wants access to \( \ell \) if they are ideologically similar. Instead, \( g \) wants access if \( \ell \) is more extreme, but not too extreme, \( \hat{x}_{\ell} \in (\hat{x}_g, x'') \). In this case, policy extremism decreases with \( \alpha \). Consequently, \( g \) strictly benefits from increasing \( \alpha \) because it (i) increases \( g \)'s lobbying opportunities and (ii) further constrains partisan legislators to propose policies that are more favorable to \( g \). Figure 5 depicts the logic.
Figure 4: Forgoing access to a more centrist legislator

Figure 4 illustrates why the group, $g$, forgoes access to legislator $\ell$ if $\hat{x}_g \in (-\pi, \pi)$ and $\hat{x}_\ell \in (x', \hat{x}_g)$. Part (a) displays equilibrium behavior if $g$ has no access, i.e. $\alpha = 0$. Part (b) illustrates behavior if $\alpha > 0$. Increasing access has two effects: (i) lobbying is more likely, and (ii) $M$’s expectations worsen. Effect (ii) expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If $\hat{x}_g$ and $\hat{x}_\ell$ are close, then effect (ii) dominates and $g$’s forgoes access.

Figure 5: Seeking access to a more extreme legislator

Figure 5 illustrates why the group, $g$, prefers strictly positive access, $\alpha > 0$, if $\hat{x}_g / \in (-\pi, \pi)$ and $\hat{x}_\ell \in (\hat{x}_g, x'')$. Part (a) displays equilibrium behavior if $\alpha = 0$. Part (b) illustrates $\alpha > 0$. Access has two effects: (i) $g$’s probability of lobbying increases, and (ii) $M$’s expectations improve. Effect (ii) causes the acceptance set to shrink, as shown in (b). Thus, partisans propose more centrist policy. Both effects improve $g$’s expected payoff.

Finally, if $\ell$ and $g$ are aligned, and $\ell$ is sufficiently extreme, then $g$ cannot profitably lobby to change $\ell$’s policy proposal, as illustrated in Figure 6 for $\hat{x}_\ell > x''$. Consequently, $g$ is indifferent over $\alpha$. 
Figure 6 illustrates why the group, $g$, is indifferent over access if it is aligned with legislator $\ell$ and $\ell$ is sufficiently extreme. Lobbying is inconsequential because both proposals are constrained by the acceptance set.

**Ideologue Interest Groups**

Next, I study ideologue interest groups. As noted previously, if $\ell$ and $g$ are aligned ideologues, then lobbying is inconsequential and $g$ is indifferent over $\alpha$. Otherwise, if $\ell$ is a non-ideologue, or $\ell$ and $g$ are opposed ideologues, then lobbying is consequential. Yet, $g$’s preferences over access are ambiguous in general. Specifically, if $\alpha$ changes the acceptance set, then either $L$ or $R$’s equilibrium proposal becomes worse for $g$ while the other’s proposal becomes more favorable to $g$. The specific balance of partisan proposal power determines which effect dominates. Thus, without restricting the balance of partisan proposal power it is difficult to draw strong conclusions about an ideologue group’s preference over access.

Accordingly, I study a substantively reasonable restriction on relative partisan power that also permits a sharp characterization of an ideologue interest group’s preferences over access. In U.S. legislatures, the majority party typically exercises substantial control over committee assignments and committee leadership positions (Cox and McCubbins, 2005, 2007). To reflect this observation, I restrict proposal power to one side of the moderate legislator.

**Definition 4.** The legislature exhibits *minority-party agenda exclusion* if one of the partisans, $L$ or $R$, has no agenda setting power, while the other, *majority*, partisan has positive recognition probability.

This restriction on the agenda power distribution reflects the widespread belief that majority parties carefully allocate agenda setting power in the U.S. Yet, the model also consistent with empirical work suggesting that individual legislators possess some degree of freedom from their party and thus can be influenced by interest groups (Fouirnaies, 2017).

**Definition 5.** Legislator $\ell$ is *majority-leaning* if she is aligned with the majority partisan.

Proposition 3 establishes that if $g$ is a majority-leaning ideologue under majority party control, then it is indifferent over $\alpha$ if $\ell$ is also a majority-leaning ideologue. But if $\ell$ is a
majority-leaning non-ideologue, then $g$ wants access. The result focuses on majority-leaning legislators because minority-leaning legislators do not have proposal power and thus it is immediate that $g$ is indifferent.

**Proposition 3.** Assume there is minority-party agenda exclusion and the interest group, $g$, is a majority-leaning ideologue.

(i) If legislator $\ell$ is a majority-leaning ideologue, then $g$ is indifferent over access.

(ii) If $\ell$ is a majority-leaning non-ideologue, then $g$ acquires access.

Part (i) of Proposition 3 follows because $g$ cannot profitably lobby to change $\ell$’s proposal. Part (ii) follows because access provides two benefits for $g$ under minority-party exclusion. First, lobbying is profitable and greater $\alpha$ increases $g$’s chances of enjoying that profit. Second, greater $\alpha$ diminishes $M$’s expectations about future policy, and thus expands the acceptance set in this case. Partisans can pass more extreme policy. Yet, minority-party partisans are unable to propose policy under minority-party agenda exclusion. Therefore $g$ benefits from emboldening aligned partisan legislators without risking more extreme proposals by opposing partisans.

**Table 1: Who acquires access to whom?**

<table>
<thead>
<tr>
<th>Group</th>
<th>Centrist</th>
<th>Nearby Centrist</th>
<th>Nearby Extreme</th>
<th>Extremist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Ideologue</td>
<td>$\times$ or $\checkmark$</td>
<td>$\times$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td>Ideologue</td>
<td>$\times$ or $\checkmark$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

Table 1 displays whether a group seeks access to a particular legislator, as a function of relative ideology. The group seeks access in pairs marked with $\checkmark$ and forgoes access in pairs marked with $\times$.

**Access and the Welfare of Legislators and Society**

*Legislator Welfare:* From a legislator’s perspective, access can be good or bad.\(^{21}\) These effects arise entirely from changes in expected extremism. If the legislator is lobbied, then she is compensated for any policy loss. Thus, access affects legislators only through the indirect effect on partisan proposals. The relative extremism of the legislator and interest group determine whether the legislator is better off. For example, certain legislators improve their

\(^{21}\)This is also true in, e.g., Schnakenberg (2017).
expected welfare by giving access to more centrist groups. Here, access is mutually desirable because it acts as a commitment device on legislators proposals. In contrast, legislators are always weakly worse off giving access to more extreme groups with whom they are aligned because extremism increases.

Social Welfare: To measure social welfare, I use $M$’s expected payoff. This approach is appropriate if the median citizen is close to $\hat{x}_M$, so the legislature effectively represents the ideological distribution of the unmodeled citizenry. Then $M$’s expected payoff corresponds to majority welfare in this ordered setting (Banks and Duggan, 2006b).

Connections shifting policy towards $\hat{x}_M$ also reduce expected extremism, and vice versa. Social welfare thus improves if groups access more extreme legislators and decreases otherwise. Therefore Figure 1 has immediate welfare implications for any aligned group-legislator pair. In some cases, the group does not acquire access and thus does not effect welfare. Highlighting this possibility, and the conditions that produce it, is a key benefit of the formal analysis.

Willingness to Acquire Access

Thus far, the analysis considers whether groups want access. What about contribution amounts? I provide two results in this direction. First, and consistent with a large body of empirical work, I demonstrate that groups are willing to pay more for access to legislators with greater proposal power. Second, I show under broad conditions that ideologically distant groups are willing to pay more to access a given legislator.

The results study $g$’s willingness to pay for access (WTP). Ideally, we would characterize $g$’s optimal amount of access and compare the cost of that access under different conditions. This task requires specifying a cost function for access. Instead of restricting this class of functions, I focus on $g$’s WTP for access.\footnote{See, e.g., Denzau and Munger (1986) and Hall and Deardorff (2006) for previous work on access-seeking campaign contributions studying willingness to pay.}

Proposition 4 shows that interest groups are willing to pay more for access to legislators with greater agenda power.

Proposition 4. All else equal, an interest group’s willingness to pay for $\alpha$ access weakly increases with the targeted legislator’s proposal power.

Proposition 4 does not depend on the respective ideologies of the legislator and interest group. Furthermore, it does not require majority party control. Proposal power amplifies the marginal benefit of access by increasing the probability that $g$ can extract surplus via lobbying. This increases the value of additional access. On the other hand, greater proposal
power also increases how sensitive the acceptance set is to increases in access, which may help or harm the interest group. The cumulative effect is proportional to the legislator’s recognition probability if $g$’s WTP is strictly positive. Thus, if the group is willing to pay for access, then it is amplified by greater proposal power.

Proposition 4 suggests that groups will pay a higher price to access more powerful legislators. This implication fits the empirical regularity that legislators on important and relevant committees, especially committee chairmen, attract more contributions (Ainsworth, 2002; Grimmer and Powell, 2016; Berry and Fowler, 2018; Fouirnaies, 2017).

Next, I analyze how $g$’s ideology affects its willingness to buy access to a majority-leaning legislator under majority party control. Proposition 5 fixes legislator $\ell$’s ideology and analyzes $g$’s willingness to pay for increasing its access from zero, which I refer to as $g$’s willingness to acquire access (WTA). Under broad conditions, $g$’s WTA weakly increases as its ideology diverges from legislator $\ell$’s ideology in either direction.

**Proposition 5.** Suppose there is minority-party agenda exclusion and legislator $\ell$ is majority-leaning. If either (i) the interest group, $g$, is more centrist than $\ell$, or (ii) $g$ is majority-leaning and more extreme than $\ell$, then $g$’s willingness to acquire access weakly increases as $g$ becomes less ideologically similar to $\ell$.

I discuss the logic for Proposition 5 using the case with right-party control, as illustrated in Figure 7. The first part assumes that $g$ is more centrist than $\ell$. Under these conditions, $g$’s WTA decreases as $|\hat{x}_g - \hat{x}_\ell|$ shrinks for two reasons. First, $g$’s lobbying surplus shrinks. It has less to gain from increasing its lobbying chances. Second, $g$’s access does not force majority-party partisans to moderate their policy proposals as much. Thus, $g$ gains less from inducing partisan moderation. These effects decrease $g$’s WTA as $\hat{x}_g$ shifts towards $\hat{x}_\ell$. 
The logic for the second part of Proposition 5 is best described in two cases.

First, suppose \( g \) is partisan when \( \alpha = 0 \). In Figure 7, this corresponds to \( \hat{x}_g \geq \pi_0 \). If \( \ell \) is centrist, as pictured in Figure 7, then \( g \)’s WTA decreases as \( \hat{x}_g \) shifts towards \( \hat{x}_\ell \) for reasons analogous to those described above: (i) \( g \)’s lobbying surplus decreases and (ii) \( g \)’s benefit from inciting more extreme partisan proposals also decreases. If \( \ell \) is partisan, which corresponds to \( \hat{x}_\ell \geq \pi_0 \) in Figure 7, then \( g \)’s lobbying is inconsequential. Therefore its willingness to acquire access is zero and thus constant as \( \hat{x}_g \) approaches \( \hat{x}_\ell \).

Second, suppose \( g \) is centrist when \( \alpha = 0 \), which corresponds to \( \hat{x}_g \in (\hat{x}_\ell, \pi_0) \) in Figure 9. Access now has competing effects. By the same logic as Proposition 2, \( g \) does not want access if \( \hat{x}_g \) is close to \( \hat{x}_\ell \). Yet, if \( g \) is sufficiently more extreme than \( \ell \) so that it wants access, then its WTA increases as \( \hat{x}_g \) moves away from \( \hat{x}_\ell \). Specifically, whenever \( g \)’s WTA is positive, lobbying surplus grows faster than the loss from inciting more extreme partisan proposals. Altogether, \( g \)’s WTA is zero if \( \hat{x}_g \) is close enough to \( \hat{x}_\ell \), and then it increases in \( \hat{x}_g \) once \( g \) is sufficiently more extreme than \( \ell \), as depicted in Figure 7.

An additional observation is that if \( g \)’s willingness to acquire access to \( \ell \) is zero, then \( g \) is not willing to pay for any amount of access. Thus, Proposition 5 implies that a majority-leaning group does not buy access if it is slightly more ideologically extreme than \( \ell \), mirroring Proposition 2.\(^{23}\)

\(^{23}\)See Lemma 7 in Appendix A for more details.
Persistent vs. Short-term Access

Thus far, I have studied persistent access. Such access has direct and indirect effects on the interest group’s ex ante welfare. The direct effect, opportunities to lobby, always benefits the group. The indirect effect, changing partisan proposals, can be good or bad. Under certain conditions, adverse indirect effects dominate, causing groups to forgo access. Any positive degree of access persistence produces these tradeoffs.

Temporary access can avoid indirect effects and produce universal access seeking. For example, suppose groups choose access once and it lasts one period. Then access today does not affect expectations about future policymaking. In turn, the acceptance set, and thus partisan proposals, do not change. The group always wants immediate, one-shot access because it provides only a direct benefit. Alternatively, suppose the group can set access each period. Then it chooses full access every period in every stationary equilibrium.

Temporary access thus encourages access-seeking. Yet, some groups prefer to commit to one period of immediate access followed by no future access. For example, consider the stationary equilibria mentioned above. Groups choose full access every period. Expected payoffs are thus equal to those from persistent full access. The main analysis implies that some groups would rather commit to forgo access. But these groups most prefer immediate, one-shot access with no chance for later access. This arrangement has a direct benefit without any indirect cost, as noted above. Thus, one of two possibilities must hold for such groups to pursue access. Either (i) access is temporary and commitment is impossible, or (ii) groups can contract against future access.

In sum, the main analysis distinguishes groups that covet access from those less inclined. The preceding discussion has focused on less access-oriented groups. But recall that some groups enjoy the indirect effect of persistent access. Such groups always want access, regardless of its durability or contractability. Moreover, legislative considerations increase their desire for access.

Multiple Groups and Conceding Access

The main analysis abstracts from groups competing over access. I take this approach to isolate how legislative considerations affect access-seeking. Interest group competition has been studied elsewhere (Chamon and Kaplan, 2013) and a full analysis is outside the scope of this paper. Yet, the analysis offers several insights about when competition is unlikely.

In many policy areas, interest groups are often unopposed (Baumgartner and Leech,
Where are the competing interests? Existing explanations for low competition include collective action problems, free-riding incentives, and entry costs. The analysis here sheds further light on when these competitive voids can arise. Legislative considerations can produce strong anti-competitive incentives.

To fix ideas, say that interest group $g$ concedes access to group $g'$ if $g$ strictly prefers letting $g'$ choose access rather than choosing access itself. Policy reasons alone can lead some groups to concede durable access. Specifically, groups may prefer to concede to more centrist groups even if both access and lobbying are free. This preference can arise regardless of whether the group wants access in isolation.

For example, consider a non-ideologue group $g$ contemplating access to a slightly more centrist legislator, $\ell$. By Proposition 2, $g$ does not want access to $\ell$. Furthermore, if there is another group $g'$ slightly more centrist than $\ell$, then $g$ strictly prefers to concede access to $g'$. Why? Group $g$ forgos access to avoid increasing expected extremism. But $g$ is even better off reducing expected extremism. Because $g'$ is more centrist than $\ell$, it seeks access by Proposition 2. This access reduces expected extremism. Thus, if $g'$ is not too far from $\ell$, then $g$ concedes access to $g'$. In this case, conceding access reduces expected extremism beyond what groups can achieve on their own by forgoing access.

The preceding example reveals that legislative forces can discourage competition. Conceding to more centrist groups can be attractive because it allows otherwise unobtainable moderation. As discussed, groups may even concede access to groups on the opposite side of a given legislator.

**Effects of Legislative Conditions on Expenditures**

Having studied access acquisition, I now characterize how equilibrium lobbying expenditures vary with legislative features. In general, equilibrium lobbying expenditures weakly increase as the acceptance set expands. Thus, the boundary characterization in (6) has direct implications for expenditures. I begin by cataloging the relevant legislative features and describing their effects on the acceptance set.

The first legislative feature I study is the distribution of agenda power. Given a distribution, $\rho$, and $g$’s access, $\alpha$, let the moderate legislator’s unconstrained extremism lottery be the lottery that puts probability $\alpha\rho_\ell$ on $|\hat{x}_M - \hat{y}|$, probability $\rho_\ell(1 - \alpha)$ on $|\hat{x}_M - \hat{x}_\ell|$, and probability $\rho_j$ on $|\hat{x}_M - \hat{x}_j|$ for each legislator $j \neq \ell$. Thus, the outcomes of an unconstrained extremism lottery are measured in terms of absolute distance between each player’s ideal proposal and $\hat{x}_M$.

Say that legislative extremism under $(\rho', \alpha')$ is higher than $(\rho, \alpha)$ if $M$’s unconstrained
extremism lottery associated with \((\rho', \alpha')\) first order stochastically dominates the lottery induced by \((\rho, \alpha)\).\(^{25}\) For example, legislative extremism increases if proposal power shifts away from \(M\) to other legislators. Increasing legislative extremism worsens \(M\)’s expectation about future policy because extreme policy proposals become more likely, without an offsetting increase in the chance of moderate policy proposals. In turn, \(M\) is willing to accept more extreme policy proposals and the acceptance set expands.

The second legislative feature is the location of the status quo, \(q\). More extreme status quo lower \(M\)’s expectations about future policy, causing the acceptance set to expand. Roughly, \(M\)’s expectations worsen because she is more averse to enduring the status quo until a new policy passes.

Third is legislator patience, \(\delta\). As \(\delta\) increases, \(M\) is less bothered by enduring the status quo. Thus, greater patience shrinks the acceptance set.

The last feature is \(g\)’s access, \(\alpha\). Greater access causes \(M\) to anticipate more frequent lobbying by \(g\). Thus, \(\alpha\)’s effect on the acceptance set depends on \(g\) and \(\ell\)’s relative ideology. If \(g\) is more extreme, then acceptance set expands because \(M\) anticipates that \(\ell\) proposes less favorable policy more often. The relationship flips if \(g\) is more centrist.

As noted above, expanding the acceptance set weakly increases ex post lobbying payments. The preceding observations thus characterize how expenditures vary. Propositions 6 and 1 collect the results.

**Proposition 6.** The interest group’s equilibrium lobbying expenditures weakly increase as either (i) legislative extremism increases, holding constant \(\hat{x}_g\) and \(\hat{x}_\ell\); (ii) the status quo policy becomes more extreme; or (iii) legislator patience decreases.

Given a group-legislator pair, the legislative extremism implication in Proposition 6 yields an immediate corollary on the relationship between access and lobbying expenditures.

**Corollary 1.** Suppose the interest group, \(g\), is aligned with legislator \(\ell\). If \(g\) is more extreme than \(\ell\), then equilibrium lobbying expenditures weakly increase with access. Otherwise, equilibrium lobbying expenditures weakly decrease with access.

Recall that \(g\)’s equilibrium transfer to \(\ell\) is \(m = u_\ell(z_\ell) - u_\ell(y)\). Therefore \(g\)’s equilibrium lobbying expenditures increase if either (i) its equilibrium policy offer becomes worse for \(\ell\) or (ii) \(\ell\) is able to pass more favorable policy after rejecting \(g\)’s overtures. Thus, there are two ways that a larger acceptance set can increase \(g\)’s lobbying expenditures: (i) more slack for \(g\) to shift \(\ell\)’s proposal, or (ii) a better outside option for \(\ell\).

\(^{25}\)In this context, the unconstrained extremism lottery \((\rho', \alpha')\) first order stochastically dominates another unconstrained extremism lottery \((\rho, \alpha)\) if: (i) for all \(x \in X\), \((\rho', \alpha')\) puts weakly greater probability on \(x'\) such that \(|x'| \geq |x|\) and (ii) for some \(x \in X\), \((\rho', \alpha')\) puts strictly greater probability on \(x'\) such that \(|x'| \geq |x|\).
First, if \( g \) is partisan and \( M \)'s acceptance constraint binds at \( g \)'s equilibrium policy offer, then greater legislative extremism gives \( g \) additional slack to lobby \( \ell \) to propose more extreme policy. Consequently, \( g \)'s lobbying expenditures increase because its policy offer is worse for \( \ell \). Figure 8 displays this case. If \( g \) and \( \ell \) are aligned extremists, then \( z_\ell = y \) and \( g \)'s lobbying expenditures are constant in legislative extremism.

Second, if \( \ell \) is partisan, then increasing legislative extremism improves \( \ell \)'s expected dynamic payoff from rejecting \( g \)'s offer because \( z_\ell \) is at the boundary of \( A(\sigma) \) closest to \( \hat{x}_\ell \). This boundary shifts towards \( \hat{x}_\ell \) as legislative extremism increases, which improves \( \ell \)'s outside option and thus forces \( g \) to provide a larger transfer to successfully lobby \( \ell \) away from \( z_\ell \). If \( \ell \) is not too extreme, and \( g \) is aligned with \( \ell \) and also extremist, then \( y \neq z_\ell \). Increasing legislative extremism makes it more expensive for \( g \) to lobby \( \ell \), even though \( g \)'s policy offer does not change, because \( \ell \) is better off from rejecting \( g \)'s overtures. Figure 9 illustrates.

Next, I state two corollaries of Proposition 6 to demonstrate how substantively meaningful features of the model affect extremism and, in turn, lobbying expenditures. First, Corollary 2 establishes that lobbying expenditures grow if \( M \) loses proposal power. Legislative extremism weakly increases and the acceptance set grows. Substantively, this result suggests that lower centrist agenda setting power encourages more vigorous lobbying.

**Corollary 2.** If proposal power transfers away from the moderate legislator, then the interest group's equilibrium lobbying expenditures weakly increase.

Corollary 3 states that lobbying expenditures grow weakly as either of the partisans, \( L \) or \( R \), shifts away from \( M \). Legislative extremism increases as a result of this change because
these legislators propose weakly more extreme policy. This result suggests that groups spend more on lobbying in legislatures that are more polarized, in the colloquial sense of having greater ideological spread among legislators.

**Corollary 3.** If either the right-leaning legislator or the left-leaning legislator move farther away from the moderate legislator ideologically, then the interest group’s equilibrium lobbying expenditures weakly increase.

**Conclusion**

I study which legislators and interest groups form connections that facilitate lobbying. To do so, I analyze a model where interest groups choose access before policymaking. Access allows groups to influence policy proposals by lobbying. The model provides a tractable window to explore how access-seeking depend on the larger legislative context.

Interest groups weigh various institutional and political factors when deciding whether to pursue access. Does greater access increase or decrease policy extremism in the legislature? Is the targeted legislator likely to have much control over policymaking? Are partisans likely to draft policy? I refine our understanding of how groups weigh these questions.

I first highlight conditions under which lobbying increases or decreases policy extremism. Then, I show that groups avoid access to particular legislators under broad conditions. Policy considerations drive this result, which arises from a neglected consequence of access. The
prospect of lobbying can spill over and affect policies proposed by other legislators.

The analysis has implications for campaign finance, *revolving door* hiring, and lobbying expenditures. First, which legislators do access-seeking interest groups direct campaign contributions towards? And whose associates do they hire through the revolving door? Second, which groups lobby which legislators? Third, what can lobbying expenditures tell us about access. Finally, why do many groups contribute so little (Tullock, 1972; Ansolabehere et al., 2003)? Distinguishing between access and lobbying is key for these implications. I now elaborate on each.

First, analyzing endogenous access suggests which connections are likely to arise. In practice, campaign contributions and revolving door hiring are prominent channels for access. Propositions 2 and 3 thus have implications both for who groups contribute to and whose staffers they hire. Data exist for both. Using revolving-door hiring data is likely a better starting point because most measures of group and legislator ideology use contributions to locate the actors on a common scale. Given group ideology, the model suggests a curvilinear, or even multimodal, relationship between hiring/contributions and legislator ideology. An especially strong prediction is that groups never access very extreme legislators.

Second, and in line with the widespread view, access is necessary for lobbying in model. Thus, Table 1 immediately suggests qualitative predictions about who lobbies whom. In some instances, detailed lobbying data specify targeted legislators. Another possibility is data on “points of contact,” which may provide information about who groups target.

Third, the relationship between observed expenditures and access is conditional on relative ideology. Thus, empirical work should control for relative ideology when evaluating the connection between access and lobbying expenditures. Otherwise, offsetting observations obscure a meaningful effect. Related, recovering a negative relationship between access and lobbying expenditures for centrist groups targeting extreme legislators need not imply groups get less for their money. Instead, expenditures may decline because the targeted legislator’s outside option is worse. Additionally, *ceteris paribus* changes in lobbying expenditures can suggest changes in access amounts. With information about relative ideology, we can infer the direction of the change.

Finally, the analysis speaks to Tullock’s puzzle: many groups do not contribute at all, and those that do rarely reach legal limits (Tullock, 1972). Some view this empirical regularity as evidence that either contributions are not valuable, or donors are unsophisticated Ansolabehere et al. (2003). Yet, previous work has shown that strategic forces can lead sophisticated groups to contribute small amounts (Chamon and Kaplan, 2013). I provide a new strategic mechanism for such behavior, legislative considerations, which can reduce contributions by sophisticated groups precisely because they are valuable for gaining access.
Appendix A

Model

I prove the main results in a more general version of the model, relaxing restrictions on the number of legislators and interest groups. There are three disjoint sets of players: voting legislators, $N^V$; committee members, $N^L$; and interest groups, $N^G$. Let $N = N^V \cup N^L \cup N^G$. There are a finite and odd number of voters, denoted $n^v$; the number of committee members in $N^L$ is $n^L \geq 3$, finite and odd; and there are $n^G \leq n^L$ interest groups. Throughout, I refer to voting legislators as voters and denote them by $i$. To align with the main text, let $M$ denote the median voter. I denote committee members by $\ell$ and interest groups by $g$.

Each $\ell \in N^L$ is associated with only one group, denoted $g_\ell$. Groups can have access to multiple legislators. Let $N^L_g$ denote the set of $\ell \in N^L$ to whom $g$ has access. Let $\alpha_\ell \in [0,1]$ denote $g_\ell$’s access to $\ell \in N^L_g$.

Legislative bargaining occurs over an infinite horizon, with periods discrete and indexed by $t \in \{1,2,\ldots\}$. The policy space $X \subseteq \mathbb{R}$ is non-empty, compact, and convex. Let $\rho = (\rho_1,\ldots,\rho_{n^L}) \in \Delta([0,1])^{n^L}$, be the distribution of recognition probabilities among committee members. Each legislative period $t$, bargaining proceeds as follows. If no policy has passed before period $t$, then $\ell$ is recognized to propose with probability $\rho_\ell > 0$. All players observe the period-$t$ proposer, $\ell_t$. With probability $1 - \alpha_\ell$, $g_\ell$ cannot lobby and $\ell_t$ freely proposes any $x_t \in X$. Conversely, with probability $\alpha_\ell$, $g_\ell$ can lobby and offers $\ell_t$ a binding contract $(y_t, m_t) \in X \times \mathbb{R}_+$. Next, $\ell_t$ accepts or rejects. Let $a_t \in \{0,1\}$ denote $\ell_t$’s period-$t$ acceptance decision, where $a_t = 1$ indicates acceptance and $a_t = 0$ if either: $\ell_t$ rejects or $g_\ell$ is unable to lobby in period $t$. If $\ell_t$ accepts, then $\ell_t$ is committed to propose $x_t = y_t$ in period $t$ and $g_\ell$ transfers $m_t$ to $\ell_t$. If $\ell_t$ rejects, then she can propose any $x_t \in X$ and $g_\ell$ keeps $m_t$. All players observe $\ell_t$’s proposal. There is a simultaneous vote using simple majority rule. Each voter chooses to support or oppose. If $x_t$ passes, then bargaining ends with $x_t$ enacted in period $t$ and all subsequent periods. If $x_t$ fails, then $q$ is enacted in period $t$ and bargaining proceeds to $t+1$.

Each player $i \in N$ has quadratic policy utility with ideal point $\hat{x}_i \in X$. Throughout, I normalize $\hat{x}_M = 0$ and assume $\hat{x}_M \neq q$. Additionally, I assume that there is not complete status quo bias among $\ell \in N^L$ and $g \in N^G$. Specifically, there exists $\ell \in N^L$ who is on the same side of $q$ as $M$ and not influenced by any group on the opposite side of $q$. To illustrate, assume $q > 0$. Then $\hat{x}_\ell < q$ and either (i) $\hat{x}_{g_\ell} < q$, or (ii) $\alpha_\ell = 0$. This assumption ensures there is some $\ell$ who wants to move policy in the same direction as $M$ and, furthermore, $g_\ell$.

---

26 An independent legislator is accommodated by giving some $g$ nominal access, i.e. $\alpha_\ell = 0$.

27 Where $\Delta([0,1])^{n^L}$ denotes the $n^L$-dimensional unit simplex.
shares this preference.

Players discount streams of stage utility by the common discount factor $\delta \in (0, 1)$. For convenience, per-period payoffs are normalized by $(1 - \delta)$. Let $I^G_t \in \{0, 1\}$ take the value one iff $t$ is the period-$t$ proposer and $g_t$ can lobby in $t$.

Given a sequence of offers $(y_1, m_1), (y_2, m_2), \ldots$, a sequence of proposers $\ell_1, \ell_2, \ldots$ a sequence of acceptance decisions $a_1, a_2, \ldots$, and a sequence of independent policy proposals $x_1, x_2, \ldots$ such that legislative proposals are rejected until $t$, the discounted sum of per-period payoffs for each $i \in N^V$ is

$$(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1} u_i(q) + \delta^{t'-1} \left[(1 - a_t)u_i(x_t) + a_t u_i(y_t)\right].$$

For each $\ell \in N^G$, the discounted sum of per-period payoffs is

$$(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1} \left[u_\ell(q) + I^G_{t'} a_t m_{t'}\right] + \delta^{t'-1} \left[(1 - a_t)u_\ell(x_t) + a_t \left(u_\ell(y_t) + I^G_{t'} m_t\right)\right].$$

The discounted sum of per-period payoffs for each $g \in N^g$ is

$$(1 - \delta) \sum_{t'=1}^{t-1} \delta^{t'-1} \left[u_g(q) - a_t m_{t'} \sum_{\ell \in N^G_{t'}} I^G_{t'}\right] + \delta^{t'-1} \left[(1 - a_t)u_g(x_t) + a_t \left(u_g(y_t) - m_t \sum_{\ell \in N^G_{t'}} I^G_{t'}\right)\right].$$

Unless otherwise noted, results are proved for this more general setting. The model presented in the text is a special case. To see this, first assume there is one voter with ideal point $\hat{x}_M$. Second, assume there are four committee members with respective ideal points $\hat{x}_L, \hat{x}_M, \hat{x}_\ell, \hat{x}_R$. Finally, assume $|N^G| = 1$, the group has ideal point $\hat{x}_g$, $\alpha_\ell \geq 0$, and $\alpha_j = 0$ for all $j \neq \ell$.

**Strategies**

I study a class of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values for each player. I then define pure strategies and the notion of no-delay pure strategy stationary legislative lobbying equilibrium with deferential voting and deferential acceptance. In Appendix B, I define mixed strategy stationary legislative lobbying equilibrium. Furthermore, I show that all mixed strategy stationary legislative lobbying equilibrium are equivalent in outcome distribution to a no-delay pure strategy stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.

Define $\Delta(X)$ to be the set of probability measures on $X$. Let $W = X \times \mathbb{R}_+$ denote
the interest group offer space, and let $\Delta(W)$ denote the set of probability measures on $W$. A stationary mixed strategy for $g \in N^G$ consists of a probability measure $\lambda_g \in \Delta(W)^{|N^L_g|}$ over $g$’s policy offers $y \in X$ and transfer offers $m \in \mathbb{R}_+$, respectively, to each $\ell \in N^L_g$. A stationary mixed legislative strategy for $\ell \in N^L_g$ is a pair $(\pi_\ell, \varphi_\ell)$; where $\pi_\ell \in \Delta(X)$ specifies a probability measure over $\ell$’s proposal in any period she is recognized to propose and reject $g_y$’s offer, and $\varphi_\ell : W \rightarrow [0, 1]$ is the probability that $\ell$ accepts each offer $(y, m) \in W$. Finally, voter $i$’s strategy is $\nu_i : X \rightarrow [0, 1]$, specifying the probability $i$ supports $x \in X$.

Let $\lambda$ denote a profile of interest group strategies, $(\pi, \varphi)$ a profile of committee member strategies, and $\nu$ a profile of voter strategies. Denote a stationary strategy profile as $\sigma = (\lambda, \pi, \varphi, \nu)$. Finally, let $\varpi_{\sigma}(x)$ represent be the probability that $x$ is passed by the legislature in each period under $\sigma$.

**Continuation Values**

Let $w = (y, m) \in W$ denote an arbitrary interest group offer. For convenience, define

$$\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_w [1 - \varphi_\ell(y, m)] \lambda_y^\ell (dw), \quad (8)$$

which is the probability under $\sigma$ that $\ell$ makes an independent policy proposal in each period she is recognized to propose until policy passes. Given $\sigma$, the continuation value of $i \in N^V$ is

$$V_i(\sigma) = \sum_{\ell \in N^\ell} \rho_\ell \left\{ \alpha_\ell \int_w \varphi_\ell(y, m) \left[ \varpi_\sigma(y) u_i(y) + [1 - \varpi_\sigma(y)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \lambda_y^\ell (dw) 
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \varpi_\sigma(x) u_i(x) + [1 - \varpi_\sigma(x)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \pi_\ell(dx) \right\}, \quad (9)$$

the continuation value of $\ell \in N^L$ is

$$\tilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \int_w \varphi_j(y, m) \left[ \varpi_\sigma(y) u_\ell(y) + [1 - \varpi_\sigma(y)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda_y^j (dw) 
+ \xi_j(\alpha, \sigma) \int_X \left[ \varpi_\sigma(x) u_\ell(x) + [1 - \varpi_\sigma(x)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_j(dx) \right\},
+ \rho_\ell \left\{ \alpha_\ell \int_w \varphi_\ell(y, m) \left[ \varpi_\sigma(y) u_\ell(y) + [1 - \varpi_\sigma(y)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m \right] \lambda_y^\ell (dw) \right\}. \quad (10)$$
\[
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \varphi_\sigma(x) u_\ell(x) + [1 - \varphi_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx),
\]

and the continuation value of \( g \in N^G \) is
\[
\tilde{V}_g(\sigma) = \sum_{\ell \in N^L_g} \rho_\ell \left\{ \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \varphi_\sigma(y) u_\ell(y) + [1 - \varphi_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda_\ell (dw) \\
+ \xi_\ell(\alpha, \sigma) \int_X \left[ \varphi_\sigma(x) u_\ell(x) + [1 - \varphi_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right\}.
\]

Stationary Legislative Lobbying Equilibrium

A stationary pure strategy for \( g \in N^G \) is \((y_g, m_g) \in X^{|N^L_g|} \times \mathbb{R}^{|N^L_g|}_+ \), where \( y_g \) denotes a profile of policy offers and \( m_g \) denotes a profile of monetary offers. The pair \((y_g, m_g)\) stipulates that \( g \) transfers \( m_\ell^g \) to \( \ell \in N^L_g \) in exchange for the policy proposal \( y_\ell^g \) in each period \( \ell \) is the proposer and accepts \( g \)’s offer. A pure stationary strategy for \( \ell \in N^L \) is \((z_\ell, a_\ell)\); where \( z_\ell \in X \) specifies \( \ell \)'s proposal in each legislative period that \( \ell \) is the proposer and rejects \( g_\ell \)'s offer, and \( a_\ell : X \times \mathbb{R} \rightarrow \{0, 1\} \) equals one iff \( \ell \) accepts \( g_\ell \)'s offer. Finally, for each voter \( i \), \( u_i : X \rightarrow \{0, 1\} \) equals one iff \( i \) supports the proposal.

Given \( \sigma \), let \( A(\sigma) \subset X \) denote the set of policies that receive majority approval in a given period. By stationarity, \( A(\sigma) \) is constant across periods. Define the dynamic policy utility of \( i \in N^V \) from proposal \( x \) under \( \sigma \) as
\[
U_i(x; \sigma) = \begin{cases} 
  u_i(x) & \text{if } x \in A(\sigma) \\
  (1 - \delta)u_i(q) + \delta V_i(\sigma) & \text{else},
\end{cases}
\]

where stage payoffs are normalized by \( 1 - \delta \) for convenience. For \( \ell \in N^L \), define \( \ell \)'s dynamic utility from \( x \) under \( \sigma \) as
\[
\tilde{U}_\ell(x; \sigma) = \begin{cases} 
  u_\ell(x) & \text{if } x \in A(\sigma) \\
  (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) & \text{else}.
\end{cases}
\]
Formally, $\sigma = (y, m, z, a, v)$ is a no-delay pure strategy stationary legislative lobbying equilibrium with deferential voting and deferential acceptance if it satisfies five conditions. First, for all $g \in N^G$ and $\ell \in N^L_g$, $(y^\ell_g, m^\ell_g)$ satisfies

$$y^\ell_g = \arg \max_{y \in A(\sigma)} u_{g\ell}(y) + u_\ell(y) - u_\ell(z_\ell)$$

and

$$m^\ell_g = u_\ell(z_\ell) - u_\ell(y^\ell_g).$$

Second, $a_\ell(y, m) = 1$ iff

$$\tilde{U}_\ell(y; \sigma) + m \geq \tilde{U}_\ell(z_\ell; \sigma)$$

for all $\ell \in N^L$ and $(y, m) \in W$. Third, $z_\ell$ solves

$$\max_{x \in A(\sigma)} u_\ell(x)$$

for each $\ell \in N^L$. Finally, $v_i(x) = 1$ iff

$$u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma).$$

for each $i \in N^V$. In Appendix B, I show that strategy profiles satisfying (14)-(18) are equivalent in outcome distribution to all mixed strategy stationary legislative lobbying equilibria.

Existence

The following shows part 1 of Proposition 1.

**Proposition 1.1.** There exists a no-delay pure strategy stationary legislative lobbying equilibrium with deferential voting and deferential acceptance.

**Proof.** There are three parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay pure interest group policy offer strategies, and (ii) no-delay pure committee member proposal strategies, to itself as the solution to optimization problems for $g \in N^G$ and $\ell \in N^L$. Part 2 then constructs a strategy profile $\sigma$. Part 3 checks that $\sigma$ satisfies (14) - (18).

**Part 1:** Let $y = (y_1, \ldots, y_n) \in X^{n_h}$ be a profile of pure interest group policy offer strategies and $z = (z_1, \ldots, z_n) \in X^{n_L}$ a profile of pure committee member proposal strategies.
Define
\[ r_i(y, z) = \sum_{\ell \in N_L} \rho_\ell \left( \alpha_\ell u_i(y_\ell) + (1 - \alpha_\ell) u_i(z_\ell) \right), \tag{19} \]
which is voter \( i \)'s continuation value if all policy proposals are accepted and passed with probability one. Thus, \( i \)'s reservation value is \((1 - \delta) u_i(q) + \delta r_i(y, z)\). In particular, \( M \)'s reservation value induces an acceptance set for \( M \), denoted \( A_M(r(y, z)) \). By Banks and Duggan (2006b) and Duggan (2014), \( M \) is decisive over lotteries, so \( A_M(r(y, z)) = A(r(y, z)) \).

Because \( u_M \) is strictly concave and \( X \subseteq \) is compact and convex, \( A(r(y, z)) \) is a non-empty, convex, and compact set. Moreover, \( A(r(y, z)) \) is continuous in \((y, z)\).

For each \( \ell \in N_L \), define
\[ \tilde{\phi}_\ell(y, z) = \arg \max_{x \in A(r(y, z))} u_{g_\ell}(x) + u_\ell(x), \tag{20} \]
which is unique for all \((y, z)\) because the objective function is strictly concave and continuous in \( x \) and \( A(r(y, z)) \) is non-empty, compact and convex. Because \( A(r(y, z)) \) is continuous, the Theorem of the Maximum implies continuity of \( \tilde{\phi}_\ell(y, z) \). Next, define
\[ \phi_\ell(y, z) = \arg \max_{x \in A(r(y, z))} u_\ell(x), \tag{21} \]
which is unique for all \((y, z)\). Continuity of \( \phi_\ell(y, z) \) also follows from the Theorem of the Maximum.

Define the mapping \( \Phi : X^{2n^L} \rightarrow X^{2n^L} \) as \( \Phi(y, z) = \prod_{\ell \in N_L} \tilde{\phi}_\ell(y, z) \times \prod_{\ell \in N_L} \phi_\ell(y, z) \), which is continuous in \((y, z)\) as a product of functions continuous in \((y, z)\). By Brouwer’s theorem, a fixed point \((y^*, z^*) = \Phi(y^*, z^*) \) exists because \( \Phi \) is a continuous function mapping a non-empty, convex, and compact set into itself.

Part 2: Define a pure strategy profile \( \sigma \) as follows. First, for each \( \ell \in N_L \), set \( z_\ell = z_\ell^* \). Second, for each \( \ell \in N_L \), set \( y_\ell^* = y_\ell^* \) and \( m_\ell^* = u_\ell(z_\ell^*) - u_\ell(y_\ell^*) \). Third, for each \( \ell \in N_L \), define
\[ a_\ell(y, m) = \begin{cases} 1 & \text{if } u_\ell(y) + m \geq u_\ell(z_\ell), \text{ for } y \in A(r(y^*, z^*)) \\ 1 & \text{if } (1 - \delta) u_\ell(q) + \delta [r_\ell(y^*) + \rho_\ell \alpha_\ell m_\ell^*] + m \geq u_\ell(z_\ell), \text{ for } y \notin A(r(y^*, z^*)) \\ 0 & \text{else.} \end{cases} \tag{22} \]

Finally, for each \( i \in N^V \) define \( v_i \) so that \( v_i(x) = 1 \) if \( u_i(x) \geq (1 - \delta) u_i(q) + \delta r_i(y^*, z^*) \) and
$u_i(x) = 0$ otherwise.

Part 3: I check that $\sigma$ satisfies (14)-(18).

First, I verify (18) to show $A(\sigma) = A(r(y^*, z^*))$. Note that $a_\ell(y^\ell, m^\ell_\ell) = 1$ and $y^\ell_\ell = y^\ell_i \in A(r(y^*, z^*))$ for all $\ell \in N^\ell$. Moreover, $z_\ell = z^\ell_\ell \in A(r(y^*, z^*))$ for all $\ell \in N^\ell$. Thus, voter $i$’s continuation value under $\sigma$ is $V_i(\sigma) = \sum_{\ell \in N^\ell} \rho_\ell [\alpha_\ell u_i(y^\ell_\ell) + (1 - \alpha_\ell)u_i(z^\ell_\ell)] = r_i(y^*, z^*)$. Each voter’s strategy satisfies (18). Thus, $A(\sigma) = A(r(y^*, z^*))$.

To check (14), consider $\ell \in N^L$ with associated $g_\ell$. Focusing on acceptable offers is without loss of generality because $\ell$ accepts $(z_\ell, 0)$. Because $A(\sigma) = A(r(y^*, z^*))$, (21) implies $\phi_\ell(y^*, z^*) = \arg\max_{x \in A(\sigma)} u_{g_\ell}(x) + u_\ell(x) - u_\ell(z^\ell_\ell)$. Thus, (14) holds because $\phi_\ell(y^*, z^*) = y^\ell_\ell = y^\ell_i$.

It is immediate that $m^\ell_\ell$ satisfies (15).

To check (16), note that $\ell$’s expected dynamic payoff from rejecting $g_\ell$’s offer is $\tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z_\ell)$. Thus, $\ell$ weakly prefers to accept any $(y, m)$ satisfying $y \in A(r(y^*, z^*))$ iff $u_\ell(y) + m \geq u_\ell(z_\ell)$. If $y \notin A(r(y^*, z^*))$, then $\ell$ weakly prefers to accept $(y, m)$ iff $(1 - \delta)u_\ell(q) + \delta r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_\ell] + m \geq u_\ell(z_\ell)$. Thus, $a_\ell$ satisfies (16) for all $(y, m) \in W$.

To check (17), note that (20) implies $\tilde{\phi}_\ell(y^*, z^*) = \arg\max_{x \in A(\sigma)} u_\ell(x)$ because $A(\sigma) = A(r(y^*, z^*))$.

Thus, (17) holds because $\tilde{\phi}_\ell(y^*, z^*) = z^* = z_\ell$ for each $\ell \in N^L$. By Lemma B.6 in Appendix B, no $\ell \in N^L$ has a profitable deviation to $x \notin A(\sigma)$.

**Equilibrium Analysis**

Appendix B shows that every mixed strategy legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay pure strategy legislative lobbying equilibrium with deferential voting and deferential acceptance. The rest of the analysis focuses on these equilibria and omits qualifiers, simply referring to equilibria.

Define

$$\hat{y}_\ell = \arg\max_{y \in X} \ u_{g_\ell}(y) + u_\ell(y) = \frac{\tilde{x}_{g_\ell} + \tilde{x}_\ell}{2},$$

(23)

where (23) follows because $u$ is quadratic.

Recall that $u_\ell(z_\ell)$ is $\ell$’s expected dynamic payoff in equilibrium, conditional on rejecting $g_\ell$’s offer. By (14), in equilibrium

$$y^\ell_\ell = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell).$$

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where (24) follows because \( u_\ell(z_\ell) \) is constant in \( y \). If \( y_\ell \in A(\sigma) \), then \( y_\ell = \hat{y}_\ell \). Otherwise, strict concavity implies \( y_\ell \) equals the boundary of \( A(\sigma) \) closest to \( \hat{y}_\ell \). As this characterization applies to every equilibrium, there is a clear connection to the characterization in Cho and Duggan (2003), where lobbying is absent.

Proposition 1.3 establishes Part 3 of Proposition 1.

**Proposition 1.3.** Every stationary legislative lobbying equilibrium has the same outcome distribution.

**Proof.** Let \( \sigma \) and \( \sigma' \) be stationary legislative lobbying equilibria. It suffices to show \( (y_g, m_g) = (y_g', m_g') \) for all \( g \in N^G \) and \( z_\ell = z_\ell' \) for all \( \ell \in N^L \). Assume \( y_\ell \neq y'_\ell \) or \( z_\ell \neq z'_\ell \) for some \( \ell \in N^L \). Arguments analogous to Proposition 1 in Cho and Duggan (2003) show a contradiction. Thus, \( A(\sigma) = A(\sigma') \) because \( \sigma \) and \( \sigma' \) are no-delay. Then \( \ell' \)'s expected dynamic payoff from rejecting \( g_\ell \)'s offer is \( u_\ell(z_\ell) \) under both \( \sigma \) and \( \sigma' \). Lemma B.1 implies \( m_\ell = u_\ell(y_\ell) - u_\ell(z_\ell) \). Therefore \( (y_g, m_g) = (y_g', m_g') \) and \( z_\ell = z'_\ell \).

**Comparative Statics on Lobbying Expenditures**

It is useful to prove Proposition 6 before analyzing endogenous access. Set \( \theta = (\hat{x}, \rho, \alpha) \). Let \( \mu_\theta \) denote the unconstrained extremism lottery, which puts probability \( \rho_j \alpha_j \) on \( |\hat{y}_j| \) and probability \( \rho_j (1 - \alpha_j) \) on \( |\hat{x}_j| \) for each \( j \in N^L \). Given \( \theta \) and \( \theta' \), say that legislative extremism is greater under \( \theta' \) if \( \mu_{\theta'} \) first order stochastically dominates \( \mu_{\theta} \).

**Lemma 1.** The equilibrium acceptance set weakly expands as legislative extremism increases.

**Proof.** Consider \( \theta \) and \( \theta' \), with legislative extremism greater under \( \theta' \). By Proposition 1.3, \( \theta \) and \( \theta' \) each induce a unique equilibrium acceptance set. Let \( \bar{x}_\theta \) and \( \bar{x}_{\theta'} \) denote the respective upper bounds of these sets. I show \( \bar{x}_{\theta'} \geq \bar{x}_{\theta} \).

For \( b \geq 0 \), let \( C_j(b) = \mathbb{I}\{x_j \leq (-b, b)\} \) and \( C_j(b) = \mathbb{I}\{\hat{y}_j \leq (-b, b)\} \). Define \( C_j(b) \) and \( \tilde{C}_j(b) \) analogously for \( \tilde{x}_j \) and \( \tilde{y}_j \). Then for all \( b \geq 0 \),

\[
(1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left\{ (1 - \alpha_j)C_j(b)u_M(\tilde{x}_j) + \alpha_j \tilde{C}_j(b)u_M(\tilde{y}_j) \right\} \\
- \delta u_M(b) \sum_{j \in N^L} \rho_j \left\{ (1 - \alpha_j)(1 - C_j(b)) + \alpha_j(1 - \tilde{C}_j(b)) \right\}
\]

(25)
\[ \geq (1 - \delta)u_M(q) + \delta \sum_{j \in N^L} \rho_j' \left\{ (1 - \alpha_j')C_j'(b)u_M(\hat{x}_j') + \alpha_j'\tilde{C}_j'(b)u_M(\hat{y}_j') \right\} 
- \delta u_M(b) \sum_{j \in N^L} \rho_j' \left\{ (1 - \alpha_j')(1 - C_j'(b)) + \alpha_j'(1 - \tilde{C}_j'(b)) \right\}, \tag{26} \]

where (26) follows because \( \mu_d \) FOSD \( \mu_\theta \). The equilibrium characterization, and construction of \( C_j \) and \( \tilde{C}_j \), implies \( \pi_\theta \) is the unique \( b \geq 0 \) such that \( u_M(b) \) equals (25). Analogously, \( \pi_{\theta'} \) is the unique \( b \geq 0 \) such that \( u_M(b) \) equals (26). Thus, \( \pi_{\theta'} \geq \pi_\theta \). \( \square \)

**Proposition 6.** For all \( \ell \in N^L \), \( g_\ell \)'s equilibrium lobbying expenditures increase as either (i) legislative extremism increases, fixing \( \hat{x}_\ell \) and \( \hat{y}_g \); (ii) \( |q| \) increases; or (iii) \( \delta \) decreases.

**Proof.** (i) Increase legislative extremism. Let \( \sigma \) denote an equilibrium, suppressing dependence on legislative extremism. By Lemma 1, \( \pi(\sigma) \) weakly increases with legislative extremism. There are two cases.

- **Case 1.** Suppose \( \hat{x}_\ell \in A(\sigma) \). Then \( z_\ell = \hat{x}_\ell \). There are two subcases.

  First, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y_\ell^f = \hat{y}_\ell \). Lemma B.1 and (14) imply \( m_\ell^f = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell) \). From Lemma 1, \( z_\ell \) and \( y_\ell^f \) are constant as legislative extremism increases. Thus, \( m_\ell^f \) is constant.

  Second, assume \( \hat{y}_\ell \notin A(\sigma) \). Since \( \hat{x}_\ell \in A(\sigma) \), this requires \( \hat{x}_g \notin [-\pi(\sigma), \pi(\sigma)] \). Without loss of generality, assume \( \hat{x}_g > \pi(\sigma) \). Then \( \hat{x}_\ell \in A(\sigma) \) and \( \hat{y}_\ell \notin A(\sigma) \) imply \( \hat{y}_\ell > \pi(\sigma) \). Thus, \( z_\ell = \hat{x}_\ell \) and \( y_\ell^f = \pi(\sigma) \). By Lemma B.1 and (14), we have \( m_\ell^f = u_\ell(\hat{x}_\ell) - u_\ell(\pi(\sigma)) \). From Lemma 1, \( \pi(\sigma) \) increases in legislative extremism. Thus, \( m_\ell^f \) increases in legislative extremism.

- **Case 2.** Suppose \( \hat{x}_\ell \notin A(\sigma) \). Without loss of generality, assume \( \hat{x}_\ell > 0 \). Then \( \hat{x}_\ell > z_\ell = \pi(\sigma) \). There are three subcases.

  First, assume \( \hat{y}_\ell < -\pi(\sigma) \). Then \( y_\ell^f = -\pi(\sigma) \). By Lemma B.1 and (14), \( m_\ell^f = u_\ell(\pi(\sigma)) - u_\ell(-\pi(\sigma)) \). Lemma 1 implies that increasing legislative extremism increases \( \pi(\sigma) \) and decreases \( -\pi(\sigma) \). Thus, \( m_\ell^f \) increases in legislative extremism because \( -\pi(\sigma) < \pi(\sigma) < \hat{x}_\ell \).

  Second, assume \( \hat{y}_\ell \in A(\sigma) \). Thus, \( y_\ell^f = \hat{y}_\ell \) and \( y_\ell^f \) is constant as legislative extremism increases. Arguments analogous to subcase 2 of Case 1 imply \( m_\ell^f \) increases.

  Third, assume \( \hat{y}_\ell \geq \pi(\sigma) \), which implies \( y_\ell^f = \pi(\sigma) \). By Lemma B.1 and (14), \( m_\ell^f = u_\ell(\pi(\sigma)) - u_\ell(\pi(\sigma)) = 0 \), which is constant in legislative extremism.
Altogether, $m_g^\ell$ weakly increases in legislative extremism.

(ii) Increase $|q|$. First, let $C_j(\hat{x}_\ell) = \mathbb{I}\{\hat{x}_j \in \text{int}A(\sigma)\}$. Similarly, let $\tilde{C}_j(\hat{y}_j) = \mathbb{I}\{\hat{y}_j \in \text{int}A(\sigma)\}$.

Then

$$
\bar{x}(\sigma) = \left(1 - \delta\right)u_M(q) + \delta \sum_{j \in N^L} \rho_j \left[ C_j(\hat{x}_\ell)(1 - \alpha_j)u_M(\hat{x}_\ell) + \tilde{C}_j(\hat{y}_\ell)\alpha_j u_M(\hat{y}_\ell) \right] \left(1 - \delta \sum_{j \in N^L} \rho_j \left[ (1 - C_j(\hat{x}_\ell))(1 - \alpha_j) + (1 - \tilde{C}_j(\hat{y}_\ell))\alpha_j \right] \right)^{\frac{1}{2}}, \tag{27}
$$

Inspection of (27) shows $\bar{x}(\sigma)$ strictly increases in $|q|$. Thus, $A(\sigma)$ expands with $|q|$. Arguments analogous to Part (i) imply $m_g^\ell$ weakly increases in $|q|$.

(iii) Decrease $\delta$. Inspection of (27) shows $\bar{x}(\sigma)$ strictly decreases in $\delta$. Thus, $A(\sigma)$ shrinks with $\delta$. Arguments analogous to Part (i) imply $m_g^\ell$ weakly increases in $\delta$. \hfill \Box

Endogenous Access

Fix $\ell \in N^L$. Recall $\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}$ for $\ell \in N^L$. For convenience, refer to $g_\ell$ as $g$. The results fix $\hat{x}_g$. Let $\sigma(\hat{x}_\ell, \alpha_\ell)$ denote an equilibrium, given $\hat{x}_\ell$ and $\alpha_\ell$. Denote the corresponding social acceptance set as $A(\sigma(\hat{x}_\ell, \alpha_\ell))$, with upper bound $\bar{x}(\sigma(\hat{x}_\ell, \alpha_\ell))$. Also, let $A(\sigma(x_g))$ denote the equilibrium acceptance set if $\hat{x}_\ell = \hat{x}_g$, suppressing $\alpha_\ell$ because it is inconsequential.

First, I establish properties used to state analogues of Propositions 2 and 3. Building upon Lemmas C.1–C.6 in Appendix C, Lemma 2 characterizes a partition that distinguishes whether $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_g))$. See Appendix C for the proof. I state the result for reference in the proof of Lemma 3.

**Lemma 2.** For all $\ell \in N^L$, there exists $\bar{x}_\ell \in (0, q]$ such that $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_g))$. Otherwise, $A(\sigma(\hat{x}_g)) = [-\bar{x}_\ell, \bar{x}_\ell]$.

**Lemma 3.** Suppose $\hat{x}_g \in (0, \bar{x}_\ell)$. There exists $\bar{\bar{x}} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\bar{\bar{x}}, \hat{x}_g)$ implies $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_\ell, \alpha_\ell))$ for all $\alpha_\ell \in [0, 1]$. A symmetric result holds if $\hat{x}_g \in (-\bar{x}_\ell, 0)$.

**Proof.** Consider $\hat{x}_g \in (0, \bar{x}_\ell)$. By Lemma 2, $\hat{x}_\ell = \hat{x}_g$ implies $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_\ell, 0))$. Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies $A(\sigma(\hat{x}_\ell, 0))$ is continuous in $\hat{x}_\ell$. Thus, there exists $\bar{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\sigma(\hat{x}_\ell, 0))$. Suppose $\hat{x}_\ell \in (\bar{x}, \hat{x}_g)$. Lemma 1 implies $A(\sigma(\hat{x}_\ell, 0)) \subset A(\sigma(\hat{x}_\ell, \alpha_\ell))$ for all $\alpha_\ell \in [0, 1]$. The desired result follows. \hfill \Box
For each $j \in N^L \setminus \{\ell\}$, define $E_j^{LB}(\hat{x}_t, \alpha_t) = \mathbb{I}\{\hat{x}_j \leq -\varpi(\sigma(\hat{x}_t, \alpha_t))\}$, $E_j^{UB}(\hat{x}_t, \alpha_t) = \mathbb{I}\{\hat{x}_j \geq \varpi(\sigma(\hat{x}_t, \alpha_t))\}$, and $C_j(\hat{x}_t, \alpha_t) = \mathbb{I}\{z_j \in \text{int}A(\sigma(\hat{x}_t, \alpha_t))\}$. Analogously, define $\tilde{E}_j^{LB}(\hat{x}_t, \alpha_t)$, $\tilde{E}_j^{UB}(\hat{x}_t, \alpha_t)$, and $\tilde{C}_j(\hat{x}_t, \alpha_t)$ for $y^g_\ell$. Let $I^g_\ell \in \{0, 1\}$ indicate whether $j \in N^L$.

I catalogue two technical assumptions.

**Assumption A.1.** There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j < 1$ and $\hat{x}_j \notin A(\sigma(\hat{x}_g))$.

**Assumption A.2.** There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j > 0$ and $\hat{y}_j \notin A(\sigma(\hat{x}_g))$.

Next, define

$$v^g_1(\hat{x}_t, \alpha_t) = \rho_\ell \left( \alpha_t \left[ u_g(\hat{y}_t) + u_t(\hat{y}_t) - u_t(\hat{x}_t) \right] + (1 - \alpha_t) u_g(\hat{x}_t) \right)$$

(28)

and

$$v^g_2(\hat{x}_t, \alpha_t) = \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \tilde{E}_j^{LB}(\hat{x}_t, \alpha_t) + (1 - \alpha_j) E_j^{LB}(\hat{x}_t, \alpha_t) \right\} u_g(-\varpi(\hat{x}_t, \alpha_t))$$

$$+ \left[ \alpha_j \tilde{E}_j^{UB}(\hat{x}_t, \alpha_t) + (1 - \alpha_j) E_j^{UB}(\hat{x}_t, \alpha_t) \right] u_g(\varpi(\hat{x}_t, \alpha_t))$$

$$+ \alpha_j \left[ \tilde{C}_j(\hat{x}_t, \alpha_t) u_g(\hat{y}_j) - I^g_\ell m_g^j(\hat{x}_t, \alpha_t) \right] + (1 - \alpha_j) C_j(\hat{x}_t, \alpha_t) u_g(\hat{x}_j) \right\}.$$  

Then, $g$’s ex ante expected dynamic payoff from $\alpha_t$ access to $\ell$ is

$$U^E_g(\hat{x}_t, \alpha_t) = v^g_1(\hat{x}_t, \alpha_t) + v^g_2(\hat{x}_t, \alpha_t).$$

(30)

**Lemma 4.** If $\hat{x}_t \neq \hat{x}_g$, then $\frac{\partial \nu^g_1(\hat{x}_t, \alpha_t)}{\partial \alpha_t} > 0$.

**Proof.** Suppose $\hat{x}_t \neq \hat{x}_g$. From (28) and $\hat{y}_t = \frac{\hat{x}_t + \hat{x}_g}{2}$, we have $\frac{\partial \nu^g_1(\hat{x}_t, \alpha_t)}{\partial \alpha_t} = \frac{\partial}{\partial \alpha_t} (\hat{x}_g - \hat{x}_t)^2 > 0$. \Box

**Lemma 5.** Suppose $0 \leq \hat{x}_t < \hat{x}_g < \varpi_\ell$ and at least one of Assumption A.1 or A.2 holds. Then $v^g_2(\hat{x}_t, \alpha_t)$ strictly decreases in $\alpha_t$. A symmetric result holds for $\hat{x}_g < 0$.

**Proof.** Assume $0 \leq \hat{x}_t < \hat{x}_g < \varpi_\ell$ and at least one of Assumption A.1 or A.2 holds. It suffices to show that

$$\left[ \alpha_j \tilde{E}_j^{LB}(\hat{x}_t, \alpha_t) + (1 - \alpha_j) E_j^{LB}(\hat{x}_t, \alpha_t) \right] u_g(-\varpi(\hat{x}_t, \alpha_t))$$

$$+ \left[ \alpha_j \tilde{E}_j^{UB}(\hat{x}_t, \alpha_t) + (1 - \alpha_j) E_j^{UB}(\hat{x}_t, \alpha_t) \right] u_g(\varpi(\hat{x}_t, \alpha_t))$$

\[37]
\[ + \alpha_j \left[ \tilde{C}_j (\hat{x}_\ell, \alpha_\ell) u_g (\hat{y}_j) - I_j^j m_g^j (\hat{x}_\ell, \alpha_\ell) \right] + (1 - \alpha_j) C_j (\hat{x}_\ell, \alpha_\ell) u_g (\hat{x}_j) \]  \tag{31}

decreases in \( \alpha_\ell \) for each \( j \in N^L \setminus \{ \ell \} \) and strictly decreases for at least one \( j \).

Without loss of generality, consider \( \hat{x}_j \geq 0 \). By inspection of (27), \( 0 \leq \hat{x}_\ell < \hat{x}_g \) implies \( \pi (\hat{x}_\ell, \alpha_\ell) \) increases in \( \alpha_\ell \). There are two implications. First, \( \hat{x}_g \in (0, \pi (\hat{x}_\ell, 0)) \) by Lemma 2, so \( u_g (\pi (\hat{x}_\ell, \alpha_\ell)) \) and \( u_g (-\pi (\hat{x}_\ell, \alpha_\ell)) \) both decrease in \( \alpha_\ell \). Second, there exists \( \alpha^j_\ell \in [0, 1] \) such that: (i) \( \alpha_\ell \in [0, \alpha^j_\ell] \) implies \( E^{UB}_j (\hat{x}_\ell, \alpha_\ell) = 1 \), (ii) \( \alpha_\ell \in (\alpha^j_\ell, 1] \) implies \( C_j (\hat{x}_\ell, \alpha_\ell) = 1 \), (iii) \( \alpha_\ell = \alpha^j_\ell \) implies \( \hat{x}_j = \pi (\hat{x}_\ell, \alpha_\ell) \). Thus, both

\[ E^{LB}_j (\hat{x}_\ell, \alpha_\ell) u_g (-\pi (\hat{x}_\ell, \alpha_\ell)) + E^{UB}_j (\hat{x}_\ell, \alpha_\ell) u_g (\pi (\hat{x}_\ell, \alpha_\ell)) + C_j (\hat{x}_\ell, \alpha_\ell) u_g (\hat{x}_j) \]  \tag{32}

and

\[ \tilde{E}^{LB}_j (\hat{x}_\ell, \alpha_\ell) u_g (-\pi (\hat{x}_\ell, \alpha_\ell)) + \tilde{E}^{UB}_j (\hat{x}_\ell, \alpha_\ell) u_g (\pi (\hat{x}_\ell, \alpha_\ell)) + \tilde{C}_j (\hat{x}_\ell, \alpha_\ell) u_g (\hat{x}_j) \]  \tag{33}

decrease in \( \alpha_\ell \). Furthermore, at least one of (32) and (33) strictly decreases for some \( j \in N^L \setminus \{ \ell \} \) because at least one of Assumptions A.1 or A.2 holds. Inspection of (27) reveals \( m_g^j (\hat{x}_\ell, \alpha_\ell) \) weakly increases in \( \alpha_\ell \) for all \( j \in N^L \). Altogether, (31) decreases in \( \alpha_\ell \) for all \( j \in N^L \setminus \{ \ell \} \) and strictly decreases for some \( j \). Thus, \( v_2 (\hat{x}_\ell, \alpha_\ell) \) strictly decreases in \( \alpha_\ell \). \( \square \)

**Lemma 6.** Assume \( \hat{x}_g \in (0, \pi_\ell) \) and at least one of Assumptions A.1 and A.2 hold. There exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U^{EB}_g (\hat{x}_\ell, \alpha_\ell) \) strictly decreases in \( \alpha_\ell \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_g \in (0, \pi_\ell) \) and at least one of Assumptions A.1 and A.2 hold. I show that if \( \hat{x}_\ell \) is sufficiently close to \( \hat{x}_g \), then \( \frac{\partial u_g (\hat{x}_\ell, \alpha_\ell)}{\partial \alpha_\ell} \) is.

Since \( \hat{x}_g \in (0, \pi_\ell) \), Lemma 3 implies existence of \( \hat{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\hat{x}, \hat{x}_g) \) implies \( \hat{x}_g \in A (\sigma (\hat{x}_\ell, \alpha_\ell)) \) for all \( \alpha_\ell \in [0, 1] \). Fix \( \hat{x}_\ell \in (\hat{x}, \hat{x}_g) \) and \( \alpha_\ell \in [0, 1] \).

First, I characterize an upper bound on \( \frac{\partial u_2 (\hat{x}_\ell, \alpha_\ell)}{\partial \pi (\hat{x}_\ell, \alpha_\ell)} \). Define

\[ \Gamma = \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \tilde{E}^{LB}_j (\hat{x}_g) + (1 - \alpha_j) E^{LB}_j (\hat{x}_g) \right\} \frac{\partial u_g (-\pi (\hat{x} \ell\ell))}{\partial \pi (\hat{x} \ell\ell)} \]
\[ + \left[ \alpha_j \tilde{E}^{UB}_j (\hat{x}_g) + (1 - \alpha_j) E^{UB}_j (\hat{x}_g) \right] \frac{\partial u_g (\pi (\hat{x} \ell\ell))}{\partial \pi (\hat{x} \ell\ell)} \].  \tag{34}
I claim \( \frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} < \Gamma < 0 \), where
\[
\frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} = \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \tilde{E}_j^{LB}(\hat{x}_\ell, \alpha_\ell) + (1 - \alpha_j) E_j^{LB}(\hat{x}_\ell, \alpha_\ell) \right\} \frac{\partial u_g(-\pi(\hat{x}_\ell, \alpha_\ell))}{\partial x(\hat{x}_\ell, \alpha_\ell)} + \left( 1 - \alpha_j \right) E_j^{UB}(\hat{x}_\ell, \alpha_\ell) \frac{\partial u_g(\pi(\hat{x}_\ell, \alpha_\ell))}{\partial x(\hat{x}_\ell, \alpha_\ell)} - \Gamma^j \delta \frac{\partial m_j g(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \right\}. \tag{35}
\]

To see \( \Gamma < 0 \), note that (i) \( \hat{x}_g \in (-\pi(\hat{x}), \pi(\hat{x})) \) implies \( \frac{\partial u_g(-\pi(\hat{x}))}{\partial x(\hat{x})} < 0 \) and \( \frac{\partial u_g(-\pi(\hat{x}))}{\partial x(\hat{x})} < 0 \), and (ii) there exists \( j \in N^L \) such that either \( \alpha_j < 1 \) and \( \hat{x}_j \notin A(\sigma(\hat{x}_g)) \), or \( \alpha_j > 0 \) and \( \hat{y}_j \notin A(\sigma(\hat{x}_g)) \) because at least one of Assumptions A.1 and A.2 hold.

To see \( \frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} < \Gamma \), note that \( \hat{x}_\ell \in (\hat{x}, \hat{x}_g) \) implies \( \pi(\hat{x}_g) \geq \pi(\hat{x}_\ell, \alpha_\ell) \). Thus, \( \tilde{E}_j^{UB}(\hat{x}_\ell, \alpha_\ell) \leq E_j^{UB}(\hat{x}_\ell, \alpha_\ell) \) and \( E_j^{LB}(\hat{x}_g) \leq E_j^{LB}(\hat{x}_\ell, \alpha_\ell) \) for all \( j \neq \ell \). Symmetrically, \( \tilde{E}_j^{LB}(\hat{x}_g) \leq E_j^{LB}(\hat{x}_\ell, \alpha_\ell) \) and \( E_j^{LB}(\hat{x}_g) \leq E_j^{LB}(\hat{x}_\ell, \alpha_\ell) \) for all \( j \neq \ell \). Next, \( \hat{x}_g < \pi(\hat{x}) < \pi(\hat{x}_\ell, \alpha_\ell) \) implies \( \frac{\partial u_g(\pi(\hat{x}, \alpha_\ell))}{\partial x(\hat{x}, \alpha_\ell)} < 0 \) and symmetrically \( -\pi(\hat{x}_\ell, \alpha_\ell) < \pi(\hat{x}) < \pi(\hat{x}_g) \) implies \( \frac{\partial u_g(-\pi(\hat{x}_\ell, \alpha_\ell))}{\partial x(\hat{x}_\ell, \alpha_\ell)} < 0 \).

Finally, \( \frac{\partial m_j g(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \geq 0 \) for all \( j \in N^L_g \) as shown in Proposition 6.

For almost all \( \alpha_\ell \in [0, 1], \frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} = \frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \frac{\partial x(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)}. \) Define \( C_j(\hat{x}_\ell, \alpha_\ell) = [(1 - \alpha_j)(1 - C_j(\hat{x}_\ell, \alpha_\ell)) + \alpha_j(1 - \tilde{C}_j(\hat{x}_\ell, \alpha_\ell))]. \) Then,
\[
\frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} < \Gamma \frac{\partial x(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \tag{36}
\]
\[
= \frac{\delta \rho_{\ell} \Gamma}{2} \frac{u_M(\hat{y}_\ell) - u_M(\hat{x}_\ell)}{\pi(\hat{x}, \alpha_\ell) \left[ 1 - \delta \left( \sum_{j \in N^L} \rho_j C_j(\hat{x}_\ell, \alpha_\ell) \right) \right]} \tag{37}
\]
\[
< \frac{\delta \rho_{\ell} \Gamma}{2} \frac{u_M(\hat{y}_\ell) - u_M(\hat{x}_\ell)}{\pi(\hat{x}, \alpha_\ell)} \tag{38}
\]
\[
= \frac{\delta \rho_{\ell} \Gamma}{2} \frac{1}{4} \left( \frac{3 \hat{x}_\ell + \hat{x}_g}{4} \right), \tag{39}
\]
where (36) follows from \( \frac{\partial x(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} > 0 \) and \( 0 > \Gamma > \frac{\partial v_2(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \); (37) from applying the implicit function theorem to \( \pi(\hat{x}_\ell, \alpha_\ell) \), which is possible for almost all \( \alpha_\ell \in [0, 1]; \) (38) because (i) \( u_M(\hat{x}_\ell) > u_M(\hat{y}_\ell) \), (ii) \( \pi(\hat{x}_\ell, \alpha_\ell) > 0 \), and (iii) \( \delta \sum_{j \in N^L} \rho_j C_j(\hat{x}_\ell, \alpha_\ell) \in (0, 1) \); and (39) because \( u_M(x) = -(x)^2 \) and \( \hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2} \).

By Lemma 4, \( \frac{\partial v_1(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} = \frac{\rho_{\ell}}{2} (\hat{x}_g - \hat{x}_\ell)^2. \) By (39), \( \frac{\partial v_1(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} < \frac{\partial v_1(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} + \Gamma \frac{\partial x(\hat{x}_\ell, \alpha_\ell)}{\partial x(\hat{x}_\ell, \alpha_\ell)} \) for
almost all \( \alpha_\ell \in [0, 1] \). Therefore \( \frac{\partial U_\ell^E(\hat{x}_\ell, \alpha_\ell)}{\partial \alpha_\ell} < 0 \) if

\[
\frac{\rho}{2} (\hat{x} - \hat{x}_\ell)^2 + \frac{\delta \rho \Gamma}{2 \hat{x}_\ell} \left[ \frac{1}{4} (\hat{x} - \hat{x}_\ell) (3 \hat{x}_\ell + \hat{x}) \right] < 0,
\]

which holds for \( \hat{x}_\ell > \hat{x}_g \left( \frac{4 \Gamma - \delta \Gamma}{4 \Gamma + 3 \delta \Gamma} \right) \). Define \( x' = \max \{ \hat{x}, \hat{x}_g \left( \frac{4 \Gamma - \delta \Gamma}{4 \Gamma + 3 \delta \Gamma} \right) \} \). Then \( x' < \hat{x}_g \) because (i) \( \hat{x} < \hat{x}_g \) and (ii) \( \delta \Gamma > 0 \) implies \( \frac{4 \Gamma - \delta \Gamma}{4 \Gamma + 3 \delta \Gamma} < 1 \). Thus, \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( \frac{\partial U_\ell^E(\hat{x}_\ell, \alpha_\ell)}{\partial \alpha_\ell} < 0 \) for almost all \( \alpha_\ell \in [0, 1] \). Continuity implies \( U_\ell^E(\hat{x}_\ell, \alpha_\ell) \) strictly decreases in \( \alpha_\ell \) for such \( \hat{x}_\ell \).

Next, I prove the analogue of Proposition 2.

**Proposition A.2** Assume \( \hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell) \) and at least one of Assumptions A.1 and A.2 hold. If \( \hat{x}_g > 0 \), then there exist \( x' \) and \( x'' \) satisfying \( x' < \hat{x}_g < \bar{x}_\ell < x'' \) such that

1. if \( \hat{x}_\ell \in [x', \hat{x}_g) \), then \( \alpha_\ell = 0 \) is uniquely optimal;
2. if \( \hat{x}_\ell \in (\hat{x}_g, x'') \), then \( \alpha_\ell = 0 \) is not optimal; and
3. if \( \hat{x}_\ell \geq x'' \), then \( g \) is indifferent over \( \alpha_\ell \).

A symmetric result holds for \( \hat{x}_g < 0 \).

**Proof.** Consider \( \ell \in N^L \) with associated \( g \in N^G \). Assume \( \hat{x}_g \in (0, \bar{x}_\ell) \) and at least one of Assumptions A.1 and A.2 hold.

1. Since \( \hat{x}_g \in (0, \bar{x}_\ell) \), Lemma 3 yields existence of \( \bar{x} \in [0, \hat{x}_g) \) such that \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \) implies \( \hat{x}_g \in A(\sigma(\hat{x}_\ell, \alpha_\ell)) \) for all \( \alpha_\ell \in [0, 1] \). Consider \( \hat{x}_\ell \in (\bar{x}, \hat{x}_g) \). Then \( z_\ell = \hat{x}_\ell \in A(\sigma(\hat{x}_\ell, \alpha_\ell)) \) and \( y_\ell^* = y_\ell \in A(\sigma(\hat{x}_\ell, \alpha_\ell)) \) for all \( \alpha_\ell \in [0, 1] \). Thus, \( g \)'s ex ante expected utility equals \( U_\ell^E(\hat{x}_\ell, \alpha_\ell) \). By Lemma 6, there exists \( x' < \hat{x}_g \) such that \( \hat{x}_\ell \in (x', \hat{x}_g) \) implies \( U_\ell^E(\hat{x}_\ell, \alpha_\ell) \) strictly decreases in \( \alpha_\ell \). Therefore \( g \) strictly prefers \( \alpha_\ell = 0 \) if \( \hat{x}_\ell \in (\max\{x', \bar{x}\}, \hat{x}_g) \).

2. Assume \( \hat{x}_\ell \in (\hat{x}_g, x'') \), where \( x'' = 2\bar{x}_\ell - \hat{x}_g \). It suffices to show that \( g \)'s ex ante expected utility strictly increases at \( \alpha_\ell = 0 \). There are two cases.

- **Case 1:** If \( \hat{x}_\ell < \bar{x}_\ell \), then \( g \)'s ex ante expected payoff equals \( U_\ell^E(\hat{x}_\ell, \alpha_\ell) \) for sufficiently small \( \alpha_\ell \). By Lemma 4, \( \frac{\partial U_\ell^E(\hat{x}_\ell, \alpha_\ell)}{\partial \alpha_\ell} > 0 \). To complete this case, I argue that \( u_\ell^1(\hat{x}_\ell, \alpha_\ell) \) increases for sufficiently small \( \alpha_\ell \). Under the maintained assumptions, \( \hat{x}_g \in (-\bar{x}(\hat{x}_\ell, 0), \bar{x}(\hat{x}_\ell, 0)) \) and \( \hat{y}_\ell \in (\hat{x}_g, \bar{x}(\hat{x}_\ell, 0)) \). Thus, \( \bar{x}(\hat{x}_\ell, \alpha_\ell) \) strictly decreases for sufficiently small \( \alpha_\ell \). Therefore \( u_\ell(-\bar{x}(\hat{x}_\ell, \alpha_\ell)) \) and \( u_\ell(\bar{x}(\hat{x}_\ell, \alpha_\ell)) \) are strictly increasing for such \( \alpha_\ell \). Arguments from Proposition 6 imply \( m_\ell^j(\hat{x}_\ell, \alpha_\ell) \) weakly decreases in \( \alpha_\ell \) for all \( j \in N^g_\ell \setminus \{\ell\} \). Thus, \( U_\ell^E(\hat{x}_\ell, \alpha_\ell) \) strictly increases at \( \alpha_\ell = 0 \).
\begin{itemize}
\item \textit{Case 2}: If \( \hat{x}_\ell > \bar{x}_\ell \), then \( \bar{\pi}(\hat{x}_\ell, 0) = \bar{x}_\ell \). Thus, \( g \)'s ex ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\bar{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\bar{x}_\ell) \right)
\]

\[
+ \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \bar{E}_j^{LB}(\hat{x}_\ell, 0) + (1 - \alpha_j) E_j^{LB}(\hat{x}_\ell, 0) \right\} u_g(-\bar{x}_\ell)
\]

\[
+ \left[ \alpha_j \bar{E}_j^{UB}(\hat{x}_\ell, 0) + (1 - \alpha_j) E_j^{UB}(\hat{x}_\ell, 0) \right] u_g(\bar{x}_\ell)
\]

\[
+ \alpha_j \bar{C}_j(\hat{x}_\ell, 0) u_g(\hat{y}_j) + (1 - \alpha_j) C_j(\hat{x}_\ell, 0) u_g(\hat{x}_j)
\]

\[
- I_j^{\tilde{x}} \alpha_j m_g(\hat{x}_\ell, \alpha_\ell) \right\} . \tag{40}
\]

Arguments analogous to Case 1 show (40) strictly increases in \( \alpha_\ell \) at \( \alpha_\ell = 0 \).

3. Assume \( \hat{x}_\ell \geq x'' \), where \( x'' \) is defined as in Case 2 of Part 2. Then \( z_\ell = y_\ell^\ell = \pi(\hat{x}_\ell, \alpha_\ell) = \bar{x}_\ell \) for all \( \alpha_\ell \in [0, 1] \). Thus, \( g \)'s ex ante expected payoff is constant in \( \alpha_\ell \).
\end{itemize}

I prove the analogue of Proposition 3 from the main text.

\textbf{Proposition A.3} Assume \( \hat{x}_g \geq \bar{x}_\ell \), and \( \hat{x}_j, \hat{y}_j > -\pi(\hat{x}_\ell, 0) \) for all \( j \in N^L \).

\begin{enumerate}
\item If \( \hat{x}_\ell \geq \bar{x}_\ell \), then \( g \) is indifferent over \( \alpha_\ell \).
\item If \( \hat{x}_\ell \in [0, \bar{x}_\ell) \), then \( \alpha_\ell = 0 \) is not optimal.
\end{enumerate}

A symmetric result holds if \( \hat{x}_j, \hat{y}_j < -\pi(\hat{x}_\ell, 0) \) for all \( j \in N^L \).

\textit{Proof.} Consider \( \hat{x}_g \geq \bar{x}_\ell \). Assume \( \hat{x}_j, \hat{y}_j > -\pi(\hat{x}_\ell, 0) \) for all \( j \in N^L \).

\begin{enumerate}
\item If \( \hat{x}_\ell \geq \bar{x}_\ell \), then arguments analogous to Part 3 of Proposition A.2 show the result.
\item Assume \( \hat{x}_\ell \in [0, \bar{x}_\ell) \). I show that \( g \)'s ex ante expected utility strictly increases in \( \alpha_\ell \) at \( \alpha_\ell = 0 \).

We have \( \hat{x}_\ell \in [0, \pi(\hat{x}_\ell, 0)) \) and \( \hat{y}_\ell > \hat{x}_\ell \). Therefore \( 0 \leq z_\ell(\hat{x}_\ell, 0) = \hat{x}_\ell < y_\ell^\ell(\hat{x}_\ell, 0) \leq \hat{y}_\ell \). Furthermore, no \( j \in N^L \) proposes \( -\pi(\hat{x}_\ell, 0) \) because \( \hat{x}_j, \hat{y}_j > -\pi(\hat{x}_\ell, 0) \). Thus, \( g \)'s ex ante expected payoff from \( \alpha_\ell = 0 \) is

\[
\rho_\ell \left( \alpha_\ell \left[ u_g(y_\ell^\ell(\hat{x}_\ell, 0)) + u_\ell(y_\ell^\ell(\hat{x}_\ell, 0)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right)
\]

41
\[
+ \sum_{j \neq \ell} \rho_j \left\{ \alpha_j \tilde{E}_j^{UB}(\hat{x}_\ell, 0) + (1 - \alpha_j) \tilde{E}_j^{LB}(\hat{x}_\ell, 0) \right\} u_g(\bar{x}(\hat{x}_\ell, 0)) \\
+ \alpha_j \left[ C_j(\hat{x}_\ell, 0) u_g(\hat{y}_j) - I_g^j m_g^j(\hat{x}_\ell, 0) \right] + (1 - \alpha_j) C_j(\hat{x}_\ell, 0) u_g(\hat{x}_j). \right\} 
\]

(41)

I now use three steps to show (41) strictly increases in \( \alpha_\ell \) at \( \alpha_\ell = 0 \).

- First, \( 0 \leq \hat{x}_\ell < y_g^f(\hat{x}_\ell, 0) \leq \hat{y}_\ell \) implies \( y_g^f(\hat{x}_\ell, 0) \) weakly increases in \( \alpha_\ell \). Therefore \( u_g(y_g^f(\hat{x}_\ell, \alpha_\ell)) \) weakly increases and \( u_\ell(y_g^f(\hat{x}_\ell, \alpha_\ell)) \) weakly decreases. Because \( u \) is quadratic and \( \hat{x}_\ell < y_g^f(\hat{x}_\ell, 0) \leq \hat{y}_\ell = \frac{x_g + \hat{x}_\ell}{2} < \hat{x}_g \), it follows that \( u_g(y_g^f(\hat{x}_\ell, \alpha_\ell)) \) increases weakly faster than \( u_\ell(y_g^f(\hat{x}_\ell, \alpha_\ell)) \) decreases. Therefore \( u_g(y_g^f(\hat{x}_\ell, 0)) + u_\ell(y_g^f(\hat{x}_\ell, 0)) - u_\ell(\hat{x}_\ell) \) weakly increases in \( \alpha_\ell \). Furthermore, \( \hat{x}_\ell < y_g^f(\hat{x}_\ell, 0) \leq \hat{y}_\ell < \hat{x}_g \) also implies \( u_g(y_g^f(\hat{x}_\ell, 0)) + u_\ell(y_g^f(\hat{x}_\ell, 0)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0 \). This shows \( \alpha_\ell \left[ u_g(y_g^f(\hat{x}_\ell, 0)) + u_\ell(y_g^f(\hat{x}_\ell, 0)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \) weakly increases at \( \alpha_\ell = 0 \).

- Second, \( 0 \leq z_\ell < y_g^f(\hat{x}_\ell, 0) \leq \bar{x}(\hat{x}_\ell, 0) \) implies \( \bar{x}(\hat{x}_\ell, 0) \) strictly increases in \( \alpha_\ell \). Since \( \bar{x}(\hat{x}_\ell, 0) < \hat{x}_g \), it follows that \( u_g(\bar{x}(\hat{x}_\ell, 0)) \) increases at \( \alpha_\ell = 0 \).

- Third, Proposition 6 implies \( m_g^j(\hat{x}_\ell, 0) \) weakly increases in \( \alpha_\ell \) for all \( j \in N_g^L \). However, \( \hat{y}_j > \bar{x}(\hat{x}_\ell, 0) \) for all \( j \in N_g^L \) such that \( m_g^j(\hat{x}_\ell, 0) \) strictly increases in \( \alpha_\ell \), which implies \( g \)’s lobbying surplus weakly increases in \( \alpha_\ell \) for any such \( j \in N_g^L \).

Thus, (41) strictly increases in \( \alpha_\ell \) at \( \alpha_\ell = 0 \).

**Willingness to Pay for Access**

The following results apply to the model in the main text. Recall \( \theta = (\hat{x}, \rho, \alpha) \). Let \( U_g^E(\theta) \) be \( g \)’s ex ante expected utility. Additionally, let \( \bar{x}_\alpha = \bar{x}(\hat{x}_\ell, \alpha) \) denote the upper bound of the social acceptance set given \( \hat{x}_\ell \) and \( \alpha \). Next, define \( \frac{\partial U_g^E}{\partial \alpha} = \frac{\partial U_g^E}{\partial \alpha} |_{\alpha=0} \), \( \frac{\partial U_g^E}{\partial x_g} = \frac{\partial U_g^E}{\partial x_g} |_{\alpha=0} \), and \( \frac{\partial^2 U_g^E}{\partial \alpha \partial x_g} = \frac{\partial^2 U_g^E}{\partial \alpha \partial x_g} |_{\alpha=0} \). Finally, let \( g \)'s *willingness to acquire access* refer to \( \frac{\partial U_g^E(\theta)}{\partial \alpha} |_{\alpha=0} \).

To state Proposition 4, I modify the baseline model to compare across distinct legislator-group pairs. Specifically, consider the baseline model but replace \( \ell \) with two arbitrary legislators, \( \ell_1 \) and \( \ell_2 \), and replace \( g \) with two groups, \( g_1 \) and \( g_2 \). To isolate differences in proposal power, assume \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), but \( \rho_{\ell_1} \neq \rho_{\ell_2} \). Also, assume \( \hat{x}_{g_1} = \hat{x}_{g_2} \). I use two identical groups to avoid complications arising if one group has access to two legislators, because the group accounts for how access to one legislator affects its offer to the other. These modifications do not qualitatively change the equilibrium characterization.
Proposition 4. Consider the modified baseline model with: (i) \( \ell_1 \) and \( \ell_2 \) such that \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), and (ii) \( g_1 \) and \( g_2 \) satisfying \( \hat{x}_{g_1} = \hat{x}_{g_2} \). For all \( \alpha \in [0,1] \), \( \rho_{\ell_2} > \rho_{\ell_1} \) implies \( g_2 \)'s WTP for \( \alpha \) access to \( \ell_2 \) is weakly greater than \( g_1 \)'s WTP for \( \alpha \) access to \( \ell_1 \). A symmetric result holds if \( \rho_{\ell_1} > \rho_{\ell_2} \).

Proof. Consider the modified setting with (i) \( \ell_1 \) and \( \ell_2 \) such that \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), and (ii) \( g_1 \) and \( g_2 \) such that \( \hat{x}_{g_1} = \hat{x}_{g_2} \). Assume \( \rho_{\ell_2} > \rho_{\ell_1} \). It suffices to show \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \geq 0 \) implies \( \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \geq 0 \) for all \( \alpha \in [0,1] \).

Because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \) and \( \hat{x}_{g_1} = \hat{x}_{g_2} \), we have \( y_{g_1} = y_{g_2} \) and \( z_{\ell_1} = z_{\ell_2} \). Thus, \( m_{g_1} = m_{g_2} \). For convenience, let \( y = y_{g_1} \), \( z = z_{\ell_1} \), and \( m = m_{g_1} \). Fix \( \alpha \in [0,1] \). Assume \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \geq 0 \).

There are five cases.

- **Case 1:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \hat{x}_\ell \) and \( y = \hat{y} \). Then,

\[
\frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} = \rho_{\ell_1} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) - \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\]

\[
\leq \rho_{\ell_2} \left( u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) - \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \left( \rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right)
\]

\[
= \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha},
\]

where (42) follows from \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} = \frac{\delta p_{\ell_1}[u_M(\hat{y})-u_M(\hat{x}_\ell)]}{\frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}[1-\delta(\rho_L+\rho_R)]} \); (43) because (i) \( \rho_{\ell_2} > \rho_{\ell_1} \) and (ii) \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} \geq 0 \) implies the bracketed expression in (42) is positive; and (44) because \( \hat{x}_{\ell_1} = \hat{x}_{\ell_2} \), \( \hat{x}_{g_1} = \hat{x}_{g_2} \), and \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} = \frac{\delta p_{\ell_2}[u_M(\hat{y})-u_M(\hat{x}_\ell)]}{\frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}[1-\delta(\rho_L+\rho_R)]} \). Therefore \( \psi_{\alpha}^2(\rho_{\ell_2}, \theta) \geq \psi_{\alpha}^1(\rho_{\ell_1}, \theta) \).

- **Case 2:** Consider \( \hat{x}_\ell \) and \( \hat{x}_g \) such that \( z = \bar{x}_\alpha \) and \( y = \hat{y} \). In this case, \( \frac{\partial U_{g_1}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} = \frac{\delta p_{\ell_1}[u_M(\hat{y})-u_M(\hat{x}_\ell)]}{\frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}[1-\delta(\rho_L+\rho_R)+1-(\rho_1+\rho_2)]} \) and \( \frac{\partial U_{g_2}^E(\theta)}{\partial \alpha_t}|_{\alpha_t=\alpha} = \frac{\delta p_{\ell_2}[u_M(\hat{y})-u_M(\bar{x}_\alpha)]}{\frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha}[1-\delta(\rho_L+\rho_R)+1-(\rho_1+\rho_2)]} \). Arguments
analogous to Case 1 show \( \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \geq \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \). The argument for \( z = -\pi_{\alpha} \) and \( y = \hat{y} \) is symmetric.

- **Case 3**: Consider \( \hat{x}_{\ell} \) and \( \hat{x}_{g} \) such that \( z = \hat{x}_{\ell} \) and \( y = \pi_{\alpha} \). In this case, \( \frac{\partial \pi_{\alpha}}{\partial \alpha_{\ell_{1}}} = \frac{\delta_{\alpha_{\ell_{1}}}[u_{M}(\pi_{\alpha})-u_{M}(\hat{x}_{\ell})]}{\partial \pi_{\alpha}} \) and \( \frac{\partial \pi_{\alpha}}{\partial \alpha_{\ell_{2}}} = \frac{\delta_{\alpha_{\ell_{2}}}[u_{M}(\pi_{\alpha})-u_{M}(\hat{x}_{\ell})]}{\partial \pi_{\alpha}} \). Arguments analogous to Case 1 show \( \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \geq \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \). The argument for \( z = \hat{x}_{\ell} \) and \( y = -\pi_{\alpha} \) is symmetric.

- **Case 4**: Consider \( \hat{x}_{\ell} \) and \( \hat{x}_{g} \) such that \( z = \pi_{\alpha} \) and \( y = -\pi_{\alpha} \). In this case, \( \frac{\partial \pi_{\alpha}}{\partial \alpha_{\ell_{1}}} = \frac{\delta_{\alpha_{\ell_{1}}}[u_{M}(\pi_{\alpha})-u_{M}(\pi_{\alpha})]}{\partial \pi_{\alpha}} \) and \( \frac{\partial \pi_{\alpha}}{\partial \alpha_{\ell_{2}}} = \frac{\delta_{\alpha_{\ell_{2}}}[u_{M}(\pi_{\alpha})-u_{M}(\pi_{\alpha})]}{\partial \pi_{\alpha}} \). Arguments analogous to Case 1 show \( \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \geq \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \). The argument for \( z = -\pi_{\alpha} \) and \( y = \pi_{\alpha} \) is symmetric.

- **Case 5**: Consider \( \hat{x}_{\ell} \) and \( \hat{x}_{g} \) such that \( z = \pi_{\alpha} \) and \( y = \pi_{\alpha} \). Then, \( \psi_{\alpha}^{2}(\rho_{\ell_{2}}, \theta) = \psi_{\alpha}^{1}(\rho_{\ell_{1}}, \theta) = 0 \). The argument for \( z = -\pi_{\alpha} \) and \( y = -\pi_{\alpha} \) is symmetric.

In each case, \( \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \geq \frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=\alpha} \).

**Proposition 5.** Assume minority-party agenda exclusion and \( \ell \) is majority-leaning. If either: (i) \( g \) is more centrist than \( \ell \), or (ii) \( g \) is majority-leaning and more extreme than \( \ell \), then \( g \)'s willingness to acquire access weakly decreases as \( |\hat{x}_{g} - \hat{x}_{\ell}| \) decreases.

**Proof.** Without loss of generality, assume \( \rho_{L} = 0 \) and \( \hat{x}_{\ell} \geq 0 \).

First, \( g \)'s ex ante expected utility for \( \alpha \in [0, 1] \) is

\[
U_{g}^{E}(\theta) = \rho_{\ell} \left( \alpha[u_{g}(y) + \ell(y) - u_{\ell}(\pi_{\alpha})] + (1 - \alpha)u_{\ell}(z_{\ell}) \right) + \rho_{M}u_{g}(0) + \rho_{R}u_{g}(\pi_{\alpha}). \tag{45}
\]

Thus, \( g \)'s willingness to acquire access to \( \ell \) is

\[
\frac{\partial U_{g}^{E}(\theta)}{\partial \alpha_{\ell}}|_{\alpha_{\ell}=0} = \rho_{\ell} \left( u_{g}(y) - u_{g}(z_{\ell}) + u_{\ell}(y) - u_{\ell}(z_{\ell}) \right) + \rho_{R} \frac{\partial u_{g}(\pi_{0})}{\partial \pi_{0}} \frac{\partial \pi_{0}}{\partial \alpha}. \tag{46}
\]

The cross-partial with respect to \( \hat{x}_{g} \) satisfies

\[
\frac{\partial^{2} U_{g}^{E}(\theta)}{\partial \alpha_{\ell} \partial \hat{x}_{g}}|_{\alpha_{\ell}=0} = \rho_{\ell} \left\{ \left( \frac{\partial u_{g}(y)}{\partial y} + \frac{\partial u_{\ell}(y)}{\partial y} \right) \frac{\partial y}{\partial \hat{x}_{g}} + \frac{\partial u_{g}(y)}{\partial \hat{x}_{g}} - \frac{\partial u_{g}(z_{\ell})}{\partial \hat{x}_{g}} \right\}.
\]
\[ \begin{align*}
+ \rho_R \left( \frac{\partial^2 u_g(x_0)}{\partial x_0^2} \frac{\partial x_0}{\partial \alpha} \frac{\partial x_0}{\partial \alpha} + \frac{\partial u_g(x_0)}{\partial x_0} \frac{\partial^2 x_0}{\partial \alpha \partial \hat{x}_g} \right) \\
= \rho_R \left( \frac{\partial u_g(y)}{\partial \hat{x}_g} - \frac{\partial u_g(z_\ell)}{\partial \hat{x}_g} \right) + \rho_R \left( \frac{\partial^2 u_g(x_0)}{\partial x_0^2} \frac{\partial x_0}{\partial \alpha} \frac{\partial x_0}{\partial \alpha} + \frac{\partial u_g(x_0)}{\partial x_0} \frac{\partial^2 x_0}{\partial \alpha \partial \hat{x}_g} \right),
\end{align*} \tag{47} \]

where (48) follows because either (i) \( y = x_0 \), which implies \( \frac{\partial y}{\partial \hat{x}_g} = 0 \), or (ii) \( y = \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \), which implies \( \frac{\partial u_g(y)}{\partial y} = -\frac{\partial u_g(y)}{\partial y} \).

**Part (i)** Consider \( \hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell] \). There are two cases.

**Case 1:** Suppose \( \hat{x}_\ell = x_0 \). Since \( \hat{x}_g \geq -\hat{x}_\ell \), we have \( \hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2} \geq 0 \). There are two subcases.

- First, assume \( \hat{x}_g \geq 2x_0 - \hat{x}_\ell \). Then \( y = z_\ell = x_0 \). For \( \alpha \in [0, 1] \), if \( y = z_\ell = x_\alpha \), then \( x_\alpha \) solves

\[ 0 = (1 - \delta) u_M(q) + \delta \rho_M u_M(0) - [1 - \delta(\rho_R + \rho_\ell)] u_M(x_\alpha). \tag{49} \]

Applying the implicit function theorem to (49) yields \( \frac{\partial x_\alpha}{\partial \alpha} = 0 \) and thus \( \frac{\partial x_0}{\partial \alpha} = 0 \). Therefore \( \frac{\partial u_{\ell M}(\theta)}{\partial \alpha} |_{\alpha=0} = 0 \), so \( \frac{\partial u_{\ell M}(\theta)}{\partial \alpha} |_{\alpha=0} \) is constant in \( \hat{x}_g \).

- Second, assume \( \hat{x}_g < 2x_0 - \hat{x}_\ell \). Then \( y = \hat{y} \). For \( \alpha \in [0, 1] \), if \( y = \hat{y} \) and \( z_\ell = x_\alpha \), then \( x_\alpha \) solves

\[ 0 = (1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \alpha \rho_\ell u_M(\hat{y}) \right) - \left( 1 - \delta(\rho_R + (1 - \alpha)\rho_\ell) \right) u_M(x_\alpha). \tag{50} \]

Applying the implicit function theorem to (50) yields

\[ \frac{\partial x_\alpha}{\partial \alpha} = \frac{\delta \rho_\ell [u_M(\hat{y}) - u_M(x_\alpha)]}{(1 - \delta(\rho_R + (1 - \alpha)\rho_\ell)) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}} \tag{51} \]

\[ \frac{\partial x_\alpha}{\partial \hat{x}_g} = \frac{\alpha \delta \rho_\ell \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta(\rho_R + (1 - \alpha)\rho_\ell)) \frac{\partial u_M(x_\alpha)}{\partial x_\alpha}} \tag{52} \]

and

\[ \frac{\partial^2 x_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho_\ell}{(1 - \delta(\rho_R + (1 - \alpha)\rho_\ell))} \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} - \frac{\partial^2 u_M(x_\alpha)}{\partial x_\alpha^2} \frac{\partial x_\alpha}{\partial \hat{x}_g} \frac{\partial x_\alpha}{\partial \alpha} \right) \left( \frac{\partial u_M(x_\alpha)}{\partial x_\alpha} \right)^{-1}. \tag{53} \]
Inspecting (52) reveals $\frac{\partial \pi_0}{\partial x_g} = 0$. Therefore,

$$\frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho_\ell \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta \rho_R + \rho_\ell) \frac{\partial u_M(x_0)}{\partial x_0}} > 0,$$

which follows because (i) $\frac{\partial \hat{y}}{\partial \hat{x}_g} > 0$ and (ii) $0 < \hat{y} < \pi_0$ implies $\frac{\partial u_M(y)}{\partial y} < 0$ and $\frac{\partial u_M(x_0)}{\partial x_0} < 0$. Thus,

$$\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha = 0} = \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(x_0)}{\partial \hat{x}_g} \right) + \rho_R \frac{\partial u_g(x_0)}{\partial x_0} \frac{\partial^2 x_0}{\partial \alpha \partial \hat{x}_g} \quad (54)$$

$$< \rho_\ell \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(x_0)}{\partial \hat{x}_g} \right) \quad (55)$$

$$< 0, \quad (56)$$

where (55) follows because (i) $\frac{\partial^2 x_0}{\partial \alpha \partial \hat{x}_g} > 0$ and (ii) $\hat{x}_g < \pi_0$ implies $\frac{\partial u_g(x_0)}{\partial x_0} < 0$; and (56) because $\hat{x}_g < \hat{y} < \pi_0$ implies $\frac{\partial u_g(\hat{x}_0)}{\partial \hat{x}_g} < \frac{\partial u_g(x_0)}{\partial x_0}$.

Thus, $g$’s willingness to acquire access to $\ell$ weakly decreases in $\hat{x}_g$.

**Case 2:** Consider $\hat{x}_\ell < \pi_0$. Then $z_\ell = \hat{x}_\ell$. Furthermore, $\hat{x}_g \in [-\hat{x}_\ell, \hat{x}_\ell]$ implies $y = \hat{y} \geq 0$. For $\alpha \in [0, 1]$, if $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$, then $\pi_\alpha$ solves

$$u_M(\pi_\alpha) = (1 - \delta) u_M(q) + \delta \left( \rho_M u_M(0) + \rho_\ell [\alpha u_M(\hat{y}) + (1 - \alpha) u_M(\hat{x}_\ell)] \right) \quad (57)$$

Applying the implicit function theorem yields

$$\frac{\partial \pi_\alpha}{\partial \alpha} = \frac{\delta \rho_\ell [u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{(1 - \delta \rho_R) \frac{\partial u_M(\pi_\alpha)}{\partial \pi_\alpha}}, \quad (58)$$

$$\frac{\partial \pi_\alpha}{\partial \hat{x}_g} = \frac{\alpha \delta \rho_\ell \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g}}{(1 - \delta \rho_R) \frac{\partial u_M(\pi_\alpha)}{\partial \pi_\alpha}}, \quad (59)$$

and

$$\frac{\partial^2 \pi_\alpha}{\partial \alpha \partial \hat{x}_g} = \left( \frac{\delta \rho_\ell}{(1 - \delta \rho_R)} \frac{\partial u_M(\hat{y})}{\partial \hat{x}_g} \frac{\partial \hat{y}}{\partial \hat{x}_g} - \frac{\partial^2 u_M(\pi_\alpha)}{\partial \pi_\alpha \partial \pi_\alpha} \frac{\partial \pi_\alpha}{\partial \hat{x}_g} \frac{\partial \pi_\alpha}{\partial \alpha} \right) \left( \frac{\partial u_M(\pi_\alpha)}{\partial \pi_\alpha} \right)^{-1}. \quad (56)$$

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Inspecting (59) reveals \( \frac{\partial^2 \pi_0}{\partial x_g} = 0 \), which implies

\[
\frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} = \frac{\delta \rho \epsilon \left( \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \right) \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta \rho R) \frac{\partial u_M(\pi_0)}{\partial \pi_0}} > 0. \tag{60}
\]

Because \( 0 \leq \hat{y} < \hat{x}_g \), a series of inequalities analogous to (54)–(56) imply \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} < 0 \).

In both cases, \( g \)'s willingness to acquire access weakly decreases in \( |\hat{x}_g - \hat{x}_\ell| \).

**Part (ii)** Assume \( \hat{x}_g \geq \hat{x}_\ell \). There are two cases.

**Case 1:** Consider \( \hat{x}_\ell \geq \pi_0 \). Then \( y = z_\ell = \pi_0 \) at \( \alpha = 0 \), which implies \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} = 0 \).

**Case 2:** Consider \( \hat{x}_\ell < \pi_0 \). Then \( z_\ell = \hat{x}_\ell \). There are three subcases.

- First, assume \( \hat{x}_g \in [\hat{x}_\ell, \pi_0] \). Then \( y = \hat{y} \). I show \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} \geq 0 \) implies \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} > 0 \). Since \( y = \hat{y} \) and \( z_\ell = \hat{x}_\ell \), Case 2 of Part 2 implies \( \frac{\partial \pi_0}{\partial x_g} \) is given by (58), \( \frac{\partial \pi_0}{\partial x_g} = 0 \), and \( \frac{\partial \pi_0}{\partial x_g} \) is (60). Therefore

\[
\frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} = \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\pi_0)}{\partial \pi_0} \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} \right)

= \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right) + \rho R \left( \frac{\partial u_g(\pi_0)}{\partial \pi_0} \delta \rho \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g} \right) \tag{61}
\]

\[
\geq \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right)

- \rho R \rho \left[ \frac{u_g(\hat{y}) - u_g(\hat{x}_\ell) + u_\ell(\hat{y})}{\rho \frac{\partial \pi_0}{\partial \alpha}} \right] \frac{\delta \rho \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g}}{(1 - \delta \rho R) \frac{\partial u_M(\pi_0)}{\partial \pi_0}} \tag{62}
\]

\[
= \rho \left( \frac{\partial u_g(\hat{y})}{\partial \hat{x}_g} - \frac{\partial u_g(\hat{x}_\ell)}{\partial \hat{x}_g} \right)

- \rho \left[ \frac{\partial u_M(\hat{y})}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial \hat{x}_g} \right] \left( \frac{u_g(\hat{y}) - u_g(\hat{x}_\ell) + u_\ell(\hat{y})}{u_M(\hat{y}) - u_M(\hat{x}_\ell)} \right) \tag{63}
\]

\[
= \frac{5 \rho \epsilon}{4} (\hat{x}_g - \hat{x}_\ell) \tag{64}
\]

\[
> 0, \tag{65}
\]

where (61) follows from using (60) to substitute for \( \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} \); (62) because (i) \( \frac{\partial^2 \pi_0}{\partial \alpha \partial \hat{x}_g} > 0 \) and (ii) \( \psi(\hat{x}_g, \theta) \geq 0 \) and \( \frac{\partial \pi_0}{\partial \alpha} > 0 \) together imply \( \frac{\partial u_M(\pi_0)}{\partial \pi_0} \geq -\frac{\rho \epsilon [u_g(\hat{y}) - u_g(\hat{x}_\ell) + u_\ell(\hat{y})]}{\rho \frac{\partial \pi_0}{\partial \alpha}} \); (63) from using (58) to substitute for \( \frac{\partial \pi_0}{\partial \alpha} \) and simplifying; (64) from simplifying; and (65) because \( \hat{x}_g > \hat{x}_\ell \). Thus, \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} \geq 0 \) implies \( \frac{\partial^2 U^E(\theta)}{\partial \alpha \partial \hat{x}_g} |_{\alpha \ell = 0} > 0 \).
Proof. Without loss of generality, assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \). There exists \( x' > \hat{x}_\ell \) such that \( \alpha_\ell > 0 \) only if \( \hat{x}_g > x' \). An analogous result holds if \( \rho_R = 0 \) and \( \hat{x}_\ell \leq 0 \).

Lemma 7. Assume \( \rho_L = 0 \) and \( \hat{x}_\ell \geq 0 \). There exists \( x' > \hat{x}_\ell \) such that \( \alpha_\ell > 0 \) only if \( \hat{x}_g > x' \). An analogous result holds if \( \rho_R = 0 \) and \( \hat{x}_\ell \leq 0 \).

Proof. Without loss of generality, assume \( \rho_L = 0 \) and \( 0 \leq \hat{x}_\ell \).

If \( \hat{x}_\ell \geq \overline{x}_0 \), then \( \hat{x}_g > \hat{x}_\ell \) implies \( \alpha_\ell = 0 \). As in the proof of Proposition 5, \( \frac{\partial U^E(\theta)}{\partial \alpha_\ell}|_{\alpha_\ell=0} = 0 \) for all \( \alpha \in [0, 1] \). Setting \( x' = \infty \) delivers the result.
Next, suppose $\hat{x}_g < \bar{x}_0$. Because $g$’s willingness to acquire access weakly increases in $\hat{x}_g$, it suffices to show that (i) $\left. \frac{\partial U^E_g(\theta)}{\partial \alpha} \right|_{\alpha = 0} \leq 0$ for $\hat{x}_g$ sufficiently close to $\hat{x}_\ell$ and (ii) $\left. \frac{\partial U^E_g(\theta)}{\partial \alpha} \right|_{\alpha = \alpha} \leq 0$ decreases in $\alpha$ for such $\hat{x}_g$.

An argument analogous to Part 1 of Proposition 2 shows existence of $\hat{x}_g' > \hat{x}_\ell$ such that $\left. \frac{\partial U^E_g(\theta)}{\partial \alpha} \right|_{\alpha = 0} \leq 0$ if $\hat{x}_g \in [\hat{x}_\ell, \hat{x}_g']$. Also, $\left. \frac{\partial U^E_g(\theta)}{\partial \alpha} \right|_{\alpha = 0} > 0$ for $\hat{x}_g \geq \bar{x}_0$ because $\rho_L = 0$, which implies $\hat{x}_g' \in (\hat{x}_\ell, \bar{x}_0)$.

Consider $\hat{x}_g \in [\hat{x}_\ell, \hat{x}_g']$. I show $\left. \frac{\partial^2 U^E_g(\theta)}{\partial \alpha^2} \right|_{\alpha = \alpha} < 0$. Applying the implicit function theorem to (57) yields

$$\frac{\partial^2 \bar{x}_g}{\partial \alpha^2} = -\frac{\frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial x^2}}{\partial \alpha^2} < 0 \quad (70)$$

because $\frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial x^2} < 0$ and $\hat{x}_g < \bar{x}_\alpha$ implies $\frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} < 0$.

We have $y = \hat{y}$ and $z_\ell = \hat{x}_\ell$, so

$$\frac{\partial^2 U^E_g(\theta)}{\partial \alpha^2} = \rho_R \left( \frac{\partial^2 u_g(\bar{x}_\alpha)}{\partial \alpha^2} \left( \frac{\partial \bar{x}_g}{\partial \alpha} \right)^2 + \frac{\partial u_g(\bar{x}_\alpha)}{\partial \alpha} \frac{\partial^2 \bar{x}_g}{\partial \alpha^2} \right)$$

$$= \rho_R \left( \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \alpha^2} \left( \frac{\partial \bar{x}_g}{\partial \alpha} \right)^2 + \frac{\partial u_g(\bar{x}_\alpha)}{\partial \alpha} \frac{\partial^2 \bar{x}_g}{\partial \alpha^2} \right)$$

$$< \rho_R \left( \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \alpha^2} \left( \frac{\partial \bar{x}_g}{\partial \alpha} \right)^2 + \frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} \frac{\partial^2 \bar{x}_g}{\partial \alpha^2} \right) \quad (71)$$

$$= 0, \quad (73)$$

where (71) follows from $\frac{\partial^2 u_g(\bar{x}_\alpha)}{\partial \alpha^2} = \frac{\partial^2 u_M(\bar{x}_\alpha)}{\partial \alpha^2}$ because $u$ is quadratic; (72) because $\frac{\partial^2 \bar{x}_g}{\partial \alpha^2} < 0$ and $0 < \hat{x}_g < \bar{x}_\alpha$ implies $\frac{\partial u_M(\bar{x}_\alpha)}{\partial \alpha} < \frac{\partial u_g(\bar{x}_\alpha)}{\partial \alpha} < 0$; and (73) from (70). Thus, $\left. \frac{\partial U^E_g(\theta)}{\partial \alpha} \right|_{\alpha = 0} \leq 0$ for all $\alpha \in [0, 1]$. Proposition 5 delivers the result.\[0x]
Appendix B

A strategy profile \( \sigma = (\lambda, \pi, \phi, \nu) \) is a \textit{mixed strategy stationary legislative lobbying equilibrium} if it satisfies four conditions. First, for all \( g \in N^G \) and \( \ell \in N^L_g \), \( \lambda^g_\ell \) places probability one solutions to

\[
\arg\max_{(y,m)} \nu(y,m) \left[ u_\ell(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] - m \\
\text{s.t.} \nu(y,m) \left[ u_\ell(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] + m \geq \int_X \left[ u_\ell(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] \pi_\ell(dx). \tag{74}
\]

Second, for all \( \ell \in N^L \) and offers \( (y,m) \in W \),

\[
\nu(y,m) \left[ u_\ell(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] + m > \int_X \left[ u_\ell(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] \pi_\ell(dx). \tag{75}
\]

implies \( \varphi_\ell(y,m) = 1 \) and the opposite strict inequality implies \( \varphi_\ell(y,m) = 0 \). Third, for each \( \ell \in N^L \),

\[
\pi_\ell \left( \arg\max_{x \in X} \nu(x) \left[ u_\ell(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] \right) = 1. \tag{76}
\]

Finally, for all \( i \in N^V \) and \( x \in X \), \( u_i(x) > (1 - \delta)u_i(q) + \delta V_i \) implies \( \nu_i(x) = 1 \) and the opposite strict inequality implies \( \nu_i(x) = 0 \).\(^{28}\)

Lemma B.1 verifies that groups never offer surplus lobby payment in equilibrium. The proof is straightforward and omitted.

Lemma B.1. In every stationary legislative lobbying equilibrium, for all \( \ell \in N^L \) every \( (y,m) \in \text{supp}(\lambda^g_\ell) \) satisfies

\[
\nu(y,m) \left[ u_\ell(y) + [1 - \nu(y)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] + m = \int_X \left[ u_\ell(x) + [1 - \nu(x)][(1 - \delta)u(q) + \delta \tilde{V}_\ell] \right] \pi_\ell(dx). \tag{77}
\]

\(^{28}\)This condition implies that voters use \textit{stage-undominated} voting strategies (Baron and Kalai, 1993; Banks and Duggan, 2006a).
Deferential Voting and Acceptance

Let \( \sigma \) be a stationary strategy profile. Recall \( \xi(\alpha; \sigma) = (1 - \alpha) + \alpha \int_W [1 - \varphi(y, m)] \lambda_g^\ell(dw) \), as defined in (8). Define

\[
\hat{\chi}(Y) = \sum_{\ell \in N^L} \rho_\ell \left\{ \xi(\alpha; \sigma) \int_X \nabla_\sigma(x) \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m) \nabla_\sigma(y) \lambda_g^\ell(dw) \right\},
\]

which is the probability some \( x \in Y \) is passed in a given period under \( \sigma \). Next, define

\[
\check{\chi} = \sum_{\ell \in N^L} \rho_\ell \left\{ \xi(\alpha; \sigma) \int_X [1 - \nabla_\sigma(x)] \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m) [1 - \nabla_\sigma(y)] \lambda_g^\ell(dw), \right\}
\]

the probability of a failed proposal in a given period under \( \sigma \).

Each player’s continuation value can be expressed as a function of a common lottery over policy, denoted \( \chi_\sigma \). Using (78) and (79), define \( \chi_\sigma \) so that for all measurable \( Y \subseteq X \): (i) if \( q \notin Y \), then \( \chi_\sigma(Y) = \hat{\chi}(Y) \frac{1 - \delta}{1 - \delta \check{\chi}} \), and (ii) if \( q \in Y \), then \( \chi_\sigma(Y) = \frac{\check{\chi}(Y) + (1 - \delta) \check{\chi}}{1 - \delta \check{\chi}} \).

Set \( V^{\text{den}}(\sigma) = 1 - \delta \check{\chi} \). Then, for each \( i \in N^V \), express \( i \)'s continuation value defined in (9) as \( V_i(\sigma) = \frac{V_i^{\text{num}}(\sigma)}{V_i^{\text{den}}(\sigma)} \), where

\[
V_i^{\text{num}}(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left\{ \xi(\alpha; \sigma) \int_X \nabla_\sigma(x), u_\ell(x) + [1 - \nabla_\sigma(x)] (1 - \delta) u_\ell(q) \right\} \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m) \left[ \nabla_\sigma(y) u_\ell(x) + [1 - \nabla_\sigma(y)] (1 - \delta) u_\ell(q) \right] \lambda_g^\ell(dw) \right\}.
\]

We can express \( V_i(\sigma) \) explicitly as a lottery over policy:

\[
V_i(\sigma) = \int_X u_\ell(x) \chi_\sigma(dx).
\]

The policy lottery \( \chi_\sigma \) is common to all players, but committee members may receive payment and interest groups may make payments. Define

\[
\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \int_W m \varphi_\ell(y, m) \lambda_g^\ell(dw),
\]

which is \( \ell \)'s expected lobby payment in each period until passage. Re-arranging (10) for each \( \ell \in N^L \) yields

\[
\tilde{V}_\ell(\sigma) = \frac{V_\ell^{\text{num}}(\sigma) + \hat{m}_\ell(\sigma)}{V_i^{\text{den}}(\sigma)}
\]

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Similarly, rearrange (11) for each $g \in N^G$ yields

$$
\hat{V}_g(\sigma) = \frac{V^\text{num}_g(\sigma) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)}{V^\text{den}(\sigma)}
$$

$$
= \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N^L_g} \hat{m}_\ell(\sigma)
$$

(82)

Finally, define

$$
\hat{U}_\ell(\pi_\ell; \sigma) = \int_X \left[ \nu_\ell(x) u_\ell(x) + \left( 1 - \nu_\ell(x) \right) \left( (1 - \delta) u_\ell(q) + \delta \hat{V}_\ell(\sigma) \right) \right] \pi_\ell(dx)
$$

(83)

which is $\ell$’s expected dynamic payoff under $\sigma$, conditional on being recognized as the proposer and rejecting $g_\ell$’s offer.

**Lemma B.2.** There does not exist a stationary legislative lobbying equilibrium $\sigma$ such that $\chi^\sigma$ is degenerate on $q$.

**Proof.** Let $\sigma$ denote an equilibrium. To show a contradiction, assume $\chi^\sigma$ is degenerate on $q$. Thus, $V_M(\sigma) = u_M(q)$. Therefore $u_M(q) \geq (1 - \delta) u_M(q) + \delta V_M(\sigma)$, so $q \in A(\sigma)$. Without loss of generality, assume $q > 0$.

By assumption, there exists $\ell \in N^L$ on the same side of $q$ as $M$ who is not influenced by a group on the opposite side of $q$. Note that $u_{g_\ell}(y) + u_\ell(y) - \hat{U}_\ell(\pi_\ell; \sigma)$ is $g_\ell$’s expected dynamic payoff from any offer $(y, m)$ such that: (i) $\nu_\ell(y) = 1$, (ii) $\ell$ is indifferent between accepting and rejecting, and (iii) $\varphi_\ell(y, m) = 1$. Furthermore, $\hat{y}_\ell = \arg \max_{g \in X} u_{g_\ell}(y) + u_\ell(y) - \hat{U}_\ell(\pi_\ell; \sigma)$ because $\hat{U}_\ell(\pi_\ell; \sigma)$ does not depend on $y$. Under the maintained assumptions, $\hat{y}_\ell < q$. Strict concavity implies that for sufficiently small $\varepsilon > 0$ there exists $y \in A(\sigma)$ such that

$$
u_{g_\ell}(y) + u_\ell(y) - \hat{U}_\ell(\pi_\ell; \sigma) + \varepsilon
$$

$$> u_{g_\ell}(q) + u_\ell(q) - \hat{U}_\ell(\pi_\ell; \sigma)
$$

$$\geq u_{g_\ell}(q) + u_\ell(q) - \hat{U}_\ell(\pi_\ell; \sigma) + \delta \left( \frac{m_\ell(\sigma)}{V^\text{den}(\sigma)} - \sum_{j \in L_{g_\ell}} \frac{m_j(\sigma)}{V^\text{den}(\sigma)} \right),
$$

(84)

where (84) follows from $\frac{m_\ell(\sigma)}{V^\text{den}(\sigma)} < \sum_{j \in N^L_{g_\ell}} \frac{m_j(\sigma)}{V^\text{den}(\sigma)}$. Notice that (84) is $g_\ell$’s expected dynamic payoff from any offer $(y, m)$ such that $\nu_\ell(y) = 0$, $\ell$ is indifferent between accepting and rejecting, and $\ell$ accepts. Thus, $g_\ell$ strictly prefers to offer $(y, m)$ such that $\varphi_\ell(y, m) = 1$, 52
Proof. Fix an equilibrium \( \sigma \). Lemma B.2 implies \( \chi^\sigma \) is not degenerate on \( q \). Let \( \chi^q \) denote the probability distribution degenerate on \( q \). Define the continuation distribution induced by \( \sigma \) as \( \chi = (1 - \delta) \chi^q + \delta \chi^\sigma \). Note that \( \chi \) is non-degenerate because \( \chi^\sigma \) is not degenerate on \( q \) and \( \delta \in (0, 1) \).

For all \( k \in N \), the expected dynamic policy payoff from a rejected policy proposal satisfies

\[
(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).
\]

Let \( x^\sigma \) denote the mean of \( \chi \). Since \( u \) is strictly concave and \( \chi \) is non-degenerate, Jensen’s Inequality implies

\[
u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma)
\]

for all players \( k \in N \).

Consider \( \ell \in N_L \). First, assume \( \varphi_\ell(y, m) = 1 \) for all \( (y, m) \in X \times \mathbb{R}_+ \) such that \( \ell \) is indifferent between accepting \( (y, m) \) and rejecting. The condition for \( g_\ell \) to strictly prefer to offer \( (y, m) \) such that \( \nu_\sigma(y) = 1 \), rather than offer \( (y', m') \) such that \( \nu_\sigma(y') = 0 \), is

\[
u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\pi_\ell; \sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\pi_\ell; \sigma).
\]

Equivalently,

\[
u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta \tilde{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma).
\]

Notice that

\[
\tilde{V}_{g_\ell}(\sigma) + \tilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{\ell' \in N_L \setminus \{\ell\}} \frac{\hat{m}_{\ell'}(\sigma)}{V_{\text{den}}(\sigma)} + \left(V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}\right)
\]

\[
\leq V_{g_\ell}(\sigma) - \frac{\hat{m}_{\ell}(\sigma)}{V_{\text{den}}(\sigma)} + \left(V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)}\right)
\]

\[
= V_{g_\ell}(\sigma) + V_\ell(\sigma),
\]

where (87) follows from substituting for \( \tilde{V}_\ell(\sigma) \) and \( \tilde{V}_{g_\ell}(\sigma) \) using (81) and (82); and (88) from
\[ \sum_{\ell' \in N^L} \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \geq \frac{\hat{m}_\ell(\tau g)}{V_{\text{den}}(\sigma)} \text{ because } \frac{\hat{m}_\ell(\sigma)}{V_{\text{den}}(\sigma)} \geq 0 \text{ for all } \ell' \in N^L. \]

From (85), it follows that for some \( \nu \), \( \sigma \) is an equilibrium. By Duggan (2014), we can assume \( \nu \) is decisive. Thus, strict concavity of \( M(\nu) \) and \( 0 \neq q \) together imply \( A(\nu) = \{ x \in X | M(\nu)(x) \geq (1 - \delta)u_M(q) + \delta V_M(\nu) \} \) is a closed, non-empty interval. Let \( A(\nu) = [\bar{x}, \overline{x}] \). Then \( A(\nu)(x) = 1 \text{ if } x \in (\bar{x}, \overline{x}) \).

Lemma B.2 implies \( \chi(g) \) is not degenerate on \( q \). Fix \( \ell \in N^L \). Lemma B.3 implies existence of \( (y, m) \in W \) such that: \( \nu(x, m) = 1 \) and \( g_\ell \) strictly prefers \( (y, m) \) over all \( (y', m') \) such that \( \nu(y', m') = 0 \). Thus, \( x \in A(\nu) \) for all \( (y, m) \in \text{supp}(\nu g) \).

Without loss of generality, assume \( \nu(x, m) < 1 \). There are two cases.

- **Case 1**: If \( \hat{x}_\ell \leq -\bar{x} \) and \( u_\ell(-\bar{x}) \geq (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\nu) \), then \( x \in A(\nu) \) for all \( x \in \text{supp}(\nu_\ell) \). Strict concavity of \( u_\ell \) and \( \nu(x, m) < 1 \) together imply existence of \( \varepsilon > 0 \) such that \( \ell \) has a profitable deviation to \( -\bar{x} + \varepsilon \), a contradiction.

- **Case 2**: Assume \( \hat{y}_\ell \leq -\bar{x} \). Continuity, Lemma B.3, and \( \nu(-\bar{x}) < 1 \) together imply existence of \( \varepsilon, \varepsilon' > 0 \) such that \( g_\ell \) has a profitable deviation to \( (y', m') = (-\bar{x} + \varepsilon, \tilde{U}_\ell(p_\ell; \sigma) - u_\ell(-\bar{x} + \varepsilon) + \varepsilon') \), a contradiction.

\[ \square \]

**Lemma B.4.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.

**Proof.** Let \( \sigma \) be an equilibrium. By Duggan (2014), \( M \) is decisive. Thus, strict concavity of \( u_M \) and \( 0 \neq q \) together imply \( A(\sigma) = \{ x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma) \} \) is a closed, non-empty interval. Let \( A(\sigma) = [\bar{x}, \overline{x}] \). Then \( A(\sigma)(x) = 1 \text{ if } x \in (\bar{x}, \overline{x}) \).

Lemma B.2 implies \( \chi(\sigma) \) is not degenerate on \( q \). Fix \( \ell \in N^L \). Lemma B.3 implies existence of \( (y, m) \in W \) such that: \( \nu(\sigma)(x, m) = 1 \) and \( g_\ell \) strictly prefers \( (y, m) \) over all \( (y', m') \) such that \( \nu(y', m') = 0 \). Thus, \( x \in A(\sigma) \) for all \( (y, m) \in \text{supp}(\lambda_{g_\ell}) \). Without loss of generality, assume \( \nu(x, m) < 1 \). There are two cases.

- **Case 1**: If \( \hat{x}_\ell \leq -\bar{x} \) and \( u_\ell(-\bar{x}) \geq (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) \), then \( x \in A(\sigma) \) for all \( x \in \text{supp}(\nu_\ell) \). Strict concavity of \( u_\ell \) and \( \nu(x, m) < 1 \) together imply existence of \( \varepsilon > 0 \) such that \( \ell \) has a profitable deviation to \( -\bar{x} + \varepsilon \), a contradiction.

- **Case 2**: Assume \( \hat{y}_\ell \leq -\bar{x} \). Continuity, Lemma B.3, and \( \nu(-\bar{x}) < 1 \) together imply existence of \( \varepsilon, \varepsilon' > 0 \) such that \( g_\ell \) has a profitable deviation to \( (y', m') = (-\bar{x} + \varepsilon, \tilde{U}_\ell(p_\ell; \sigma) - u_\ell(-\bar{x} + \varepsilon) + \varepsilon') \), a contradiction.

\[ \square \]

**Lemma B.5.** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.

**Proof.** Let \( \sigma \) denote an equilibrium. By Lemma B.4, we can assume \( \nu(\sigma)(x) = 1 \text{ iff } x \in A(\sigma) \). Fix \( \ell \in N^L \) and define \( y^*_\ell = \arg \max_{y \in A(\sigma)} u_\ell(y) - \tilde{U}_\ell(p_\ell; \sigma) \), which is uniquely defined, and \( m^*_\ell = \tilde{U}_\ell(p_\ell; \sigma) - u_\ell(y^*_\ell) \).

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By Lemma B.2, \( \chi^* \) is not degenerate on \( q \). Lemma B.3 implies that \( g \) strictly prefers \((y^*_g, m^*_g)\) over any \((y', m')\) such that \( y' \notin A(\sigma) \). Thus, if \( \pi_\ell \) is not degenerate on \( y^*_g \) and \( \varphi_\ell(y^*_g, m^*_g) < 1 \), then continuity implies existence of \( \varepsilon > 0 \) such that \( g_\ell \) has a profitable deviation to offer \((y^*_g, m^*_g + \varepsilon)\) which is accepted with probability one. This contradicts that \( \sigma \) is an equilibrium. Thus, \( \sigma \) must satisfy either (i) \( \pi_\ell(y^*_g) = 1 \), or (ii) \( \varphi_\ell(y^*_g, m^*_g) = 1 \). \( \square \)

A strategy profile \( \sigma \) is no-delay if \( \bar{\sigma}(x) = 1 \) for all \( x \in \text{supp}(\pi_\ell) \) and \( \bar{\sigma}(y) = 1 \) for all \((y, m) \in \text{supp}(\lambda_\ell^\prime)\).\(^{29}\)

**Lemma B.6.** Every stationary legislative lobbying equilibrium is no-delay.

**Proof.** Fix an equilibrium \( \sigma \). By Lemma B.2, \( \chi^* \) is not degenerate on \( q \). Thus, Lemma B.3 implies \( g \) strictly prefers \((y, m) \in W \) such that \( \bar{\sigma}(y) = 1 \). Lemma B.4 implies we can assume \( \bar{\sigma}(x) = 1 \) iff \( x \in A(\sigma) \). Also, Lemma B.5 implies we can assume all \( \ell \in N^L \) use deferential acceptance strategies under \( \sigma \).

Fix \( \ell \in N^L \). The preceding argument and Lemma B.1 imply \( \lambda_\ell^\prime \) puts probability one on \((y^*, m^*)\) such that \( y^* = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell; \sigma) \), which is unique. Lemmas B.4 and B.5 imply we can assume \( \bar{\sigma}(y^*) = 1 \) and \( \varphi_\ell(y^*, m^*) = 1 \).

The proof verifies \( z_\ell \notin A(\sigma) \) cannot be optimal. To show a contradiction, assume proposing \( z_\ell \notin A(\sigma) \) is optimal for some \( \ell \in N^L \). Let \( z^* = \arg \max_{x \in A(\sigma)} u_\ell(x) \). There are two steps. Step 1 establishes useful properties of \( \ell \)'s preferences over lotteries. Step 2 shows a contradiction.

**Step 1:** Recall \( \chi = (1 - \delta) + \delta \chi^* \). Let \( x^\sigma \) to be the mean of \( \chi \). Jensen’s inequality implies

\[
 u_i(x^\sigma) > \int_X u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) \tag{90}
\]

for all \( i \in N \). Thus, \( x^\sigma \in \text{int}A(\sigma) \).

Next, let \( \chi^z^* \) denote the policy lottery that is nearly equivalent to \( \chi \), but transfers probability \( \frac{\delta \rho_i \alpha_i}{V_{\text{den}}(\sigma)} \) from \( y^* \) to \( z^* \). Let \( x^z^* \) denote the mean of \( \chi^z^* \). For all \( i \in N \), Jensen’s inequality implies

\[
 u_i(x^z^*) > \int_X u_i(x) \chi^z^*(dx) \\
= (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho_i \alpha_i u_i(y^*)}{V_{\text{den}}(\sigma)} + \frac{\delta \rho_i \alpha_i u_i(z^*)}{V_{\text{den}}(\sigma)}.
\]

Moreover, \( x^z^* \) is located weakly between \( x^\sigma \) and \( z^* \), implying \( x^z^* \in A(\sigma) \).

\(^{29}\)This definition shares the spirit of no-delay strategy profiles as defined in Banks and Duggan (2006b).
Step 2: Since $z_\ell \notin A(\sigma)$ is optimal, Lemma B.1 implies

$$m^* = (1 - \delta)u_\ell(q) + \delta \widehat{V}_\ell(\sigma) - u_\ell(y^*)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)} - u_\ell(y^*). \quad (91)$$

Accordingly, $\hat{m}_\ell(\sigma)$ is recursively defined as

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)} - u_\ell(y^*) \right)$$

$$= \frac{\rho_\ell \alpha_\ell V^{\text{den}}(\sigma)[(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell}. \quad (92)$$

Because $z_\ell \notin A(\sigma)$ is optimal, we have

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \widehat{V}(\sigma)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)}$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell[(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell}, \quad (93)$$

$$\text{where (93) follows from the definition of } \widehat{V}_\ell(\sigma); \text{ and (94) from using (92) to substitute for } \hat{m}_\ell(\sigma) \text{ and simplifying. Next,}$$

$$V^{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in \mathcal{N}_\ell} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell$$

$$\geq 1 - \delta \sum_{j \in \mathcal{N}_\ell} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell$$

$$> 0, \quad (95)$$

where (95) follows because Lemma B.3 implies that all lobby offers are accepted and passed under $\sigma$; and (96) from rearranging and $\rho_\ell + \sum_{j \neq \ell} \rho_j (1 - \alpha_j) \leq 1$. Thus, $V^{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell > 0$. Simplifying (94) yields

$$0 \leq V^{\text{den}}(\sigma) \left( (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \right) - \delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*) \left( V^{\text{den}}(\sigma) - \delta \rho_\ell \alpha_\ell \right)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*)}{V^{\text{den}}(\sigma)} - u_\ell(z^*)$$

$$= \int_X u_\ell(x) \chi^{z^*}(dx) - u_\ell(z^*),$$

a contradiction because $u_\ell(z^*) \geq u_\ell(x^{z^*}) > \int_X u_\ell(x) \chi^{z^*}(dx)$. Therefore $\ell$ strictly prefers $z_\ell \in$
$A(\sigma)$ and $\pi_\ell$ places probability zero on $z_\ell \notin A(\sigma)$. Thus, $\overline{\nu}_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$. □

Lemma B.7. Every stationary legislative lobbying equilibrium is such that (i) $\lambda_g$ is degenerate for all $g \in N^G$ and (ii) $\pi_\ell$ is degenerate for all $\ell \in N^L$.

Proof. Let $\sigma$ denote an stationary legislative lobbying equilibrium. By Duggan (2014), $A_M(\sigma) = A(\sigma)$. Thus, $A(\sigma)$ is nonempty, compact and convex.

(i) Consider $g \in N^g$ and $\ell \in N^L_g$. Recall $\overline{U}_\ell(\pi_\ell; \sigma)$ as defined in (83). Lemma B.1 and Lemma B.6 imply $\lambda^g_\ell$ puts probability one on $(y^*, m^*)$ satisfying

$$y^* = \arg\max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \overline{U}_\ell(\pi_\ell; \sigma).$$

(97)

and

$$m^* = \overline{U}_\ell(\pi_\ell; \sigma) - u_\ell(y^*).$$

(98)

The objective function in (97) is continuous and strictly concave in $y$, and $A(\sigma)$ is nonempty and compact. Thus, $y^*$ is unique. Then $\lambda^g_\ell$ is degenerate because $m^*$ is uniquely determined by (98).

(ii) Consider $\ell \in N^L$. Lemma B.6 implies $\pi_\ell$ puts probability one on $x^* = \arg\max_{x \in A(\sigma)} u_\ell(x)$. Because $u_\ell$ is quadratic and $A(\sigma)$ is nonempty and compact, $x^*$ is unique. Thus, $\pi_\ell$ is degenerate.

□

Proposition 1.2 corresponds to Part 2 of Proposition 1 in the text.

**Proposition 1.2** Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay pure strategy stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.

Proof. Consider a stationary legislative lobbying equilibrium $\sigma$. Lemma B.7 implies $\lambda$ and $\pi$ are degenerate. Lemma B.5 and (14) imply $\sigma$ is equivalent in outcome distribution to an equilibrium with all $\ell \in N^L$ using deferential acceptance strategies. Lemmas B.4 - B.6 thus deliver the result. □
Appendix C

First, I define several components to construct a function $\zeta^\ell$ that is closely tied to $M$’s equilibrium voting decision. Lemmas C.3 - C.6 then characterize $\zeta^\ell$. Finally, Lemma 2 delivers the partitional characterization on $\hat{x}_g$ facilitating Proposition 2.

Preliminaries to define $\zeta^\ell$: Recall $\pi(0) = \pi(\hat{x}_g)$ for $\hat{x}_g = 0$ Let $\hat{D}^{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \pi(0), j \neq \ell\}$ and $\hat{D}^{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \pi(0), j \neq \ell\}$. Next, set $D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y}\}$ and $D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x}\}$. Define $D^\ell$ as the unique elements of $D^{\ell,y} \cup D^{\ell,x} \cup \{\pi(0)\}$. Let $K^\ell + 1 = |D^\ell|$. Denote the $k$-th element of $D^\ell$ as $d^\ell_k$. Index elements $k = 0, \ldots, K^\ell$ of $D^\ell$ in ascending order so that $d^\ell_0 = \pi(0)$ and $k' > k$ implies $d^\ell_k > d^\ell_{k'}$.

For each $k$ and $j \neq \ell$, let $C^k_j = \{\hat{x}_j \in [-d^\ell_k, d^\ell_k]\}$ and $\hat{C}^k_j = \{\hat{y}_j \in [-d^\ell_k, d^\ell_k]\}$, suppressing dependence on $\ell$. Define

$$I^k_j = (1 - \alpha_j)C^k_j u_M(\hat{x}_j) + \alpha_j \hat{C}^k_j u_M(\hat{y}_j)$$

and

$$O^k_j = (1 - \alpha_j)(1 - C^k_j) + \alpha_j (1 - \hat{C}^k_j),$$

again suppressing dependence on $\ell$. Let

$$\hat{x}^\ell_k = \left\{ \frac{1}{\rho_\ell} \left[ (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_k) \left( 1 - \delta \sum_{j \neq \ell} O^k_j \right) \right] \right\}^{\frac{1}{2}}. \quad (99)$$

Because $d^\ell_0 = \pi(0)$, rearranging (99) yields $\hat{x}^\ell_0 = 0$.

Lemma C.1 establishes a useful identity.

Lemma C.1. For all $\ell \in N^\ell$ and each $k = 0, \ldots, K^\ell - 1$, we have

$$\delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j) = \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j). \quad (100)$$

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell - 1$. We have

$$\delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j)$$

$$= \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j)$$

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where (101) follows from the construction of $I^k_j$, $O^k_j$, $I^{k+1}_j$, $O^{k+1}_j$, and $d^k_{k+1}$.

Lemma C.2 shows that $\hat{x}^\ell_k$ strictly increase in $k$.

**Lemma C.2.** For all $\ell \in N^\ell$ and each $k = 0, \ldots, K^\ell$, we have $\hat{x}^\ell_k < \hat{x}^\ell_{k'}$ for all $k' > k$.

**Proof.** Consider $\ell \in N^L$ and fix $k < K^\ell$. We have

\[
\delta \sum_{j \neq \ell} \rho_j I^{k+1}_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j) = \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_{k+1})(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j) > \delta \sum_{j \neq \ell} \rho_j I^k_j - u_M(d^\ell_k)(1 - \delta \sum_{j \neq \ell} \rho_j O^k_j)
\]

where (103) follows from Lemma C.1; and (104) follows from $0 > u_M(d^\ell_k) > u_M(d^\ell_{k+1})$. From (99), $\hat{x}^\ell_k < \hat{x}^\ell_{k+1}$. Iterating yields $\hat{x}^\ell_k < \hat{x}^\ell_{k'}$ for all $k' > k$.

**Definition of $\zeta^\ell$:** For each $k = 0, \ldots, K^\ell$, define

\[
\bar{x}^\ell_k(x) = \left( -\frac{(1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_J I^k_j}{1 - \delta \sum_{j \neq \ell} \rho_j O^k_\ell} \right) ^{\frac{1}{2}}
\]

and

\[
\zeta^\ell_k = u_M(x) - \left( (1 - \delta)u_M(q) + \delta \rho_k u_M(x) + \delta \sum_{j \neq \ell} \rho_j I^k_j + \delta u_M(\bar{x}^\ell_k(x)) \sum_{j \neq \ell} \rho_j O^k_j \right).
\]

By construction, $\bar{x}^\ell_k(\hat{x}^\ell_k) = d^\ell_k$. Adopt the convention $d^\ell_{K^\ell+1} = \infty$. Define the piecewise
The function
\[
\zeta^\ell(x) = \zeta^\ell_k(x) \text{ if } x \in [d^\ell_k, d^\ell_{k+1}).
\]

**Properties of \(\zeta^\ell\):** Lemmas C.3 - C.5 prove useful properties of \(\zeta^\ell\).

**Lemma C.3.** For all \(\ell \in N^L\), \(\zeta^\ell\) satisfies \(\zeta^\ell(0) > 0\) and \(\zeta^\ell(q) \leq 0\).

**Proof.** Consider \(\ell \in N^L\). First, I show \(\zeta^\ell(0) > 0\). We have
\[
\zeta^\ell(0) = \zeta^\ell_0(0) = u_M(0) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(0) - \delta \sum_{j \neq \ell} \rho_j I^0_j - \delta u_M(\bar{x}_0(0)) \sum_{j \neq \ell} \rho_j O^0_j \right)
\]
\[
= - \left( (1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^0_j + \delta u_M(\bar{x}_0(0)) \sum_{j \neq \ell} \rho_j O^0_j \right) \geq 0,
\]
where (107) follows from \(u_M(0) = 0\) and \(\bar{x}_0(0) = \tilde{x}_0\); and (108) because \(\delta \in (0, 1)\) and \(u_M(x) \leq 0\) for all \(x \in X\).

Next, I show \(\zeta^\ell(q) \leq 0\). Let \(k'\) denote the largest \(k\) such that \(\bar{x}^\ell_k \leq q\). There are three steps.

- **Step 1:** Because \(\bar{x}^{k'}(\bar{x}^\ell_k) = d^\ell_{k'}\), we have
\[
u_{m}(d^\ell_{k'}) = \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(\bar{x}^\ell_k) + \delta \sum_{j \neq \ell} \rho_j I^j_{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O^j_{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I^j_{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O^j_{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(d^\ell_{k'}) \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C^j_{k'} + \alpha_j \tilde{C}^j_{k'}]}{1 - \delta \sum_{j \neq \ell} \rho_j O^j_{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(d^\ell_{k'}) (1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O^j_{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O^j_{k'}},
\]
Rearranging and simplifying (112) yields
\[
\sum_{j \neq \ell} \rho_j I_j^{k'} \geq \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j) C_j^{k'} u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^{k'} u_M(\hat{y}_j) \right]
\geq u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j \left[ (1 - \alpha_j) C_j^{k'} + \alpha_j \tilde{C}_j^{k'} \right]
= u_M(d_{k'}^\ell)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})
\geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})
\]
where (115) follows from the definition of $I_j^{k'}$; (116) from $u_M(\hat{x}_j) \geq u_M(d_{k'}^\ell)$ for all $j$ such that $C_j^{k'} = 1$ and $u_M(\hat{y}_j) \geq u_M(d_{k'}^\ell)$ for all $j$ such that $\tilde{C}_j^{k'} = 1$; (117) because $\sum_{j \neq \ell} \rho_j [(1 - \alpha_j) C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction; and (118) from $q > \hat{x}_{k'}^\ell$.

- **Step 2**: We have
\[
u_M(x_{k'}^\ell(q)) = \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}
= u_M(q),
\]
where (119) follows from $\sum_{j \neq \ell} \rho_j I_j^{k'} \geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})$ and (120) from simplifying.

- **Step 3**: We show $\zeta^\ell(q) \leq 0$. We have
\[
\zeta^\ell(q) = u_M(q) - \left( (1 - \delta)u_M(q) + \delta \rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'} + \delta u_M(\bar{x}_{k'}^\ell(q)) \sum_{j \neq \ell} \rho_j O_j^{k'} \right)
\]
where (121) follows from Lemma C.1. Then
\[
\sum_{j \neq \ell} \rho_j O_j^k \geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^k).
\]

Lemma C.4. For all \( \ell \in N^L \), \( \zeta^\ell \) is continuous.

Proof. Consider \( \ell \in N^L \) and fix \( k \). Because \( \pi^\ell_k(x) \) is continuous, \( \zeta^\ell \) is continuous over \((\hat{x}^\ell_k, \hat{x}^\ell_{k+1})\). It suffices to show \( \zeta^\ell_k(\hat{x}^\ell_{k+1}) = \zeta^\ell_{k+1}(\hat{x}^\ell_{k+1}) \).

First, I establish \( d^\ell_{k+1} = \pi^\ell_k(\hat{x}^\ell_{k+1}) \). Rearranging (99) yields
\[
0 = u_M(d^\ell_{k+1}) \left( 1 - (1 - \delta)u_M(q) - \delta \rho_\ell u_M(\hat{x}^\ell_{k+1}) - \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} \right)
\]
\[
= u_M(d^\ell_{k+1}) \left( 1 - (1 - \delta)u_M(q) - \delta \rho_\ell u_M(\hat{x}^\ell_{k+1}) - \delta \sum_{j \neq \ell} \rho_j I_j^{k} \right),
\]

where (123) follows from Lemma C.1. Then \( u_M(d^\ell_{k+1}) = \frac{(1 - \delta)u_M(q) - \delta \rho_\ell u_M(\hat{x}^\ell_{k+1}) - \delta \sum_{j \neq \ell} \rho_j I_j^{k}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \), which implies \( d^\ell_{k+1} = \pi^\ell_k(\hat{x}^\ell_{k+1}) \). Thus,
\[
\zeta^\ell_k(\hat{x}^\ell_{k+1}) = u_M(\hat{x}^\ell_{k+1}) - \left( 1 - (1 - \delta)u_M(q) + \delta \rho_\ell u_M(\hat{x}^\ell_{k+1}) + \delta \sum_{j \neq \ell} \rho_j I_j^{k} + \delta u_M(\pi^\ell_k(\hat{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O_j^k \right)
\]
\[
= u_M(\hat{x}^\ell_{k+1}) - \left( 1 - (1 - \delta)u_M(q) + \delta \rho_\ell u_M(\hat{x}^\ell_{k+1}) + \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} + \delta u_M(\pi^\ell_{k+1}(\hat{x}^\ell_{k+1})) \sum_{j \neq \ell} \rho_j O_j^{k+1} \right)
\]
\[
= \zeta^\ell_{k+1}(\hat{x}^\ell_{k+1}),
\]

where (124) follows from Lemma C.1 because \( d^\ell_{k+1} = \pi^\ell_k(\hat{x}^\ell_{k+1}) \).

Lemma C.5. For all \( \ell \in N^L \), \( \zeta^\ell \) is strictly decreasing.

Proof. Consider \( \ell \in N^L \). The proof shows that the derivative of \( \zeta^\ell \) is strictly negative at every \( x \in (\hat{x}^\ell_k, \hat{x}^\ell_{k+1}) \) for all \( k \). Thus, \( \zeta^\ell \) strictly decreases over \((\hat{x}^\ell_k, \hat{x}^\ell_{k+1})\) for all \( k \). Continuity then implies that \( \zeta^\ell \) is strictly decreasing.
Fix $k$ and consider $x \in (\bar{x}^k_k, \bar{x}^k_{k+1})$. Then

$$\zeta(x) = u_M(x) - \left((1 - \delta)u_M(q) + \delta \rho \epsilon u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\bar{x}^k_k(x)) \sum_{j \neq \ell} \rho_j O_j^k \right)$$

and

$$\frac{\partial \zeta(x)}{\partial x} = -2x + 2x \delta \rho \epsilon + \frac{\partial u_M(\bar{x}^k_k(x))}{\partial \bar{x}^k_k(x)} \frac{\partial \bar{x}^k_k(x)}{\partial x} \left( \delta \sum_{j \neq \ell} \rho_j O_j^k \right)$$

(126)

$$= -2x + 2x \delta \rho \epsilon + \frac{2x \delta \rho \epsilon (\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$$

(127)

$$\propto \delta \rho \epsilon + \frac{\delta \rho \epsilon (\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} - 1$$

(128)

$$\propto \delta \rho \epsilon - \delta \rho \epsilon (\delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \rho \epsilon (\delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j O_j^k - 1$$

(129)

$$< 0,$$

(130)

where (126) follows because $u_M = -(x)^2$; (127) from $\frac{\partial u_M(\bar{x}^k_k(x))}{\partial \bar{x}^k_k(x)} \frac{\partial \bar{x}^k_k(x)}{\partial x} = -\frac{2x \delta \rho \epsilon}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$; (128) from factoring out $2x > 0$; (129) from factoring out $1 - \delta \sum_{j \neq \ell} \rho_j O_j^k > 0$; and (130) because $\delta \in (0, 1)$ and $\rho \epsilon + \delta \sum_{j \neq \ell} \rho_j O_j^k \leq 1$. Thus, $\zeta(x)$ strictly decreases over $(\bar{x}^k_k, \bar{x}^k_{k+1})$. It follows that $\bar{\epsilon}$ is strictly decreasing.

**Lemma C.6.** For all $\ell \in N^L$, there is a unique $\bar{x}^\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}^\ell)$, $\zeta^\ell(\bar{x}^\ell) = 0$, and $\zeta^\ell(x) < 0$ for all $x > \bar{x}^\ell$.

**Proof.** Consider $\ell \in N^L$. Lemma C.3 implies $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. By Lemma C.5, $\zeta^\ell$ is strictly decreasing. Thus, there is a unique $\bar{x}^\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}^\ell)$ and $\zeta^\ell(x) < 0$ for all $x > \bar{x}^\ell$. Because $\zeta^\ell$ is continuous by Lemma C.4, $\zeta^\ell(\bar{x}^\ell) = 0$.

**Lemma 2.** For all $\ell \in N^L$, there exists $\bar{x}^\ell \in (0, q]$ such that $\hat{x}_g \in (-\bar{x}^\ell, \bar{x}^\ell)$ implies $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_g))$ and otherwise $A(\sigma(\hat{x}_g)) = [-\bar{x}^\ell, \bar{x}^\ell]$.

**Proof.** Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_\ell = \hat{x}_g$. There are two parts. Part 1 shows $\hat{x}_g \in (-\bar{x}^\ell, \bar{x}^\ell)$ implies $\hat{x}_g \in \text{int}A(\sigma(\hat{x}_g))$. Part 2 shows $\hat{x}_g \notin (-\bar{x}^\ell, \bar{x}^\ell)$ implies $A(\sigma(\hat{x}_g)) = [-\bar{x}^\ell, \bar{x}^\ell]$.

**Part 1.** Assume $\hat{x}_g \in (-\bar{x}^\ell, \bar{x}^\ell)$ and suppose $\hat{x}_g \geq 0$ without loss of generality. Let $k'$ be the largest $k$ such that $\hat{x}^k_k \leq \hat{x}_g$. Define the strategy profile $\sigma'$ such that it puts probability $\rho^k_k$ on
\(\hat{x}_g\) and for each \(j \neq \ell\) it (i) puts probability \((1 - \alpha_j)\rho_j\) on \(\hat{x}_j\) if \(\hat{x}_j \in [-d^\ell_k, d^\ell_k]\) and otherwise puts that probability on \(\pi^{\ell}_k(\hat{x}_g)\), and (ii) puts probability \(\alpha_j\rho_j\) on \(\hat{y}_j\) if \(\hat{y}_j \in [-d^\ell_k, d^\ell_k]\) and otherwise puts that probability on \(\pi^{\ell}_k(\hat{x}_g)\). By construction, \(\pi(\sigma') = \pi^{\ell}_k(\hat{x}_g)\). Furthermore, legislator proposal strategies are optimal given \(A(\sigma') = \pi(\sigma')\).

I now check optimality for \(M\). Because \(\hat{x}_g \in [\hat{x}^\ell_k, \hat{x}^\ell_{k+1}]\), we have \(\pi(\sigma') = \pi^{\ell}_k(\hat{x}_g)\). Thus, \(M\) optimally accepts all offers by \(j \neq \ell\). Next, I verify \(\hat{x}_g \in \text{int}A(\sigma')\). By Lemma C.6, \(\hat{x}_g \in (-\pi_\ell, \pi_\ell)\) implies \(\zeta(\hat{x}_g) > 0\), which is equivalent to \(u_M(\hat{x}_g) \geq \frac{(1 - \delta)u_M(q) + \delta \rho_map_M(\hat{x}_g) + \delta \sum_{\ell' \neq \ell} \rho_{\ell'} I^{\ell'}_k}{1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell}_j}\). Under \(\sigma'\), this is equivalent to \(\hat{x}_g \in \text{int}A(\sigma')\).

Thus, \(\sigma'\) is equivalent to the equilibrium \(\sigma(\hat{x}_g)\) and \(\hat{x}_g \in \text{int}A(\sigma(\hat{x}_g))\), as desired.

**Part 2.** Assume \(\hat{x}_g \notin (-\pi_\ell, \pi_\ell)\) and suppose \(\hat{x}_g \geq 0\) without loss of generality. There are two steps. Step 1 shows \(\pi(\sigma(\hat{x}_g)) \geq \pi_\ell\). Step 2 shows \(\pi(\sigma(\hat{x}_g)) \leq \pi_\ell\).

- **Step 1.** Suppose \(\pi(\sigma(\hat{x}_g)) < \pi_\ell\). Let \(k'\) be the largest \(k\) such that \(\hat{x}^\ell_k \leq \pi(\sigma(\hat{x}_g))\). Because \(\hat{x}_g > \pi_\ell > \pi(\sigma(\hat{x}_g))\), it follows that \(\sigma(\hat{x}_g)\) puts probability \(\rho_\ell\) on \(\pi(\sigma(\hat{x}_g))\). Thus, \(u_M(\pi(\sigma(\hat{x}_g))) = \frac{(1 - \delta)u_M(q) + \delta \sum_{\ell' \neq \ell} \rho_{\ell'} I^{\ell'}_k}{1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell}_j}\) and rearranging yields \(\zeta(\pi(\sigma(\hat{x}_g))) = 0\). Lemma C.6 implies \(\pi(\sigma(\hat{x}_g)) = \pi_\ell\), a contradiction.

- **Step 2.** Suppose \(\pi(\sigma(\hat{x}_g)) > \pi_\ell\). If \(\hat{x}_g \geq \pi(\sigma(\hat{x}_g))\), then the argument from Step 1 shows a contradiction. Assume \(\hat{x}_g < \pi(\sigma(\hat{x}_g))\). Let \(k'\) be the largest \(k\) such that \(\hat{x}^\ell_k \leq \pi(\sigma(\hat{x}_g))\). Then \(\sigma(\hat{x}_g)\) puts probability \(\rho_\ell\) on \(\hat{x}_g\). Next, \(M\) optimally accepts \(\hat{x}_g\) under \(\sigma(\hat{x}_g)\) if \(u_M(\hat{x}_g) \geq \frac{(1 - \delta)u_M(q) + \delta \rho_map_M(\hat{x}_g) + \delta \sum_{\ell' \neq \ell} \rho_{\ell'} I^{\ell'}_k}{1 - \delta \sum_{j \neq \ell} \rho_j O^{\ell}_j}\). Rearranging, this condition is equivalent to \(\zeta(\hat{x}_g) \geq 0\). Lemma C.6 implies \(\hat{x}_g \leq \pi_\ell\), a contradiction.

\(\square\)
References


